THE 2-MODULAR CHARACTERS OF THE FISCHER GROUP $\mathbf{Fi}_{23}$

GERHARD HISS, MAX NEUNHÖFFER, AND FELIX NOESKE

Dedicated to Professor Bernd Fischer on the occasion of his 70th birthday

Abstract. We determine the 2-modular character table of the Fischer group Fi_{23}.

1. Introduction

Using computational methods, we construct the 2-modular character table of Fischer's second sporadic simple group Fi_{23}. By previous results published in [7, 8], this leaves only the 3-modular character table of Fi_{23} to be determined.

All modular character tables of the first Fischer group Fi_{22} and its covering groups are also known. The third author has computed the 2- and 3-modular tables for these groups in his PhD thesis [16]. The situation for the third Fischer group Fi_{24}' is less favorable. Here, the p-modular character tables for the primes $p \in \{2, 3, 5, 7\}$ are still not available.

The degrees of the irreducible 2-modular characters $\varphi_1, \ldots, \varphi_{25}$ of Fi_{23} are displayed in Table 1. The first twenty of these belong to the principal block, φ_{21} is the unique irreducible character in a block of defect 1, the two characters φ_{22} and φ_{23} belong to a block of defect 3, and φ_{24} and φ_{25} are of defect 0. The decomposition matrices of the principal block and the block of defect 3 are given in Tables 2 and 3, respectively. The ordering of the columns follows the numbering of the characters in Table 1. As usual, zero entries have been replaced by dots.

Our work adds to the program of computing the modular character tables for all sporadic groups. The methods used are by now standard: The Meat-Axe enhanced by condensation. An excellent survey on these techniques can be found in the article by Lux and Pahlings [12, Section 4]. To actually perform the computations described in the sequel, we had to use several new tuning tricks and reimplement major parts of the programs for tensor condensation and induced condensation. This work will be published elsewhere.

A novel feature of our approach is the fact that we work with a condensation algebra which we know to be Morita equivalent to the group algebra. More precisely, we produce an idempotent $e \in \mathbb{F}_2\mathrm{Fi}_{23}$ such that $e(\mathbb{F}_2\mathrm{Fi}_{23})e$ is Morita equivalent to $\mathbb{F}_2\mathrm{Fi}_{23}$, and group elements $g_1, \ldots, g_{38}$ such that $e(\mathbb{F}_2\mathrm{Fi}_{23})e$ is generated, as an $\mathbb{F}_2$-algebra, by $eg_1e, \ldots, eg_{38}e$. The idempotent e is of the form $e = 1/|K| \sum_{y \in K} y$ for a subgroup K of order 3^9.

2. The blocks

Using GAP [4], it is easy to determine the invariants of the 2-blocks of Fi_{23} and the distribution of the ordinary irreducible characters into blocks. The invariants

TABLE 1. The degrees of the irreducible 2-modular characters of Fi_{23}

No.	Degree
1	1
2	782
3	1 494
4	3 588
5	19 940
6	57 408
7	94 588
8	94 588
9	79 442
10	583 440

No.	Degree
11	1 951 872
12	724 776
13	979 132
14	1 997 872
15	1 997 872
16	7 821 240
17	8 280 208
18	5 812 860
19	17 276 520
20	34 744 192

No.	Degree
21	73 531 392
22	97 976 320
23	166 559 744
24	504 627 200
25	504 627 200

of the blocks are collected in Table 4 (we use a standard convention where $d(B)$, $k(B)$, and $\ell(B)$ denote the defect, the number of ordinary irreducible characters, and the number of modular irreducible characters of the block B, respectively). Block 2 contains the two ordinary characters χ_{56} and χ_{57}, Block 3 contains the five irreducible characters χ_{60}, χ_{64}, χ_{65}, χ_{76}, and χ_{77}, and the two characters χ_{94} and χ_{95} are of 2-defect 0.

Each of the Blocks 2, 4 and 5 only has one irreducible Brauer character and thus their decomposition matrices are trivial. Peter Landrock has shown [9, Section 8], that the defect group of Block 3 is dihedral of order 8. Karin Erdmann gives three possible decomposition matrices for blocks B with this defect group, $k(B) = 5$ and $\ell(B) = 2$ (see [3, Propositions (3.2), (3.3), (3.6)]). In all of these, at least one of the two irreducible Brauer characters is liftable. It follows that the ordinary character χ_{60}, the one with the smallest degree in Block 3, remains irreducible modulo 2. Since it occurs only once in Block 3, whereas in the possibility described in [3, Proposition (3.3)] there are two ordinary characters of the same smallest degree, the decomposition matrix of Block 3 is described by one of the remaining possibilities. In each of these, both irreducible Brauer characters are liftable. This, together with the relations of the ordinary characters in Block 3 on elements of odd order, determines the decomposition matrix of Block 3 as displayed in Table 3.

3. IRRATIONALITIES

There are 25 conjugacy classes of elements of odd order in Fi_{23}. Exactly one pair of these, $23A$, $23B$ is not real (i.e., the inverses of the elements of $23A$ lie in $23B$). Hence, by Brauer's permutation lemma, there is exactly one pair of complex conjugate irreducible Brauer characters. This must belong to the principal block, since the irreducible Brauer characters in the other blocks are real valued.

