Solution to Exercise 3.1.2

Assume that E is an elementary abelian subgroup of $G := \mathrm{M}_{11}$ of order 2^n. For convenience we reproduce the character table of G;

	1a	2a	3a	4a	5a	6a	8a	8b	11a	11b
χ_1	1	1	1	1	1	1	1	1	1	1
χ_2	10	2	1	2	0	-1	0	0	-1	-1
χ_3	10	-2	1	0	0	1	α	$-\alpha$	-1	-1
χ_4	10	-2	1	0	0	1	$-\alpha$	α	-1	-1
χ_5	11	3	2	-1	1	0	-1	-1	0	0
χ_6	16	0	-2	0	1	0	0	0	β	$\overline{\beta}$
χ_7	16	0	-2	0	1	0	0	0	$\overline{\beta}$	β
χ_8	44	4	-1	0	-1	1	0	0	0	0
χ_9	45	-3	0	1	0	0	-1	-1	1	1
χ_{10}	55	-1	1	-1	0	-1	1	1	0	0

Since there is just one class of involutions we find

$$(\,\chi_3|_E, \mathbf{1}_E\,)_E = \frac{1}{2^n}\,(10 - 2\cdot(2^n - 1)).$$

Hence

$$12 - 2^{n+1} \equiv \quad \bmod 2^n.$$

It follows that $n \leq 2$.

Computing the symmetric class multiplication coefficient $\alpha_{\mathtt{2a},\mathtt{2a},\mathtt{4a}} = \frac{|G|}{2}$ (see Example 2.5.17) we see from Remark 2.5.10 and Remark 2.5.11 that G contains dihedral groups of order eight; hence the rank of G cannot be one. One could also use e.g. Glauberman's $\mathbf{Z}^*$-theorem (Theorem 4.11.18) to show that the rank of G is not one, but this would be by far too complicated.