Solution to Exercise 4.5.8

Note first that $V = e_H V \oplus (1-e_h)V$ is a decomposition of V as an $e_H FGe_H$-module. Moreover, since $e_H FGe_H$ annihilates $(1 - e_H)V$ we conclude that $\mathrm{trace}_V(e_H g e_H) = \mathrm{trace}_{e_H V}(e_H g e_H)$. On the other hand, since $\mathrm{trace}(ab) = \mathrm{trace}(ba)$ for $a, b \in FG$, we get

$$\begin{array}{l} \mathrm{trace}_{e_H V}(e_H g e_H) = \mathrm{trace}_V(e_H g e_H) = \mathrm{trace}_V(e_H^2 g) \\ = \mathrm{trace}_V(e_H g) = \mathrm{trace}_V(\frac{1}{|H|}\sum_{h\in H} hg) \\ = \frac{1}{|H|}\sum_{h\in H} \mathrm{trace}(gh) \end{array} .$$