The principal block contains ordinary characters with values $b13 = (-1+\sqrt{13})/2$ on some elements of odd order. Thus there is an irreducible Brauer character φ taking non-rational values in the field $\mathbb{Q}(b13)$. The minimal polynomial of $b13$

Table 2. The 2-decomposition matrix of the principal block of Fi_{23}

1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
782	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
3588	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
5083	1	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
25806	2	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
30888	2	1	2	2	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
60996	.	.	.	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
106743	3	1	2	1	1	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.
111826	4	1	3	2	1	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.
274482	.	1	.	2	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.
279565	1	1	1	3	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.
752675	5	1	3	2	1	1	.	.	1	1	.	.	.	.	.	.	.	.	.	.
789360	.	.	.	2	.	1	.	.	.	.	.	1	.	.	.	.	.	.	.	.
812889	1	.	1	2	.	.	.	.	1	.	.	1	.	.	.	.	.	.	.	.
837200	.	.	.	2	.	1	1	1	.	1	.	.	.	.	.	.	.	.	.	.
837200	.	.	.	2	.	1	1	1	.	1	.	.	.	.	.	.	.	.	.	.
850850	4	1	3	3	1	1	1	.	1	1	.	.	.	.	.	.	.	.	.	.
850850	4	1	3	3	1	1	.	1	1	1	.	.	.	.	.	.	.	.	.	.
1677390	4	1	3	5	1	1	1	1	1	1	.	1	.	.	.	.	.	.	.	.
1951872	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.
2236520	6	3	4	6	2	2	1	1	2	.	.	1	1	.	.	.	.	.	.	.
2322540	2	2	2	6	1	2	2	2	1	.	.	1	1	.	.	.	.	.	.	.
3913910	.	1	.	3	.	1	1	1	.	.	1	1	1	.	.	.	.	.	.	.
5533110	4	3	3	9	2	3	3	3	1	.	1	1	2	.	.	.	.	.	.	.
6709560	4	1	3	6	2	1	1	1	.	1	.	.	.	.	.	.	.	1	.	.
7468032	2	.	1	6	1	2	1	1	.	1	.	1	.	.	.	.	.	1	.	.
8783424	10	4	7	16	3	5	4	4	2	3	.	1	1	1	1	.	.	.	.	.
9108736	10	4	7	12	3	3	3	3	2	2	1	.	1	1	1	.	.	.	.	.
9108736	10	4	7	12	3	3	3	3	2	2	1	.	1	1	1	.	.	.	.	.
10567557	5	4	4	10	3	3	3	3	.	.	1	.	2	.	.	.	.	1	.	.
10674300	10	4	7	15	3	4	4	4	2	3	1	1	1	1	1	.	.	.	.	.
12077208	14	6	10	19	4	6	5	5	3	3	1	1	2	1	1	.	.	.	.	.
15096510	14	7	10	20	4	5	5	5	3	2	2	2	3	1	1	.	.	.	.	.
17892160	20	8	14	28	6	8	7	7	4	5	1	1	2	2	2	.	.	.	.	.
18812574	4	3	2	11	1	3	3	3	.	1	2	1	1	1	1	1	.	.	.	.
20322225	19	7	13	20	6	7	5	5	3	4	1	.	2	1	1	.	1	.	.	.
21135114	6	5	4	17	2	5	5	5	1	1	2	2	2	1	1	1	.	.	.	.
21348600	2	4	1	11	1	3	4	4	.	.	3	1	2	1	1	1	.	.	.	.
22644765	21	9	15	26	7	9	7	7	4	4	1	1	3	1	1	.	1	.	.	.
26838240	10	5	7	12	4	4	3	3	1	.	1	.	2	.	.	1	1	1	.	.
28464800	10	5	7	16	4	6	4	4	1	1	1	1	2	.	.	1	1	1	.	.
29354325	9	5	6	17	3	6	5	5	1	2	2	1	2	1	1	1	1	.	.	.
35225190	4	3	2	13	1	5	4	4	.	1	2	1	1	1	1	2	1	.	.	.
37573536	20	9	14	28	7	9	7	7	3	3	2	1	3	1	1	1	1	1	.	.
40840800	28	13	20	38	9	12	10	10	5	5	3	1	4	2	2	1	1	.	.	.

