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Abstracts

Almkvist Zeilberger Algorithm and (Generalized) Harmonic Sums

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In this talk I will present algorithms for indefinite nested sums and products, like, e.g., harmonic sums, S-sums or cyclotomic sums that play an important role in Quantum field theory. Special emphasis will be put, e.g., on integral representations of these sums and on algorithms that compute the asymptotic expansion of such objects. Moreover, I will present enhancements of the multivariate Almkvist-Zeilberger algorithm to certain types of Feynman integrals. The underlying algorithms are supplemented by concrete examples using the packages HarmonicSums and MultiIntegrate.

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Linear differential and difference systems: EG$_\delta$ - and EG$_\sigma$ - eliminations

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Ordinary systems of the form

$$A_r(x)\xi^r y(x) + \cdots + A_1(x)\xi y(x) + A_0(x)y(x) = b(x)$$

are considered for the differential ($\xi = \delta$: $\delta y(x) = y'(x)$) and difference ($\xi = \sigma$: $\sigma y(x) = y(x+1)$) cases. $A_i(x)$, $i = 0, \ldots, r$, are square matrices of order m with entries from $\mathbb{C}[x]$ ($A_r(x), A_0(x)$ are nonzero), $b(x) \in \mathbb{C}[x]^m$. We assume that equations of $A_r(x)\xi^r y(x) + \cdots + A_1(x)\xi y(x) + A_0(x)y(x) = 0$ are independent over $k[x, \xi]$.

For any system S of this form the algorithm EG$_\delta$ in the differential case and the algorithm EG$_\sigma$ in the difference case construct, in particular, an *l-embracing* system $\bar{S}$ of the same form, but with the leading matrix $\bar{A}_r(x)$ being invertible, and with the set of solutions containing all the solutions of S. EG$_\delta$ and EG$_\sigma$ are used for finding solutions of the considered systems and for finding and investigating singular points of analytic solutions of such systems. In our talk we discuss some concrete problems which can be solved with EG$_\delta$ and EG$_\sigma$. The Maple code of EG$_\delta$ and EG$_\sigma$ is available from `http://www.ccas.ru/ca/doku.php/eg`.

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On the Exponential Parts of Linear Differential Equations with Meromorphic Coefficients and Their Computation

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In this talk we shall study the relationship between the exponential parts of formal solutions of a differential system $dY/dz = A(z)Y$ having a singularity of pole type at $z = 0$ and the singular parts of the eigenvalues of the matrix $A(z)$ considered as Puiseux series in z.

Given a linear differential system $dY/dz = A(z)Y$ of size n with meromorphic coefficients of order p at the origin, we first show that the exponential parts of the system and the eigenvalues of its matrix $A(z)$ do agree up to a certain order that depends on n and p. We then show that under suitable conditions on the matrix $A(z)$, some formal invariants of the differential system $dY/dz = A(z)Y$ can be computed from the characteristic polynomial of $A(z)$. We conclude by showing how these results can be used to compute efficiently the formal solutions of the system.

Some computable objects in $\mathcal{D}$-module theory

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$\mathcal{D}$-modules theory studies modules over a ring $\mathcal{D}$ of linear differential operators. For computational purposes we can restrict ourselves to the case of finitely generated D–modules over the complex Weyl algebra $D = A_n$. Elements in the Weyl algebra A_n are linear differential operators with coefficients in the polynomial ring $\mathbb{C}[x]$ of complex polynomials in n variables $x = (x_1, \ldots, x_n)$. The Weyl algebra A_n is (for $n > 0$) a non-commutative noetherian ring. For a given non-zero polynomial f, the ring of rational functions $\mathbb{C}[x]_f$ is *finitely generated* considered as a module over A_n (this main result is due to J. Bernstein). Finding a system of generators of such a module is a computationally difficult task and it is related to the computation of the annihilating ideal of the rational function $1/f$. This annihilating ideal can be computed by using algorithms by T. Oaku and N. Takayama. These algorithms use Groebner basis computations in the Weyl algebra A_n. In this talk we will describe the role of logarithmic D–modules in the *logarithmic comparison conjecture*, an open problem concerning both the annihilating ideal of $1/f$ and the comparison between the cohomology of the meromorphic and the logarithmic de Rham complexes associated to the polynomial f. I will also talk about recent results in a joint work with A. Leykin on this subject and then on a work in progress with M. Barakat and A. Leykin.

