

Solution to Exercise 4.10.1

We perform the character calculations in GAP.

```
gap> l211:=CharacterTable("L2(11)");
CharacterTable( "L2(11)" )
gap> m:=Maxes(l211);
[ "A5", "A5", "11:5", "S3x2" ]
gap> a5:=CharacterTable(m[1]);
CharacterTable( "A5" )
gap> Display(l211);
L2(11)
```

2	2	2	1	.	.	1	.	.
3	1	1	1	.	.	1	.	.
5	1	.	.	1	1	.	.	.
11	1	.	.	.	.	.	1	1

	1a	2a	3a	5a	5b	6a	11a	11b
2P	1a	1a	3a	5b	5a	3a	11b	11a
3P	1a	2a	1a	5b	5a	2a	11a	11b
5P	1a	2a	3a	1a	1a	6a	11a	11b
11P	1a	2a	3a	5a	5b	6a	1a	1a

X.1		1	1	1	1	1	1	1
X.2		5	1	-1	.	.	1	B /B
X.3		5	1	-1	.	.	1	/B B
X.4		10	-2	1	.	.	1	-1 -1
X.5		10	2	1	.	.	-1	-1 -1
X.6		11	-1	-1	1	1	-1	.
X.7		12	.	.	A	*A	.	1 1
X.8		12	.	.	*A	A	.	1 1

```
A = E(5)+E(5)^4
  = (-1+ER(5))/2 = b5
B = E(11)+E(11)^3+E(11)^4+E(11)^5+E(11)^9
  = (-1+ER(-11))/2 = b11
gap> Display(a5);
A5
```

2	2	2	.	.	.
3	1	.	1	.	.
5	1	.	.	1	1

	1a	2a	3a	5a	5b
2P	1a	1a	3a	5b	5a

```

3P 1a 2a 1a 5b 5a
5P 1a 2a 3a 1a 1a

```

```

X.1      1  1  1  1  1
X.2      3 -1  .  A *A
X.3      3 -1  . *A  A
X.4      4  .  1 -1 -1
X.5      5  1 -1  .  .

```

```

A = -E(5)-E(5)^4
   = (1-ER(5))/2 = -b5
gap> i:=InducedClassFunctions(a5,[Irr(a5)[4]],l211);
gap> MatScalarProducts(Irr(l211),i);
[ [ 0, 0, 0, 1, 1, 0, 1, 1 ] ]
gap> PrimeBlocks(l211,3);
rec( block := [ 1, 2, 2, 1, 2, 1, 3, 4 ], defect := [ 1, 1, 0, 0 ],
     height := [ 0, 0, 0, 0, 0, 0, 0, 0 ], relevant := [ 2, 4, 7 ],
     centralcharacter :=
       [ [ , 55,, 132,, 60 ], [ , 11,, 0,, 12*E(11)+12*E(11)^3+12*E(11)^4
         +12*E(11)^5+12*E(11)^9 ], [ , 0,, 11*E(5)+11*E(5)^4,, 5 ],
         [ , 0,, 11*E(5)^2+11*E(5)^3,, 5 ] ] )

```

The results show that $\eta_4^G = \chi_4 + \chi_5 + \chi_7 + \chi_8$. Since η_4 is a trivial source character, (which can be easily verified using the natural permutation character of A_5) and the four characters $\chi_4, \chi_5, \chi_7, \chi_8$ all lie in different blocks, the trivial source SG -module corresponding to η_4^G decomposes as a direct sum of four trivial source SG -modules $L_1, \dots, L_4$ with characters χ_4, χ_5, χ_7 , and χ_8 .