Table 2. The 2-decomposition matrix of the principal block of Fi_{23}, continued

42270228	26	12	18	39	8	12	10	10	5	5	3	3	4	2	2	1	1	.	.	.
48034350	18	9	12	34	5	11	9	9	3	4	3	3	3	2	2	2	1	.	.	.
48308832	30	13	21	44	10	14	11	11	5	6	3	2	4	2	2	1	1	1	.	.
55740960	8	5	6	10	3	4	3	3	1	.	1	.	2	.	.	1	1	.	.	1
56360304	12	10	9	32	7	11	10	10	1	3	2	1	4	1	1	1	1	1	1	.
56360304	10	8	6	30	3	10	9	9	1	2	4	3	3	2	2	3	1	.	.	.
57254912	20	10	14	28	8	10	7	7	2	1	3	1	4	.	.	2	2	2	.	.
58708650	16	11	11	30	7	11	9	9	1	1	4	1	5	1	1	2	2	1	.	.
58708650	12	11	9	32	7	11	10	10	1	2	3	1	5	1	1	1	1	1	1	.
65875680	30	14	21	48	10	17	13	13	4	6	3	2	5	2	2	2	2	1	.	.
78278200	40	20	28	63	14	20	17	17	6	7	5	3	7	3	3	2	2	1	.	.
93933840	38	18	26	62	13	21	17	17	5	8	5	3	6	3	3	3	3	1	.	.
133398252	28	20	20	60	13	21	18	18	4	4	5	3	8	2	2	3	2	1	1	1
153014400	24	18	16	62	10	23	19	19	2	3	7	4	7	3	2	6	3	1	.	1
153014400	24	18	16	62	10	23	19	19	2	3	7	4	7	2	3	6	3	1	.	1
176125950	34	23	24	74	16	28	22	22	3	5	6	3	9	2	2	5	4	2	1	1
176125950	32	21	21	72	12	27	21	21	3	4	8	5	8	3	3	7	4	1	.	1
203802885	49	30	34	101	20	36	29	29	6	9	8	5	11	4	4	6	4	2	1	1
207793431	57	34	39	111	22	39	32	32	8	11	9	6	12	5	5	6	4	1	1	1
211351140	32	24	21	87	13	32	26	26	3	6	9	6	9	4	4	8	4	1	1	1
216154575	45	31	31	102	19	36	30	30	5	7	10	6	12	4	4	7	4	2	1	1
216770400	60	36	42	116	23	39	33	33	8	11	10	6	13	5	5	6	4	2	1	1
244563462	58	37	40	123	23	44	36	36	7	10	11	7	14	5	5	8	5	2	1	1
263376036	62	40	42	134	24	47	39	39	7	11	13	8	15	6	6	9	5	2	1	1
264188925	69	42	48	136	28	48	39	39	8	11	12	7	16	5	5	8	6	3	1	1
286274560	54	34	37	122	22	46	36	36	6	11	10	7	12	5	5	9	6	1	1	2
287721720	60	37	41	126	24	47	37	37	8	11	10	7	13	5	5	9	6	1	1	2
289027200	42	32	29	110	20	42	33	33	4	7	9	6	12	3	4	9	5	2	2	2
289027200	42	32	29	110	20	42	33	33	4	7	9	6	12	4	3	9	5	2	2	2
289103904	74	43	51	150	29	55	43	43	9	14	12	8	15	6	6	10	7	2	1	1
313112800	64	39	44	134	26	50	39	39	7	11	11	7	14	5	5	10	7	2	1	2
313112800	58	40	41	132	27	49	39	39	6	9	10	6	15	4	4	9	6	3	2	2
313112800	56	38	38	130	23	48	38	38	6	8	12	8	14	5	5	11	6	2	1	2
322058880	48	36	32	126	20	48	38	38	4	7	13	8	14	5	5	12	6	2	1	2
336061440	72	44	50	144	29	54	42	42	8	11	13	7	16	5	5	11	8	2	1	2
341577600	74	44	51	150	30	56	43	43	8	12	12	8	16	5	5	11	8	3	1	2
343529472	74	44	51	150	30	56	43	43	8	12	13	8	16	5	5	11	8	3	1	2
352251900	62	44	43	148	28	55	44	44	6	10	13	8	17	5	5	11	7	3	2	2
362316240	58	44	40	150	27	56	45	45	5	9	14	9	17	5	5	12	7	3	2	2
476702577	93	63	64	209	39	76	62	62	11	15	19	12	24	8	8	15	9	3	2	3
496897335	81	58	56	199	36	74	59	59	7	12	19	11	22	7	7	17	10	4	2	3
526752072	94	66	64	223	39	81	66	66	9	15	22	13	25	9	9	18	10	4	2	3
528377850	90	65	62	222	40	82	66	66	9	15	20	13	25	8	8	17	10	4	3	3
559458900	110	74	75	251	45	91	74	74	12	18	24	15	28	10	10	19	11	4	2	3

TABLE 3. The decomposition matrix of Block 3

97 976 320	1	.
166 559 744	.	1
166 559 744	.	1
264 536 064	1	1
264 536 064	1	1

TABLE 4. Blocks of Fi_{23}

B	$d(B)$	$k(B)$	$\ell(B)$
1	18	89	20
2	1	2	1
3	3	5	2
4	0	1	1
5	0	1	1

equals $x^2 + x - 3$. Since this is irreducible modulo 2, the simple module (over an algebraically closed field of characteristic 2) with Brauer character φ is not realizable over $\mathbb{F}_2$. Hence there is at least one pair of irreducible Brauer characters in the principal block, whose underlying simple modules are conjugate by the non-trivial Galois automorphism of $\mathbb{F}_4/\mathbb{F}_2$.

4. Applying the Meat-Axe

To begin with, we consider the permutation representation of Fi_{23} of degree 31 671 on the cosets of $2.\mathrm{Fi}_{22}$. This representation is available from Robert Wilson's Atlas of Finite Group Representations [17].

Using the Meat-Axe (version 2.4.3 of Ringe's C-Meat-Axe) we find that the corresponding matrix representation over $\mathbb{F}_2$ has 10 irreducible constituents, namely

$$1a, 1a, 1a, 782a, 782a, 1\,494a, 1\,494a, 3\,588a, 3\,588a, 19\,940a. \tag{1}$$

5. Applying condensation

From now on we write G for Fi_{23}. Let N denote the seventh maximal subgroup of G, which is the normalizer of a subgroup of order 3 containing an element of the conjugacy class $3B$ (see [2, p. 177]).

The permutation character of G on the cosets of N is described in Table 5. The norm of this character, and hence the number of N-N-double cosets of G, equals 36.

Let K denote the largest normal 3-subgroup of N. Then K is an extraspecial group of order 3^9 (see [2, p. 177]). We take K as our condensation subgroup. Thus

Table 5. The permutation character on the cosets of N

No.	Degree	Mult.	No.	Degree	Mult.
1	1	1	36	20 322 225	1
6	30 888	2	40	26 838 240	1
8	106 743	2	44	37 573 536	2
14	812 889	1	53	58 708 650	1
24	5 533 110	1	56	73 531 392	1
26	7 468 032	1	71	216 154 575	1
27	8 783 424	1	72	216 770 400	2
32	12 077 208	3	82	289 103 904	1

let

$$e := \frac{1}{|K|} \sum_{y \in K} y \in \mathbb{F}_2 G,$$

end put $\mathcal{H} := e\mathbb{F}_2 Ge$. Then $\mathcal{H}$ is isomorphic to the endomorphism ring of the $\mathbb{F}_2$-permutation module of G on the right cosets of K. We obtain an exact functor (called condensation)

$$\text{mod-}\mathbb{F}_2 G \to \text{mod-}\mathcal{H},$$

sending an $\mathbb{F}_2 G$-module M to the $\mathcal{H}$-module Me. By producing enough simple $\mathcal{H}$-modules explicitly, we will find (see the remark at the end of Section 7), that K is a faithful condensation subgroup for G, i.e., that $Se \neq 0$ for every simple $\mathbb{F}_2 G$-module S. This implies that $\mathcal{H}$ is Morita equivalent to $\mathbb{F}_2 G$ (see [5, Section 6]).