Solving third order linear difference equations in terms of second order linear difference equations

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In this talk we show how to give a closed form solution for third order difference operators in terms of solutions of second order operators. This work is an extension of previous results on finding closed form solutions of recurrence equations and a counterpart to existing results on differential operators. As motivation and application for this work, we discuss the problem of proving positivity of sequences given merely in terms of their defining recurrence relation. The main advantage of the present approach to earlier methods attacking the same problem is that our algorithm provides human-readable and verifiable, i.e., certified proofs.

Local integrability in three dimensional Lotka-Volterra equations.

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We consider various aspects of the problem of showing local integrability in three dimensional Lotka-Volterra equations. In contrast with the two-dimensional case, the subject is richer and computationally much harder. We report on calculations done together with Waleed Aziz which give necessary and sufficient conditions for integrability in the case of $(a : -b : c)$-resonance where $a + b + c \leq 4$. Integrability, as usual, is provided by the Darboux method, but with some twists. We also consider the applicability of the monodromy method to proving integrability in these systems.

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An invitation to Sage

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The open source computer mathematics system Sage creates a novel platform for computational algebra by combining the widely popular programming language Python and state of the art mathematics software such as Singular, PARI/GP, GAP, Maxima, LinBox, FLINT. This short introduction to Sage will show examples involving the recent interface to Plural, the noncommutative component of Singular, as well as important building blocks such as fast linear algebra (FFLAS - FFPACK), polynomial arithmetic and factorization (NTL, FLINT, Singular-factory) and Gröbner basis computation (Singular).

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Bifurcation values and uniqueness of the limit cycle for a family of quintic planar polynomial vector fields

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In this talk we study the number of limit cycles and the bifurcation diagram in the Poincaré sphere of an 1-parameter family of planar differential equations of degree five, $\dot{\mathbf{x}} = X_b(\mathbf{x})$, which has been already considered in [3,5]. We prove that there is a value $b^* > 0$ such that the limit cycle exists only when $b \in (0, b^*)$ and that it is unique and hyperbolic by using a rational Dulac function. Moreover we provide an interval of length $27/1000$ where b^* lies. As far as we know the tools used to determine this interval are new and are based on the construction of algebraic curves without contact for the flow of the differential equation. These curves are obtained using analytic information about the separatrices of the infinite critical points of the vector field.

To prove that the Bendixson-Dulac Theorem ([1,2]) works we develop a method for studying when one-parameter families of polynomials in two variables do not vanish based on the computation of the so called double discriminant. For our problem this discriminant turns out to be a huge polynomial in $b2$ with rational coefficients. In particular we need to control, on a given interval with rational extremes, how many reals roots has a polynomial of degree 965, with enormous rational coefficients. Although Sturm type algorithms theoretically work, in practice our computers can not deal with this problem using them. Fortunately we can utilize a kind of bisection procedure based on the Descartes rule([4]) to overcome this difficulty.

This talk is based on a joint with with J. D. García-Saldaña and H. Giacomini.

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Constructions of Solutions of the Yang-Baxter Equation

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Set-theoretic solutions of the Yang–Baxter equation (YBE) form a meeting-ground of mathematical physics, algebra and combinatorics. Such a solution consists of a set X and a function $r : X \times X \to X \times X$ which satisfies the braid relation. In a series of works we have studied the close relation between the set-theoretic solutions of the Yang–Baxter equation (YBE), the skew polynomial rings with binomial relations and the Artin-Schelter regular algebras. Recently we proved that the problem of classifying Artin-Schelter regular PBW algebras with quantum binomial relations and global dimension n is equivalent to finding the classification of involutive square-free set-theoretic solutions of YBE, (X, r), on sets X of order n. Even under these strong restrictions on the shape of the relations, the problem remains highly nontrivial. In this talk we present new constructions of solutions of YBE based on strong twisted unions, and wreath products of solutions with an investigation of retracts and the multipermutation level of the solutions. Clearly, this way we also generate algebras with explicit quadratic relations. These algebras are noncommutative but they posses the "good" algebraic and homological properties of the commutative polynomial rings, in particular they provide conditions good for symbolic computation and the use of noncommutative Groebner bases.

