

Low degree cohomology of integral Specht modules - Tables and graphs

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In the following table we have $\lambda \vdash n$ and $k = \text{rk}(S_{\mathbb{Z}}^{\lambda})$. In the column of "e.d." we list the nontrivial elementary divisors of $H^2(\Sigma_n, S_{\mathbb{Z}}^{\lambda})$ if existent. If $H^2(\Sigma_n, S_{\mathbb{Z}}^{\lambda})$ is trivial, that is, cyclic of order 1, the column has the entry 1.

The elementary divisors were determined in GAP as the elementary divisors of the Zassenhaus matrix Z_{λ} corresponding to the presentation G_n and the representation given by the matrices from the library `spechtmats.g`. In some cases the elementary divisors could not be determined explicitly, but only their prime divisors. In these cases, the entries are set in brackets.

The partitions are ordered lexicographically for every n . If one or more partitions are skipped, this is marked by three dots.

n	λ	k	$e.d.$
2	(1^2)	1	1
	(2)	1	2
3	(1^3)	1	3
	$(2, 1)$	2	1
	(3)	1	2
4	(1^4)	1	3
	$(2, 1^2)$	3	2
	(2^2)	2	2
	$(3, 1)$	3	2
	(4)	1	2
5	(1^5)	1	1
	$(2, 1^3)$	4	3
	$(2^2, 1)$	5	4
	$(3, 1^2)$	6	10
	$(3, 2)$	5	2
	$(4, 1)$	4	1
	(5)	1	2
6	(1^6)	1	1
	$(2, 1^4)$	5	3
	$(2^2, 1^2)$	9	4
	(2^3)	5	1

n	λ	k	$e.d.$
6	$(3, 1^3)$	10	2
	$(3, 2, 1)$	16	5
	(3^2)	5	6
	$(4, 1^2)$	10	3
	$(4, 2)$	9	2
	$(5, 1)$	5	2
	(6)	1	2
7	(1^7)	1	1
	$(2, 1^5)$	6	1
	$(2^2, 1^3)$	14	6
	$(2^3, 1)$	14	1
	$(3, 1^4)$	15	2
	$(3, 2, 1^2)$	35	2
	$(3, 2^2)$	21	1
	$(3^2, 1)$	21	10
	$(4, 1^3)$	20	3
	$(4, 2, 1)$	35	2
	$(4, 3)$	14	3
	$(5, 1^2)$	15	14
	$(5, 2)$	14	2
	$(6, 1)$	6	1
	(7)	1	2

n	λ	k	$e.d.$
8	(1^8)	1	1
	$(2, 1^6)$	7	1
	$(2^2, 1^4)$	20	6
	$(2^3, 1^2)$	28	1
	(2^4)	14	1
	$(3, 1^5)$	21	2
	$(3, 2, 1^3)$	64	1
	$(3, 2^2, 1)$	70	2
	$(3^2, 1^2)$	56	2, 10
	$(3^2, 2)$	42	2
	$(4, 1^4)$	35	1
	$(4, 2, 1^2)$	90	2
	$(4, 2^2)$	56	2
	$(4, 3, 1)$	70	3
	(4^2)	14	2
	$(5, 1^3)$	35	6
	$(5, 2, 1)$	64	7
	$(5, 3)$	28	2
	$(6, 1^2)$	21	4
	$(6, 2)$	20	2
	$(7, 1)$	7	2
	(8)	1	2
9	(1^9)	1	1
	$(2, 1^7)$	8	1
	$(2^2, 1^5)$	27	2
	$(2^3, 1^3)$	48	3
	$(2^4, 1)$	42	1
	$(3, 1^6)$	28	2
	$(3, 2, 1^4)$	105	1
	$(3, 2^2, 1^2)$	162	2
	$(3, 2^3)$	84	1
	$(3^2, 1^3)$	120	2, 2
	$(3^2, 2, 1)$	168	1
	(3^3)	42	2
	$(4, 1^5)$	56	1
	$(4, 2, 1^3)$	189	2
	$(4, 2^2, 1)$	216	1
	$(4, 3, 1^2)$	216	5
	$(4, 3, 2)$	168	3
	$(4^2, 1)$	84	1
	$(5, 1^4)$	70	6
	$(5, 2, 1^2)$	189	1
	$(5, 2^2)$	120	2
	$(5, 3, 1)$	162	14
	$(5, 4)$	42	2
	$(6, 1^3)$	56	1
	$(6, 2, 1)$	105	8
	$(6, 3)$	48	3
	$(7, 1^2)$	28	18
	$(7, 2)$	27	2