We shall condense the $\mathbb{F}_2 G$-modules $S_i \otimes_{\mathbb{F}_2} S_j$, for $2 \leq i < j \leq 5$. Here, $S_2, \ldots, S_5$ denote the simple $\mathbb{F}_2 G$-modules $782a$, $1\,494a$, $3\,588a$, and $19\,940a$, respectivly. (See Section 4, where we obtained these as constituents of the permutation representation of G on the cosets of the maximal subgroup $2.\mathrm{Fi}_{22}$.) The second maximal subgroup $M_2 := O_8^+(3) \colon S_3$ (see [2, p. 177]) of G has an absolutely irreducible module of degree 596 over $\mathbb{F}_2$. We construct it as a tensor product of a 2-dimensional by a 298-dimensional $\mathbb{F}_2 M_2$-module. The first is a constituent of the restriction to M_2 of $782a$, the second of the restriction of $1\,494a$. Generators of M_2 as words in the standard generators of G were taken from Rob Wilson's Web Atlas [17].

The algorithms to condense tensor products have been developed by Lux and Wiegelmann in [13]. The condensation of induced modules is described in a paper [14] by Müller and Rosenboom.

To give the reader an idea of the computational magnitude involved, we comment on some timings. Let M denote the $\mathbb{F}_2 G$-module $19\,940a \otimes 19\,940a$. Then the dimension of Me equals 25 542. The computation of a single matrix describing the action of ege on Me for a group element g took about a week on a machine with a Pentium 4 processor at 3.2 GHz. If M is the $\mathbb{F}_2 G$-module obtained by inducing the 596-dimensional $\mathbb{F}_2 M_2$-module to G, the dimension of Me equals 3 694. Here, the condensation took about two hours per matrix.

In order to exploit the full strength of the Morita equivalence between $\mathbb{F}_2G$ and $\mathcal{H}$, we need a set of algebra generators for $\mathcal{H}$. The following argument is taken from the PhD-thesis of the third author [16]. Since K is a normal subgroup of N, the idempotent e is in the center of $\mathbb{F}_2N$, and so $e\mathbb{F}_2Ge$ is generated, as an $e\mathbb{F}_2Ne$-$\mathbb{F}_2Ne$-bimodule, by the elements eg_ie, where $\{g_1, \dots, g_{36}\}$ denotes a set of representatives for the N-N-double cosets of G. Also, $e\mathbb{F}_2Ne \cong \mathbb{F}_2(N/K)$, again since K is normal in N. Hence $e\mathbb{F}_2Ne$ is generated, as an $\mathbb{F}_2$-algebra, by $\{en_1e, \dots, en_re\}$, for every collection of elements $n_1, \dots, n_r \in N$ generating N as a group. In our actual computation we had $r = 3$ and $g_1 = 1$, so that we had 38 elements generating $e\mathbb{F}_2Ge$.

6. The construction of the double coset representatives

For our computations we need a set $\{g_1, \dots, g_{36}\}$ of N-N-double coset representatives in different representations. Therefore we look for a straight line program that takes as input a pair of standard generators for G and produces the elements g_1 to g_{36}. We use the term "straight line program" in the sense of GAP, see [4, Reference Manual, Chapter 35.8]. That is, the basic operations are multiplication, powering, and inversion of group elements.

The first step is to find a straight line program that takes as input a pair of standard generators of G and produces generators of N. Such a straight line program for the maximal subgroup N was not available from the ATLAS of Finite Group Representations [17]. To this end, we choose an element b from class $3B$ in G and an element c from class $2C$ (available as straight line programs from [17]). We then seek to find generators of $N = N_G(\langle b \rangle)$. Using the information about conjugacy class fusions available in the character table library (see [1]) we use GAP to compute that class $2C$ of G has 12 839 581 755 elements, intersects N non-trivially and that the intersection is the disjoint union of three conjugacy classes of N with 6 561, 26 244, and 419 904 elements, respectively. This shows that if we conjugate c with pseudo random (uniformly distributed) elements of G, the probability of producing an element of N is $(6\,561 + 26\,244 + 419\,904)/12\,839\,581\,755 \approx 3.53 \cdot 10^{-5}$ and in most cases we will reach the biggest conjugacy class of N, which happens to lie outside the normal subgroup $C_G(b)$ of N of index 2. Thus we have a good chance to find generators for N. Of course, we check whether an element c^x lies in N by checking whether it centralizes b or conjugates b to b^{-1}.

With this random approach we find three elements n_1, n_2, and n_3 of N as straight line programs in the standard generators of G. By computing with GAP in the permutation representation of G on 31 671 points, using standard permutation group methods, we verify that these elements generate N by comparing group orders.

To find N-N-double coset representatives we consider the permutation action of G on the set of right N-cosets of G, restricted to N, and find elements $\{g_1, \dots, g_{36}\}$ in G to reach the 36 N-suborbits. We compute in this permutation representation in the following way: We restrict the irreducible $\mathbb{F}_2G$-module $1\,494a$ to N and find a non-zero invariant vector v_0. Its G-orbit v_0G has length $[G : N]$ because N is maximal in G. Thus v_0G is isomorphic as a G-set to the set of right N-cosets in G. We can act with generators of G or N on vectors by vector-matrix-multiplication and can compare and store points, without writing down permutations on $[G : N]$ elements.

One vector in v_0G needs about 200 bytes of storage. Therefore storing v_0G completely would require $200\cdot[G : N] \approx 250$ GB of main memory. Also, enumerating that many vectors would take much too long.