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Computer algebra application to numerical solving of nonlinear KdV-type equations

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In the present talk we consider some nonlinear partial differential equations of Korteweg-de Vries (KdV) type with one temporal and one spatial independent variables. We analyze first a scalar equation with one dependent variable. This equation describes dynamics of viscous incompressible liquid interacting with cylinder shell under propagation of deformation waves. Then we study a system of coupled equations with two dependent variables that describes the nonlinear wave dynamics in physically nonlinear elastic cylindrical shells with viscous incompressible liquid inside them. By using our algorithmic approach [1] which combines the finite volume method and difference elimination by means of Gröbner bases, we construct finite-difference approximations to both problems and present the results of numerical simulation based on those approximations. In the case of the coupled system of two equations we also discuss consistency of its finite-difference approximation by using the ideas and methods of paper [2]. For computation of the relevant difference Gröbner bases one can apply the algorithmic approach proposed recently in [3].

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Quasideterminants, Degree Bounds and "Fast" Algorithms for Matrices of Ore Polynomials

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We look at the problem of computational linear algebra over the ring of Ore polynomials. Many new algorithms for computing normal forms and solving systems of equations with matrices of Ore polynomials have been proposed over the past few years. Examples of such computations include the Hermite (triangular) form and Jacobson (diagonal) form, which are canonical for one-sided and two-sided unimodular equivalence respectively. While many of these new algorithms are effective in practice, the complexity of most of them has not been established.

Our goal has been to develop provably polynomial-time algorithms for these problems, as well as establish tools by which to analyze other algorithms. We will outline algorithms for the Hermite and Jacobson form which require time polynomial in the dimension, degree and coefficient-size of the input matrices. One aspect which has made the problem for Ore polynomials more difficult than the analogous problems for commutative polynomials has been the lack of the usual determinantal theory, and basic theorems such as Hadamard's bound, Cramer's rule and Sylvester's identity. We instead apply the *quasideterminantal* theory of Gelfand and Retakh to Ore polynomials and establish tight degree bounds on these determinant-like objects.

We also explore the rigorous use of randomization in our computations, which has been highly effective for *preconditioners* for fast algorithms in linear algebra over the commutative polynomials. Our algorithm for the Jacobson form of a matrix of Ore polynomials is ultimately a simple randomized preconditioner, which gives a reduction to the computation of the Hermite form, for which we also have a provably polynomial-time algorithm.

This work is in collaboration with Albert Heinle (RWTH) and Myung Sub Kim (U. Waterloo)

Some computational problems in noncommutative polynomials

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The factorization theory and the structure of finitely generated modules over an Ore extension of a division ring are well understood since the decade of 1940. However, a computational approach to this topic leads to specific difficulties that are not present in the algorithms for commutative polynomials. We will discuss some of them.

Nash conjecture for binomial varieties and multidimensional Euclidean algorithm

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Nash blow-up of a variety X is defined as the closure of the graph of the Gauss map on X. John Nash has formulated a conjecture that a sequence of Nash blow-ups starting with X (over a field of characteristic zero) always terminates. In the latter case the resulting variety is smooth due to Lipman's theorem, and thereby provides a desingularization of X. In case of dimension 2 Hironaka-Sprivakovsky have proved a normalized version of the Nash conjecture when the Nash blow-ups are alternated with normalizations. The complexity of this procedure (as well as of any other known resolution of singularities) is very high.

We establish an equivalence of the Nash conjecture (respectively, of its normalized version) for binomial varieties with the termination of a multidimensional Euclidean algorithm (respectively, normalized). The main result gives a polynomial complexity bound for 2-dimensional normalized Euclidean algorithm.

(joint work with P. Milman)

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Algebraic Methods for Systems Biology

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Mathematical models are an essential part of the new field of systems biology as they are the only way to formalize and analyze models that capture the dynamics and provide insights at the system level. Recently polynomial dynamical systems over finite fields have been introduced as a new framework for modeling biological networks as multi-states finite dynamical systems, generalizing Boolean networks and logical models. Within this algebraic framework, using tools from computational algebra and algebraic geometry, the whole model space is presented and different algebraic methods are proposed for identifying a particular model from the model space. Furthermore, methods for analyzing the dynamics of classes of polynomial systems have been developed. In this talk I will present methods for the development of polynomial dynamical systems models as well as methods for the analysis of their dynamics.

