Solution to Exercise 4.6.3

We use GAP to verify the claim. First, we get the character table of $G := \mathrm{U}_3(3)$.

```
gap> u33:=CharacterTable("U3(3)");
CharacterTable( "U3(3)" )
gap> Display(u33);
U3(3)

      2  5  5  2  .  5  5  4  2  .  .  3  3   2   2
      3  3  1  3  2  1  1  .  1  .  .  .  .   1   1
      7  1  .  .  .  .  .  .  .  1  1  .  .   .   .

        1a 2a 3a 3b 4a 4b 4c 6a 7a 7b 8a 8b 12a 12b
     2P 1a 1a 3a 3b 2a 2a 2a 3a 7a 7b 4a 4b  6a  6a
     3P 1a 2a 1a 1a 4b 4a 4c 2a 7b 7a 8b 8a  4b  4a
     7P 1a 2a 3a 3b 4b 4a 4c 6a 1a 1a 8b 8a 12b 12a

X.1      1  1  1  1  1  1  1  1  1  1  1  1   1   1
X.2      6 -2 -3  . -2 -2  2  1 -1 -1  .  .   1   1
X.3      7 -1 -2  1  3  3 -1  2  .  . -1 -1   .   .
X.4      7  3 -2  1  A /A  1  .  .  .  E -E   F  /F
X.5      7  3 -2  1 /A  A  1  .  .  . -E  E  /F   F
X.6     14 -2  5 -1  2  2  2  1  .  .  .  .  -1  -1
X.7     21  5  3  .  1  1  1 -1  .  . -1 -1   1   1
X.8     21  1  3  .  B /B -1  1  .  .  E -E  -E   E
X.9     21  1  3  . /B  B -1  1  .  . -E  E   E  -E
X.10    27  3  .  .  3  3 -1  . -1 -1  1  1   .   .
X.11    28 -4  1  1  C -C  . -1  .  .  .  .   E  -E
X.12    28 -4  1  1 -C  C  . -1  .  .  .  .  -E   E
X.13    32  . -4 -1  .  .  .  .  D /D  .  .   .   .
X.14    32  . -4 -1  .  .  .  . /D  D  .  .   .   .

A = -1-2*E(4)
  = -1-2*ER(-1) = -1-2i
B = -3+2*E(4)
  = -3+2*ER(-1) = -3+2i
C = 4*E(4)
  = 4*ER(-1) = 4i
D = -E(7)-E(7)^2-E(7)^4
  = (1-ER(-7))/2 = -b7
E = E(4)
  = ER(-1) = i
F = -1+E(4)
  = -1+ER(-1) = -1+i
```

Since the order of G is $2^5 \cdot 3^3 \cdot 7$ and defect zero characters of G are 0 on all 2-

singular classes, we see that there are 2 blocks of defect zero: B_1, which contains χ_{13}, and B_2, which contains χ_{14}. Using equation (1) from the solution of the previous Exercise 4.6.2, we see that the class `3b` is the only defect class for the block B_1 and it is the only defect class for the block B_2.

This exercise is due to Thomas Breuer. It provides a counterexample to an assertion made in Exercise (15.6) of [92].