By using the methods described in [10] we can enumerate N-suborbits "by U-orbits" for a helper subgroup $U < N$ with $|U| = 6\,561$, that is, we can archive all vectors in an N-suborbit by storing the so called "U-minimal" vectors of each U-suborbit. Given any vector, we can then recognize whether it lies in one of the stored U-suborbits: we just "U-minimalize" it by applying an element of U and look up the result in our database. The specific subgroup $U \leq N$ we used is recorded on the web page [15] through straight line programs expressing generators of U in terms of standard generators of G.

Using these techniques we can enumerate all 36 N-suborbits of v_0G on a machine with 2 GB of main memory. Thus, once we have candidates for $\{g_1, \ldots, g_{36}\}$, written in terms of n_1, n_2, and n_3, we can verify that they are in fact N-N-double coset representatives by enumerating the N-suborbits v_0g_iN and checking that they are disjoint and their lengths sum up to $[G : N]$. This verification takes about 3 hours of CPU time on a computer equipped with two Opteron processors at 2.2 GHz.

However, finding such candidates is not completely trivial, because it takes a very long time to find the short suborbits. Especially the smallest non-trivial suborbit of length 768 (see Table 6) could not be found by using standard orbit enumeration or randomized methods.

Therefore we took a different approach: once we had the other 35 suborbits and knew that we were looking for a suborbit of length 768, we first guessed the stabilizer T in N of a vector in the last suborbit, a subgroup of index 768 in N. For this subgroup T we computed the subspace of invariant vectors, which is only 3-dimensional. To all of these invariant vectors we applied a generator g of G. Using the stored knowledge about the first 35 suborbits we found at last a T-invariant vector v not lying in one of these suborbits, such that v is mapped into v_0G by g, thereby proving that $v \in v_0G$.

Once we had one and thus all vectors in the last suborbit, we found a straight line program in our standard generators for G mapping v_0 to v by performing a breadth-first search with storing all vectors starting at v_0 until the main memory was full, and then doing a depth-first backward search starting at v. The verified N-suborbit lengths in v_0G can be found in Table 6.

Straight line programs for the N-N-double coset representatives as well as for the generators of N can be found on the web page [15].

7. Results of the condensation

The results of the condensation are reported in Table 7. The first column of this table just enumerates the composition factors, the second columns names these by their degrees and a letter. The remaining eleven columns correspond to the tensor products of (in this order) $782a \otimes 782a$, $782a \otimes 1\,494a$, $782a \otimes 3\,588a$, $782a \otimes 19\,940a$, $1\,494a \otimes 1\,494a$, $1\,494a \otimes 3\,588a$, $1\,494a \otimes 19\,940a$, $3\,588a \otimes 3\,588a$, $3\,588a \otimes 19\,940a$, $19\,940a \otimes 19\,940a$, and the induced module of degree 596 of M_2, the second maximal subgroup of G. As usual, zero entries are replaced by dots. Except for the two pairs of modules of dimensions 6 and 22, the simple $\mathcal{H}$-modules occurring in the condensed modules have pairwise different dimension. The two 6-dimensional modules can easily be distinguished by their fingerprints, i.e., a vector of nullities of certain

Table 6. Lengths of N-suborbits in v_0G

1	186 624	5 038 848	30 233 088
768	944 784	7 558 272	34 012 224
3 888	944 784	10 077 696	68 024 448
15 552	1 679 616	10 077 696	90 699 264
15 552	1 679 616	10 077 696	90 699 264
19 683	3 359 232	20 155 392	90 699 264
62 208	3 779 136	22 674 816	136 048 896
78 732	3 779 136	30 233 088	272 097 792
124 416	5 038 848	30 233 088	272 097 792

elements of $\mathcal{H}$. All simple $\mathcal{H}$-modules are absolutely simple, except the one of dimension 88, which splits over the field with four elements, and thus gives rise to two absolutely simple modules.

We have already mentioned in Section 2, that the irreducible Brauer characters of the non-principal blocks are all liftable to characteristic 0. From Table 8 we can thus read off the dimensions of the condensed simple modules lying in non-principal blocks. We find that all these are larger than 1 248. Hence all $\mathcal{H}$-modules occurring in Table 7 are condensed principal block modules. Since there are 19 such $\mathcal{H}$-modules up to isomorphism, and 19 simple $\mathbb{F}_2G$-modules in the principal block, it follows that K is a faithful condensation subgroup, i.e., that $Se \neq 0$ for all simple $\mathbb{F}_2G$-modules S.

8. A basic set of Brauer characters

It is easy to find a basic set of Brauer characters consisting of ordinary characters restricted to the elements of odd order. Table 8 lists the Atlas numbers, the degrees and the condensed degrees of such a basic set. The first twenty of these characters form a basic set for the principal block. The basic set for the other blocks consists of their irreducible Brauer characters, which are all liftable.

If φ is a Brauer character, its condensed degree $\varphi^c(1)$ is the dimension of Me for any module M with Brauer character φ. It can be computed as the scalar product $\varphi^c(1) = (\varphi_K, 1_K)$.

It can easily be checked with GAP that the displayed set of characters is indeed a basic set. Extract the indicated characters from the ordinary character table, restrict them to the conjugacy classes of elements of odd order, and test whether every restricted ordinary character of the principal block is a $\mathbb{Z}$-linear combination of these.

9. Computing the irreducible Brauer characters

We now indicate how to compute the set of irreducible Brauer characters from the results of the condensation.