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Solving Polynomial Systems Using Linear Algebra

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The eigenvalue and eigenvector methods for solving zero-dimensional polynomial systems reduce the task to a question of Linear Algebra: computing the joint eigenvectors of a family of commuting matrices. We discuss the relevant algebraic theory of families of commuting matrices: coherent families of eigenvalues, joint eigenspaces and joint generalized eigenspaces, cyclic and non-derogatory families, multiplication families, and their relations to more familiar algebraic objects such as the coordinate ring of the 0-dimensional affine scheme and its canonical module. The main point of the talk is that there are large, non-trivial and interesting parts of Linear Algebra which few algebraists know about, but which are very useful for Symbolic Computation and its applications.

Gröbner bases and gradings for partial difference ideals

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In this talk we introduce a working generalization of the theory of Gröbner bases for the algebras of partial difference polynomials with constant coefficients. Such algebras are free objects in the category of commutative algebras endowed with the action by endomorphisms of a monoid isomorphic to $\mathbb{N}^r$. Since they are not Noetherian algebras, we propose a theory for grading them that provides a Noetherian subalgebras filtration. This implies that the variants of the Buchberger algorithm we developed for partial difference ideals terminate in the finitely generated graded case when truncated up to some degree. Moreover, even in the non-graded case, we provide criterions for certifying completeness of eventually finite Gröbner bases when they are computed within sufficiently large bounded degrees. We generalize also the concepts of homogenization and saturation, and related algorithms, to the context of partial difference ideals. The feasibily of the proposed methods is shown by an implementation in Maple and a test set based on the discretization of concrete systems of non-linear partial differential equations.

Elements of Computer Algebraic Analysis

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Algebraic Analysis has been coined as a term in the mid 50's by the Japanese group led by Mikio Sato. In recent years many constructions of Algebraic Analysis have been approached from the computer-algebraic point of view, with algorithms and their implementations. Extension of such an interaction from linear differential operators to linear difference, q-difference and q-differential operators we call Computer-Algebraic Analysis. The major object of study are systems of linear functional equations, their properties, solutions (including those in terms of generalized functions) and behaviour. Modern techniques of the ring theory, such as Gel'fand-Kirillov dimension and Ore localization as well as methods of homological algebra such as the purity filtration have been recently turned into algorithms with the help of computer algebra, where Gröbner bases play a prominent role. We will show the merits of these emerged techniques and discuss their applicability to concrete problems with the help of available implementations. In particular, applications of purity filtration of a module, modelling of polynomial signals and the computation of Bernstein-Sato polynomials of affine varieties will be discussed.

Prime fuzzy ideals over noncommutative rings

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The theory of Fuzzy sets was introduced in the pioneer work of Zadeh [1]. This construction has been a major tool for computational treatment on uncertainty in the last forty years, and it has produced many applications in a wide range of areas in Artificial Intelligence. Since the well-known paper of Rosenfeld [2] dealing with fuzzy sets of a group, many researchers have focused on giving an algebraic structure to the universe space, defining algebraic topics on a fuzzy environment and studying their properties. See references.

Focusing on rings, the early paper of Liu [9] defining fuzzy ideals initiated the investigation of rings by expanding the class of ideals with these fuzzy objects. Some years later, during the late eighties and nineties, many works of different authors were published in order to develop a Fuzzy Ring Theory. Most of these authors limit their attention to commutative rings or, even, fail to mention that this requirement is necessary. Nevertheless, the commutativity condition becomes too restrictive when we realize that noncommutative rings can be found in a wide range of knowledge areas as Particle Physics, Quantum Field Theory, Gauge Theory, Cryptography or Coding Theory.

In this work we study the notion of primeness on fuzzy ideals clarifying relationships between various definitions appearing in the literature, and we propose a new definition of primeness over arbitrary rings. It is generally accepted that the concept of fuzzy primeness considered in [5,6] is the most appropriate since, in commutative rings, it is equivalent to the level cuts being crisp prime ideals (when dealing with fuzzy sets, the word crisp is reserved to denote classical sets), although other notions of prime fuzzy ideal are studied in [15,11,7,4]. Nevertheless, when working over arbitrary rings, this is no longer valid. Our definition fills in this gap. It generalizes the one given in [5,6] and verifies the aforementioned property.

