Solution to Exercise 4.2.4

$\psi := \varphi|_{\langle g\rangle}$ is an ordinary character of $H := \langle g\rangle$ because $g \in G_{p'}$. Clearly $\mathrm{Fix}_V(g)$ is an $\mathbb{F}_{p^n}$-vector space of dimension $(\psi, \mathbf{1}_H)_H = \frac{1}{m}\sum_{j=1}^{m}\varphi(g^j) =: n$. Thus by Example 2.1.3 $\theta(g) = p^{nk} - 1$.

We display the 2-Brauer character table of $\mathrm{S}_6(2)$:

```
gap> t := CharacterTable( "S6(2)" ) mod 2
gap> Display( t );
S6(2)mod2

      2   9   4  3  2  1  .  .   .
      3   4   3  4  3  1  .  2   1
      5   1   1  .  .  1  .  .   1
      7   1   .  .  .  .  1  .   .

         1a  3a 3b 3c 5a 7a 9a 15a
     2P  1a  3a 3b 3c 5a 7a 9a 15a
     3P  1a  1a 1a 1a 5a 7a 3b  5a
     5P  1a  3a 3b 3c 1a 7a 9a  3a
     7P  1a  3a 3b 3c 5a 1a 9a 15a

X.1       1   1  1  1  1  1  1   1
X.2       6   3 -3  .  1 -1  .  -2
X.3       8  -4 -1  2 -2  1 -1   1
X.4      14   2  5 -1 -1  . -1   2
X.5      48 -12  3  . -2 -1  .  -2
X.6      64   4 -8 -2 -1  1  1  -1
X.7     112  -8 -5 -2  2  .  1   2
X.8     512 -16  8 -4  2  1 -1  -1
```

Assuming that $g \in$ `5a` and $\varphi :=$ `X.3` we see that $(\psi, \mathbf{1}_H)_H = \frac{1}{5}(8 + 4\,(-2)) = 0$. Hence $\mathrm{Fix}_V(g) = \{0\}$.