We begin by computing the irreducible Brauer characters contained in the permutation module on the cosets of the first maximal subgroup $2.\mathrm{Fi}_{22}$. The ordinary

TABLE 7. Condensation results

Nr.	Name	1	2	3	4	5	6	7	8	9	10	11
1	$1a$	6	.	.	8	14	2	10	16	8	120	6
2	$8a$	5	.	.	4	4	.	8	10	6	70	4
3	$10a$	4	.	.	7	9	1	7	12	4	88	5
4	$6a$	8	.	4	16	12	2	18	27	28	206	14
5	$34a$	4	.	.	5	4	.	6	6	1	53	2
6	$6b$	2	.	2	4	4	.	7	8	9	76	7
7	$22a$	2	1	.	5	2	1	5	6	10	56	5
8	$22b$	2	1	.	5	2	1	5	6	10	56	5
9	$36a$	.	.	.	1	4	.	.	4	.	16	.
10	$42a$	.	.	.	1	2	2	.	.	4	14	.
11	$48a$	.	.	1	.	.	.	1	2	6	10	2
12	$54a$	.	.	1	.	.	.	.	4	2	6	.
13	$72a$	.	1	.	2	.	.	4	4	2	26	4
14/15	$88a$	.	.	.	.	.	1	.	.	3	2	.
16	$124a$	.	.	.	.	.	.	.	.	4	8	2
17	$374a$	.	.	.	.	.	.	2	.	1	8	2
18	$534a$	.	.	.	2	.	.	1	.	.	8	1
19	$814a$	.	.	.	.	.	.	.	.	.	4	.
20	$1248a$	.	.	.	.	.	.	.	.	.	2	1

TABLE 8. A basic set of Brauer characters

i	$\chi_i(1)$	$\chi_i^c(1)$
1	1	1
2	782	8
3	3 588	6
4	5 083	17
5	25 806	60
7	60 996	12
8	106 743	107
10	274 482	104
12	752 675	173
13	789 360	72

i	$\chi_i(1)$	$\chi_i^c(1)$
17	850 850	200
20	1 951 872	48
21	2 236 520	428
25	6 709 560	772
27	8 783 424	928
35	18 812 574	774
36	20 322 225	1721
49	55 740 960	2400
50	56 360 304	3652
62	153 014 400	7136

i	$\chi_i(1)$	$\chi_i^c(1)$
56	73 531 392	5 120
60	97 976 320	4 608
64	166 559 744	9 472
94	504 627 200	24 576
95	504 627 200	24 576

character of this permutation module equals

$$1a + 782a + 30\,888a \tag{2}$$

(see [2, p. 177]). The list (1) of modulo 2 composition factors of this permutation module shows that the ordinary character of degree 782 remains irreducible modulo 2.

Every non-trivial simple module of $\mathbb{F}_2G$ condenses to a module of degree at least 6. This implies that the ordinary character of degree 3 588 remains irreducible modulo 2 (cf. Table 8), giving an irreducible $\mathbb{F}_2G$-module of degree 3 588 which condenses to an $\mathcal{H}$-module of dimension 6. We shall show below that this $\mathcal{H}$-module is in fact $6a$.

As ordinary characters, the tensor product $782a \otimes 3\,588a$ decomposes as

$$782a \otimes 3\,588a = 3\,588a + 60\,996a + 789\,360a + 1\,951\,872a. \tag{3}$$

By Table 8, the ordinary character $60\,996a$ condenses to a module of dimension 12, and by Table 7 (the column labelled 3), its reduction modulo 2 splits into two modules, each of which condenses to a 6-dimensional $\mathcal{H}$-module. Using GAP we find that

$$(60\,996a, 106\,743a \otimes 504\,627\,200a) = 1. \tag{4}$$

Since $504\,627\,200a$ is of 2-defect zero, the product $106\,743a \otimes 504\,627\,200a$ is projective, and so the scalar product (4) implies that the reduction modulo 2 of $60\,996a$ is not twice an irreducible Brauer character. Thus there is exactly one irreducible Brauer character of G of degree 3 588 which condenses to a 6-dimensional $\mathcal{H}$-module. Moreover, there is an irreducible Brauer character $57\,408a := 60\,996a - 3\,588a$ which condenses to the other 6-dimensional $\mathcal{H}$-module.

We have, on elements of odd order, the relation

$$1a + 782a + 30\,888a = 782a + 5\,083a + 25\,806a \tag{5}$$

between restricted ordinary characters. Now $5\,083a$ condenses to a module of dimension 17 (see Table 8). By (1), (2), and (5), the reduction modulo 2 of $5\,083a$ has at most three trivial composition factors. Looking at the degrees of the condensed modules in Table 7, and using the fact that $8a$ is the condensed module of $782a$, it follows that $5\,083a$ has a composition factor condensing to a 6-dimensional $\mathcal{H}$-module. By what we have said above, this must be the condensation of $3\,588a$. Hence $3\,588a$ is a composition factor, with multiplicity 1, of the reduction modulo 2 of $5\,083a$. By (1) and (5), the Brauer character of $1\,494a$ equals

$$1\,494a = 5\,083a - 3\,588a - 1a,$$

where on the right hand side we understand the restricted ordinary characters of these degrees. From this, (1) and (2), it is clear how to compute the Brauer character of $19\,940a$.

In order to show that the irreducible $\mathbb{F}_2G$-module $3\,588a$ condenses to the $\mathcal{H}$-module $6a$, we consider the product of the Brauer character $782a$ with itself. We find that

$$782a \otimes 782a = 6\times 1a + 3\times 782a + 4\times 1\,494a + 4\times 3\,588a + 2\times 19\,940a + 2\times 274\,482a, \tag{6}$$

as Brauer characters, where the last of these characters on the right hand side is the basic set character obtained by restricting the ordinary irreducible character of this degree to the elements of odd order (cf. Table 8). Since $6b$ occurs only twice in the condensed module of $782a \otimes 782a$ (cf. Table 7), it follows from (6) that $3\,588a$ indeed condenses to $6a$.

By Table 7, the tensor product $782a \otimes 3\,588a$ contains a unique condensed constituent of degree 48. Thus it follows from (3), together with Tables 7 and 8, that the ordinary character $1\,951\,872a$ remains irreducible modulo 2.