This work has appeared in [1].

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On Lie algebras of derivations of fields and commutative algebras

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Let $\mathbb{K}$ be an algebraically closed field of characteristic 0 and $\mathbb{K}(x_1, x_2)$ the field of rational functions in two variables. I. Dolgachev and V. Iskovskikh [1] described all finite subgroups of the group of $\mathbb{K}$-automorphisms of $\mathbb{K}(x_1, x_2)$. Let $\widetilde{W_2}(\mathbb{K})$ be the Lie algebra of all derivations of $\mathbb{K}(x_1, x_2)$. We obtain a description of all finite-dimensional subalgebras of $\widetilde{W_2}(\mathbb{K})$ up to isomorphism (as Lie algebras) using only algebraic tools (finite dimensional algebras of analytical vector fields in one real, one and two complex variables were classified by S. Lie, this classification was extended to real case in [2] and [3].

More generally, let $F \supset \mathbb{K}$ be a field extension, L be a two-dimensional F-subspace of the Lie algebra $Der_{\mathbb{K}}(F)$ closed under the Lie bracket. In this case, we obtain a description of all maximal nilpotent and maximal solvable subalgebras of L up to isomorphism.

Further, let R be a commutative $\mathbb{K}$-algebra, L be a submodule of the R-module $Der_{\mathbb{K}}(R)$ with n generators, closed under Lie multiplication (in particular, $L = W_n(\mathbb{K}) = Der(\mathbb{K}[x_1 ... x_n])$). We have proved that any nilpotent subalgebra of L has derived length $\leq n$.

The results come from the joint work with A. P. Petravchuk.

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Primary decomposition of polynomial ideals and its efficient implementation

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Recently we found a new algorithm for computing primary decomposition of polynomial ideals without producing any intermediate redundant component. In the new algorithm, the set of all associated primes are divided into disjoint subsets according to inclusion relation and these subsets can be successively extracted as the set of minimal associated primes of certain ideal quotients. Then primary components can be computed from the associated primes by using simple ideal operations.

In this talk, we give the details of the algorithm and its efficient implementation including parallelization.

Two Applications of Cylindrical Algebraic Decomposition

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Cylindrical Algebraic Decomposition (CAD) is a well-established tool in Symbolic Computation, but it is not yet so widely appreciated by other mathematical communities. Partly this is because of the high computational complexity of CAD-computations so that for many problems it is not sufficient to merely plug in the the given problem, but some reformulation may be necessary.

In this talk we report on two recent collaborations, where CAD entered in a non-trival way. In one case, we carried out a convergence analysis for a certain numerical method. In order to obtain exact bounds for the convergence rate, the supremum over the given quantities was determined symbolically using CAD. This is joint work with Stefan Takacs. In the other case, the dominance property of Sugeno-Weber t-norms was settled in joint work with Susanne Saminger-Platz and Manuel Kauers. This problem originates in the field of fuzzy logic. In either of these cases, the input was too large to be handled directly by CAD and we present how we circumvented these hurdles to prove the result despite of the high complexity.

Deformation cohomology of algebraic and geometric structures

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In 1953 the physicists E. Inonü and E.P. Wigner introduced the concept of *deformation of a Lie algebra* by considering the composition law for speed $(u, v) \rightarrow (u+v)/(1+(uv/c^2))$ existing in special relativity (Poincaré group) when c is the speed of light, claiming that the limit $1/c \rightarrow 0$ should produce the composition $(u, v) \rightarrow u + v$ used in classical mechanics (Galilée group). However, this result has no meaning independently of the choice of length and time units because the dimensionless law $(\bar{u} = u/c, \bar{v} = v/c) \rightarrow (\bar{u}, \bar{v}) \rightarrow (\bar{u} + \bar{v})/(1 + \bar{u}\bar{v})$ does not involve any parameter. Nevertheless, this idea brought the birth of the theory of *deformation of algebraic structures* culminating in the use of the Chevalley-Eilenberg cohomology of Lie algebras and one of the first applications of computer algebra for exhibiting counterexamples in dimension ≥ 8.

A few years later, a deformation theory was introduced for *structures on manifolds*, generally represented by *fields of geometric objects* like tensors and we may quote riemannian, symplectic or complex analytic structures with works by V. Guillemin, K. Kodaira, L. Nirenberg, D.C. Spencer or S. Sternberg. The idea is to make the underlying geometric objects depending on a parameter while satisfying prescribed integrability conditions. The link betwen the two approaches, though often conjectured, has never been exhibited by the above authors and our aim is to sketch the solution of this problem, in particular to study the possibility to use computer algebra for such a purpose. Accordingly, this work can be considered as a natural continuation of symbolic computations done at RWTH-Aachen university by M. Barakat and A. Lorenz in 2008.

The general solution involves new tools mixing differential geometry and homological algebra. However, the key argument is to acknowledge the fact that the *Vessiot structure equations* (1903, still unknown today) must be used in place of the *Cartan structure equations*, along the following diagram described in the reference below.

$$\begin{array}{ccccc} & & CARTAN & \longrightarrow & SPENCER \\ & \nearrow & & & \\ LIE & & \updownarrow & & \updownarrow \\ & \searrow & & & \\ & & VESSIOT & \longrightarrow & JANET \end{array}$$

In a word, one has to replace Lie algebras by Lie algebroids with a bracket now depending on the Spencer operator and use the corresponding canonical *linear Janet sequence* in order to induce a new sequence locally described by finite dimensional vector spaces and linear maps, *even for structures providing infinite dimensional Lie algebroids* (contact structure is a good example). The cohomology of this sequence only depends on the so-called "*structure constants*" appearing in the Vessiot structure equations (constant riemannian curvature is an example with only one constant having nothing to do with any Lie algebra). Finally, the simplest case of a principal homogeneous space for a Lie group (for example itself) gives back the Chevalley-Eilenberg cohomology.

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Module structure of rings of partial differential operators

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The purpose of this talk is to develop constructive versions of Stafford's theorems on the module structure of Weyl algebras $A_n(k)$ (i.e., the rings of partial differential operators with polynomial coefficients) over a base field of characteristic 0 [5] (see also [1]). Using the very simplicity of the ring $A_n(k)$, we show how to explicitly compute a unimodular element of a finitely generated left $A_n(k)$-module of rank at least 2. This result is used to constructively decompose any finitely generated left $A_n(k)$-module into a direct sum of a free left $A_n(k)$-module and a left $A_n(k)$-module of rank at most 1. If the latter is torsion-free, then we explicitly show that it is isomorphic to a left ideal of $A_n(k)$ which can be generated by 2 elements. Then, we give an algorithm which reduces the number of generators of a finitely presented left $A_n(k)$-module having a module of relations of rank at least 2 to 2. In particular, any finitely generated torsion left $A_n(k)$-module can always be generated by at most 2 generators and is the image of a projective ideal whose construction is explicitly given. Moreover, a non-torsion left $A_n(k)$-module of rank r, which is not free, can be generated by $r+1$ elements but no fewer. Similar results hold for right $A_n(k)$-modules which can easily be obtained from the above results. These results are implemented in the STAFFORD package [3], and their system-theoretical interpretations are given within an algebraic analysis (D-module) approach [1,3,4]. Finally, using a result due to Caro and Levcovitz [2], we show that the above results also hold for the ring of partial differential operators with either formal power series or locally convergent power series coefficients. For more details, see [4].

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Polynomial solutions and zero divisors of linear ordinary integro-differential operators

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This is a joint work with Alban Quadrat. We present ongoing work on algorithmic aspects of linear ordinary integro-differential operators with polynomial coefficients. Even though this algebra is not Noetherian, Bavula recently proved that it is coherent (i.e. syzygy modules of finitely generated left/right ideals are finitely generated). Computing syzygies is crucial for an algorithmic approach to systems of integro-differential equations with boundary conditions. As a first step, we have to find zero divisors, which is, in turn, related to polynomial solutions.

We discuss an algorithmic approach for computing polynomial solutions (kernel) and compatibility conditions (cokernel) for a general class of linear operators including integro-differential operators. With this data, a generating set for right zero divisors can be constructed. In the case of initial value problems, an involution on the algebra of integro-differential operators allows us to compute also left zero divisors, which can be interpreted as compatibility conditions. We illustrate our approach with some sample computations.