For the computations below we modify our original basic set, described in Table 8, by replacing the restricted ordinary characters $5\,083a$ and $25\,806a$ with the Brauer characters of $1\,494a$ and $19\,940a$, respectively. Moreover, the first of these two characters is moved to position 3, and the Brauer character of $3\,588a$ to position 4 in the new basic set, whose elements will be denoted by $\vartheta_1, \ldots, \vartheta_{20}$.

Having found the irreducible Brauer characters of the simple modules $1a$, $782a$, $1\,494a$, $3\,588a$, and $19\,940a$, we can compute the products of these and express them as $\mathbb{Z}$-linear combinations of the (new) basic set characters. The result is displayed in Table 9. The columns numbered 1–10 contain the ten products of characters in the same order as in Table 7. Column 11 gives the multiplicities in the basic set of Brauer characters obtained by inducing the character of degree 596 of the second maximal subgroup. For later use, columns 12 and 13 give the basic set expressions of the two restricted ordinary characters $850\,850b$ and $153\,014\,400b$, algebraically conjugate to $850\,850a$ and $153\,014\,400a$ (which are contained in the basic set), respectively.

The remaining irreducible Brauer characters can be found by solving a system of linear equations. Namely, let $\varphi_1, \ldots, \varphi_{20}$ denote the set of absolutely irreducible 2-modular characters of G, where the numbering corresponds to the one of the rows in Table 7. Let us choose an algebraic closure $\bar{\mathbb{F}}_2$ of $\mathbb{F}_2$ and write S_i for the simple $\bar{\mathbb{F}}_2G$-module with Brauer character φ_i, $i = 1, \ldots, 20$.

Let φ, φ' denote one of the two pairs of non-rational valued irreducible Brauer characters (see Section 3). Since the only possible non-rational value of φ and φ' involve the quadratic irrationalities $b23 = (-1 + \sqrt{-23})/2$ or $b13 = (-1 + \sqrt{13})/2$, it follows that φ and φ' have the same scalar product with the trivial character of the condensation subgroup K. This implies that the underlying $\bar{\mathbb{F}}_2G$-modules condense to $\bar{\mathbb{F}}_2 \otimes_{\mathbb{F}_2} \mathcal{H}$-modules of the same degree.

Since the two $\bar{\mathbb{F}}_2 \otimes_{\mathbb{F}_2} \mathcal{H}$-modules of degrees 6 arise from $\bar{\mathbb{F}}_2G$-modules of different degrees, and since $88a$ is the only $\mathcal{H}$-module which is not absolutely irreducible, it follows that φ_7 and φ_8 are complex conjugate, and that φ_{14} and φ_{15} are conjugate under the Galois automorphism of the field $\mathbb{Q}((-1 + \sqrt{13})/2)$. The $\bar{\mathbb{F}}_2G$-modules S_{14} and S_{15} with Brauer characters φ_{14} and φ_{15} condense to two modules $44a$ and $44b$ of $\bar{\mathbb{F}}_2 \otimes_{\mathbb{F}_2} \mathcal{H}$.

Recall that our basic set characters are denoted by $\vartheta_1, \ldots, \vartheta_{20}$, the ordering being the one indicated by the rows of Table 9. There are non-negative integers a_{ij}, $1 \leq i, j \leq 20$, such that

$$\vartheta_i = \sum_{j=1}^{20} a_{ij} \varphi_j, \qquad i = 1, \ldots, 20.$$

The matrix $(a_{ij}) \in \mathbb{Z}^{20 \times 20}$ is invertible over the integers. Since we already know seven of the irreducible Brauer characters, we are left with 260 unknowns.

Suppose that M is an $\bar{\mathbb{F}}_2G$-module whose Brauer character ϑ is known explicitly. Suppose also that we know the composition factors of M, including their multiplicities. This is the case for the tensor products as well as for the induced module we have condensed.

Suppose we have

$$[M] = \sum_{j=1}^{20} s_j \, [S_j]$$

in the Grothendieck group of $\bar{\mathbb{F}}_2G$. This implies that

$$\vartheta = \sum_{j=1}^{20} s_j \, \varphi_j. \tag{7}$$

On the other hand, since ϑ is explicitly known as a class function, we can compute integers t_i, $1 \leq i \leq 20$, such that

$$\vartheta = \sum_{i=1}^{20} t_i \vartheta_i. \tag{8}$$

Equations (7) and (8) yield 20 linear equations for the unknowns a_{ij}, namely

$$\sum_{i=1}^{20} t_i \, a_{ij} = s_j, \qquad j = 1, \ldots, 20.$$

Thus we obtain 220 linear equations from our eleven condensed modules.

We obtain further equations from the symmetries arising from complex conjugation of Brauer characters and applying the non-trivial Galois automorphism of $\mathbb{Q}((-1+\sqrt{13})/2)$. For example, the column headed 12 of Table 9 gives the coefficients u_i in the expression

$$\bar{\vartheta}_{11} = \sum_{i=1}^{20} u_i \, \vartheta_i,$$

where $\bar{\vartheta}_{11}$ is the Brauer character complex conjugate to ϑ_{11}. Since φ_7 and φ_8 is the only pair of complex conjugate irreducible Brauer characters in the principal block of $\bar{\mathbb{F}}_2G$, we obtain equations

$$\sum_{i=1}^{20} u_i \, a_{ij} - a_{11,j} = 0, \qquad j = 1, \ldots, 20, j \neq 7, 8,$$

$$\sum_{i=1}^{20} u_i \, a_{i,7} - a_{11,8} = 0, \qquad \sum_{i=1}^{20} u_i \, a_{i,8} - a_{11,7} = 0.$$

Similar equations arise from column 13 of Table 9 which gives the coefficients in the basic set of the Galois conjugate of ϑ_{20}.