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Updates to DE solvers in Maple 16 with Applications in MapleSim

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We present some of the recent new features in Maple's symbolic ODE and PDE solvers and related functionality, e.g. the extended Physics package. Some differential equations that were previously intractable are now solved in Maple.

Also, we give a short demo on DAE simplification and solving in MapleSim, the multi-domain physical modeling and simulation tool based on Maple. Engineers can obtain all the equations for a block diagram and manipulate them in the Maple environment. For specific applications, we present some of the MapleSim API commands.

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Invariants and Time-reversibility in Polynomial Systems of ODE's

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Consider a dynamical system described by ODE's of the form

$$\frac{d\mathbf{z}}{dt} = F(\mathbf{z}),$$

where F is a vector field on a manifold. A time-reversible symmetry of the system is an invertible map R, such that

$$\frac{d(R(\mathbf{z}))}{dt} = -F(R(\mathbf{z})).$$

We first discuss the case of two-dimensional polynomial systems, that is, $\mathbf{z} = (x, y)$ and F is a polynomial vector-function. If the coefficients of F are parameters, then the action of the group SL(2,C) on $\mathbf{z}$ induces a transformation of the coefficients of the system. These transformations also form a group, which we denote by U. We call polynomial invariants of U the Sibirsky invariants of our system. In the talk we describe an efficient algorithm to compute a generating set of these invariants. Using methods of computational algebra we show an interconnection of the physically important phenomena of time-reversibility, involution and the Sibirsky invariants. Furthermore, we characterize the set of all time-reversible systems within a particular family of complex two-dimensional polynomial differential systems and give an efficient computational algorithm for finding this set.

We then discuss application of the invariants to studying limit cycles bifurcations in polynomial systems of ODE and a generalization to the case of higher dimensional systems.

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Computational Aspects of Center and Focus Problems on Center Manifolds

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Consider a singularity of a vector field (or equilibrium of a system of autonomous ordinary differential equations) in $\mathbb{R}^3$ at which the linear part has a pair of purely imaginary eigenvalues and a negative real eigenvalue. The asymptotic stability of the singularity of the full system is determined by the behavior of the system restricted to an invariant two-dimensional manifold, the center manifold, through the singularity. We discuss theoretical and practical aspects for determining the behavior of the system on the local center manifold when the vector field (or right hand sides of the differential equations) is composed of polynomials. In particular, we address the existence and computability of polynomial functions of the coefficients of the vector field whose vanishing and sign indicate the stability of the singularity of interest.

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ISOLDE — A Maple package for systems of linear functional equations

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The ISOLDE package contains algorithms for symbolically solving various classes of linear functional systems. In this presentation, we focus on the local analysis of systems of linear differential and difference equations. We illustrate the use of some reduction algorithms and sketch their link with the computation of a formal fundamental matrix solution at a regular or irregular singularity. The work was done in collaboration with Moulay Barkatou and Eckhard Pflügel.

The limit cycle bifurcation curve for the Bogdanov-Takens system

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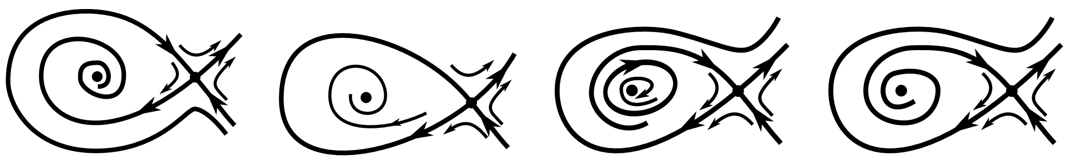
The Bogdanov-Takens' normal form

$$\begin{cases} x' = y, \\ y' = -n + by + x^2 + xy. \end{cases}$$

is a well-known quadratic differential system that exhibits a homoclinic connection for some values of the parameters. Fixed $n > 0$, it can be seen that a unique limit cycle emerges from the origin for $b = \sqrt{n}$, from a Hopf bifurcation, and it disappears on a homoclinic connection for a special value $b^*(n)$. Next figures shows this bifurcation phenomenon, for $n, b > 0$.



$b < b^*(n)$ $\qquad$ $b = b^*(n)$ $\qquad$ $b^*(n) < b < \sqrt{n}$ $\qquad$ $b \geq \sqrt{n}$

From [3] it can be seen that $b^*(n) = H(\sqrt{n})$, where H is an analytic function. In this work we will give a global description of the bifurcation curve $b = b^*(n)$.

This bifurcation curve, close to the origin, writes as

$$b^*(n) = \frac{5}{7}n^{1/2} + \frac{72}{2401}n - \frac{30024}{45294865}n^{3/2} - \frac{2352961656}{11108339166925}n^2 + O(n^{5/2}).$$

and for n big enough we get

$$b^*(n) = \sqrt{n} - 1 + O(n^{-1}).$$

Last expression proves the conjecture of Perko, [3], about how this function goes to infinity.

The explicit expression of function b^* can not be done but, for all $n > 0$, we can determine two functions $b_d(n)$ and $b_u(n)$ that give upper and lower global bounds for this bifurcation curve in the parameter space.

This is a joint work with A. Gasull, H. Giacomini and S. Pérez-González.

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Some Commutative and Non-commutative Problems

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We consider several problems where Gröbner basis methods could help but the approach is not straightforward.

How to describe a finite set of numbers? How to find a center in a non-commutative algebra? How to solve linear equations in such algebras?

How many homogeneous relations we need to get a finite dimensional algebra?

The difficulties and some approaches for these and other questions is the subject of this talk.

Invariant sets and integrating factors of planar polynomial vector fields

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The Darboux theory of integrability for planar polynomial differential equations is a classical field, with connections to Lie symmetries, differential algebra and other areas of mathematics. In the talk we present notions, problems and some answers. The focus will be on an overview of recent results on inverse problems, most of which were achieved jointly with C. Christopher (Plymouth), J. Llibre (Barcelona), and C. Pantazi (Barcelona). The inverse problem for curves (i.e., to determine all planar polynomial vector fields which admit a given algebraic curve as invariant set) can be solved by a straightforward application of (algorithmic) commutative algebra. Solving an inverse problem to determine all polynomial vector fields which admit a given Darboux integrating factor can be reduced, in principle, to algorithmic problems involving a Weyl algebra. We will discuss some examples where shortcuts work, and sketch the proof of a general finiteness theorem.

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Algebraic Stability and Bifurcation Analysis for Certain Dynamical Systems

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In this talk, we show how to analyze the qualitative behaviors, in particular stability and bifurcation, of certain dynamical systems using algebraic methods with symbolic computation. Our general approach is based on algebraic settings of some well-known stability criteria and bifurcation techniques, makes use of algebraic methods of parametric curve implicitization, real root isolation, symbolic integration, real solution classification, and quantifier elimination, and allows one to derive exact stability and bifurcation conditions. It has been successfully applied to the analysis of biological networks and flight dynamics modeled as autonomous systems of ordinary differential equations and closed-loop linear control systems with linear or nonlinear feedback. Several examples are given to illustrate our approach and its effectiveness. Some numerical stability results obtained previously are also confirmed.

Most of the results presented in this talk are published in the papers listed below.

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Semi-algebraic Methods for Computing and Excluding Oscillations of Parametric Dynamical Systems

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Investigating oscillations for parametric ordinary differential equations (ODEs) has many applications in science and engineering but is a very hard problem. Hopf bifurcations in general yield oscillations, and we will discuss methods to parametrically compute Hopf bifurcations (of rational vector fields) by reductions to quantifier elimination problems on the ordered field of the reals and hence to semi-algebraic problems. Using current software systems the problem of existence of Hopf bifurcations can be handled fully algorithmically in general for about 3 to 4 dimensional problems (in theory, the method will work for arbitrary dimensions). In the context of chemical reaction networks we will discuss possible variations of the method by computing the flux cone first that might allow the computation of Hopf bifurcations in much higher dimensions - and also allows to handle systems with fractional exponents.

For exlcuding oscillations many researchers presume that in the case of chemical reaction networks Hopf bifurcations are the mechanism associated with oscillations. However, in general the problem of excluding osciallations is more general than the one of excluding Hopf bifurcations. We review some recently developed criteria which give sufficient conditions to exclude oscillations by reducing them to problems on semi-algebraic sets (for polynomial or rational vector fields). We will give some examples and we will discuss possible future work in the form of problems to be solved. Some of these problems might be rather immediate to be solved, some others might pose major challenges.