The system of linear equations we obtain this way has rank 398. The solution matrix for the a_{ij}, with two unknowns α_1 and α_2, is displayed in Table 10. The determinant of this matrix equals

$$4\alpha_2\alpha_1 - 10\alpha_1 - 2\alpha_2 + 5.$$

Since the determinant of (a_{ij}) is ± 1 and the a_{ij} are non-negative integers, we conclude that $\alpha_1 \in \{0, 1\}$ and $\alpha_2 \in \{2, 3\}$. Any two solution matrices can be transformed into each other by swapping columns 7 and 8 or 14 and 15 or both.

TABLE 9. Some characters expressed in the basic set

$\vartheta_i(1)$	1	2	3	4	5	6	7	8	9	10	11	12	13
1	6	.	.	−1	−2	.	.	4	−14	−6	−9	−2	−36
782	3	−1	.	−2	−2	1	1	.	−7	−10	−5	−1	−22
1 494	4	.	.	.	−1	−1	−1	4	−7	2	−5	.	−16
3 588	4	−1	1	−2	2	−3	.	5	−6	2	−5	1	−8
19 940	2	.	.	−1	−2	1	.	.	−7	−9	−4	−1	−20
60 996	.	−1	1	.	.	.	1	−2	−3	4	.	−1	−5
106 743	.	−2	.	−2	2	1	−3	−4	3	−2	−2	.	1
274 482	2	.	.	1	2	−2	.	2	3	8	.	1	15
752 675	.	.	.	−1	2	−1	−3	.	2	6	.	2	5
789 360	.	−1	1	−2	.	.	.	.	2	.	.	.	6
850 850	.	.	.	.	.	.	.	.	.	.	.	−1	.
1 951 872	.	.	1	.	.	.	−1	2	−3	−6	−2	.	−12
2 236 520	.	1	.	2	.	−1	2	4	−2	6	1	.	1
6 709 560	.	.	.	2	.	.	1	.	.	4	1	.	2
8 783 424	.	.	.	.	.	1	−2	.	−2	−6	−2	.	−9
18 812 574	.	.	.	.	.	.	.	.	4	2	1	.	10
20 322 225	.	.	.	.	.	.	2	.	1	2	1	.	4
55 740 960	.	.	.	.	.	.	.	.	.	2	1	.	2
56 360 304	.	.	.	.	.	.	.	.	.	4	.	.	.
153 014 400	.	.	.	.	.	.	.	.	.	.	.	.	−1

This amounts to renaming φ_7 and φ_8 or φ_{14} and φ_{15} or both. We choose notation such that $\alpha_1 = 1$ and $\alpha_2 = 3$.

From the inverse of the matrix (a_{ij}) and the table of the basic set characters $\vartheta_1, \ldots, \vartheta_{20}$, we can compute the irreducible Brauer characters $\varphi_1, \ldots, \varphi_{20}$, and from these the decomposition matrix of Table 2. We omit further details.

We should like to mention that the antique MOC-system (cf. [6, 11, 12]) was used in the first place to perform numerous experiments with products of Brauer characters, induced Brauer characters and projective characters, in order to find suitable modules to condense.

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Table 10. The matrix of the a_{ij} ($\alpha_1' = -\alpha_1 + 1$, $\alpha_2' = -\alpha_2 + 5$)

$$
\begin{bmatrix}
1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
. & . & . & 1 & . & 1 & . & . & . & . & . & . & . & . & . & . & . & . & . & . \\
3 & 1 & 2 & 1 & 1 & . & . & . & 1 & . & . & . & . & . & . & . & . & . & . & . \\
. & 1 & . & 2 & 1 & 1 & 1 & 1 & . & . & . & . & . & . & . & . & . & . & . & . \\
5 & 1 & 3 & 2 & 1 & 1 & . & . & 1 & 1 & . & . & . & . & . & . & . & . & . & . \\
. & . & . & 2 & . & 1 & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . \\
4 & 1 & 3 & 3 & 1 & 1 & \alpha_1 & \alpha_1' & 1 & 1 & . & . & . & . & . & . & . & . & . & . \\
. & . & . & . & . & . & . & . & . & . & 1 & . & . & . & . & . & . & . & . & . \\
6 & 3 & 4 & 6 & 2 & 2 & 1 & 1 & 2 & . & . & 1 & 1 & . & . & . & . & . & . & . \\
4 & 1 & 3 & 6 & 2 & 1 & 1 & 1 & . & 1 & . & . & . & . & . & . & . & 1 & . & . \\
10 & 4 & 7 & 16 & 3 & 5 & 4 & 4 & 2 & 3 & . & 1 & 1 & 1 & 1 & . & . & . & . & . \\
4 & 3 & 2 & 11 & 1 & 3 & 3 & 3 & . & 1 & 2 & 1 & 1 & 1 & 1 & 1 & . & . & . & . \\
19 & 7 & 13 & 20 & 6 & 7 & 5 & 5 & 3 & 4 & 1 & . & 2 & 1 & 1 & . & 1 & . & . & . \\
8 & 5 & 6 & 10 & 3 & 4 & 3 & 3 & 1 & . & 1 & . & 2 & . & . & 1 & 1 & . & . & 1 \\
12 & 10 & 9 & 32 & 7 & 11 & 10 & 10 & 1 & 3 & 2 & 1 & 4 & 1 & 1 & 1 & 1 & 1 & 1 & . \\
24 & 18 & 16 & 62 & 10 & 23 & 19 & 19 & 2 & 3 & 7 & 4 & 7 & \alpha_2 & \alpha_2' & 6 & 3 & 1 & . & 1
\end{bmatrix}
$$

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Lehrstuhl D für Mathematik, RWTH Aachen University, 52056 Aachen, Germany
E-mail address: `G.H.: Gerhard.Hiss@Math.RWTH-Aachen.DE`
E-mail address: `M.N.: Max.Neunhoeffer@Math.RWTH-Aachen.DE`
E-mail address: `F.N.: Felix.Noeske@Math.RWTH-Aachen.DE`