n	λ	k	$e.d.$
9	$(8, 1)$	8	1
	(9)	1	2
10	(1^{10})	1	1
	$(2, 1^8)$	9	1
	$(2^2, 1^6)$	35	2
	$(2^3, 1^4)$	75	3
	$(2^4, 1^2)$	90	1
	(2^5)	42	1
	$(3, 1^7)$	36	2
	$(3, 2, 1^5)$	160	1
	$(3, 2^2, 1^3)$	315	2
	$(3, 2^3, 1)$	288	1
	$(3^2, 1^4)$	225	2, 2
	$(3^2, 2, 1^2)$	450	2
	$(3^2, 2^2)$	252	1
	$(3^3, 1)$	210	2
	$(4, 1^6)$	84	1
	$(4, 2, 1^4)$	350	2
	$(4, 2^2, 1^2)$	567	1
	$(4, 2^3)$	300	1
	$(4, 3, 1^3)$	525	1
	$(4, 3, 2, 1)$	768	15
	$(4, 3^2)$	210	3
	$(4^2, 1^2)$	300	1
	$(4^2, 2)$	252	1
	$(5, 1^5)$	126	2
	$(5, 2, 1^3)$	448	3
	$(5, 2^2, 1)$	525	1
	$(5, 3, 1^2)$	567	2
	$(5, 3, 2)$	450	2, 2
	$(5, 4, 1)$	288	7
	(5^2)	42	10
	$(6, 1^4)$	126	1
	$(6, 2, 1^2)$	350	4
	$(6, 2^2)$	225	2
	$(6, 3, 1)$	315	2
	$(6, 4)$	90	2
	$(7, 1^3)$	84	6
	$(7, 2, 1)$	160	3
	$(7, 3)$	75	6
	$(8, 1^2)$	36	5
	$(8, 2)$	35	2
	$(9, 1)$	9	2
	(10)	1	2
11	(1^{11})	1	1
	$(2, 1^9)$	10	1
	$(2^2, 1^7)$	44	2
	$(2^3, 1^5)$	110	1
	$(2^4, 1^3)$	165	3
	$(2^5, 1)$	132	1

n	λ	k	$e.d.$
11	$(3, 1^8)$	45	2
	$(3, 2, 1^6)$	231	1
	$(3, 2^2, 1^4)$	550	2
	$(3, 2^3, 1^2)$	693	1
	$(3, 2^4)$	330	1
	$(3^2, 1^5)$	385	2, 2
	$(3^2, 2, 1^3)$	990	1
	$(3^2, 2^2, 1)$	990	2
	$(3^3, 1^2)$	660	2, 2
	$(3^3, 2)$	462	2
	$(4, 1^7)$	120	1
	$(4, 2, 1^5)$	594	2
	$(4, 2^2, 1^3)$	1232	1
	$(4, 2^3, 1)$	1155	1
	$(4, 3, 1^4)$	1100	1
	$(4, 3, 2, 1^2)$	2310	1
	$(4, 3, 2^2)$	1320	3
	$(4, 3^2, 1)$	1188	5
	$(4^2, 1^3)$	825	1
	$(4^2, 2, 1)$	1320	1
	$(4^2, 3)$	462	1
	$(5, 1^6)$	210	2
	$(5, 2, 1^4)$	924	3
	$(5, 2^2, 1^2)$	1540	1
	$(5, 2^3)$	825	1
	$(5, 3, 1^3)$	1540	2
	$(5, 3, 2, 1)$	2310	1
	$(5, 3^2)$	660	2, 6
	$(5, 4, 1^2)$	1155	1
	$(5, 4, 2)$	990	2
	$(5^2, 1)$	330	14
	$(6, 1^5)$	252	1
	$(6, 2, 1^3)$	924	2
	$(6, 2^2, 1)$	1100	1
	$(6, 3, 1^2)$	1232	1
	$(6, 3, 2)$	990	1
	$(6, 4, 1)$	693	4
	$(6, 5)$	132	5
	$(7, 1^4)$	210	2
	$(7, 2, 1^2)$	594	2
	$(7, 2^2)$	385	2
	$(7, 3, 1)$	550	2, 18
	$(7, 4)$	165	2
	$(8, 1^3)$	120	3
	$(8, 2, 1)$	231	10
	$(8, 3)$	110	1
	$(9, 1^2)$	45	22
	$(9, 2)$	44	2
	$(10, 1)$	10	1
	(11)	1	2

n	λ	k	$e.d.$
12	(1^{12})	1	1
	$(2, 1^{10})$	11	1
	$(2^2, 1^8)$	54	2
	$(2^3, 1^6)$	154	1
	$(2^4, 1^4)$	275	3
	$(2^5, 1^2)$	297	1
	(2^6)	132	1
	$(3, 1^9)$	55	2
	$(3, 2, 1^7)$	320	1
	$(3, 2^2, 1^5)$	891	2
	$(3, 2^3, 1^3)$	1408	1
	$(3, 2^4, 1)$	1155	1
	$(3^2, 1^6)$	616	2, 2
	$(3^2, 2, 1^4)$	1925	1
	$(3^2, 2^2, 1^2)$	2673	2
	$(3^2, 2^3)$	1320	1
	$(3^3, 1^3)$	1650	2, 2
	$(3^3, 2, 1)$	2112	1
	(3^4)	462	2
	$(4, 1^8)$	165	1
	$(4, 2, 1^6)$	945	2
	$(4, 2^2, 1^4)$	2376	1
	$(4, 2^3, 1^2)$	3080	1
	$(4, 2^4)$	1485	1
	$(4, 3, 1^5)$	2079	1
	$(4, 3, 2, 1^3)$	5632	?
	$(4, 3, 2^2, 1)$	5775	?
	$(4, 3^2, 1^2)$	4158	?
	$(4, 3^2, 2)$	2970	1
	$(4^2, 1^4)$	1925	1
	$(4^2, 2, 1^2)$	4455	?
	$(4^2, 2^2)$	2640	1
	$(4^2, 3, 1)$	2970	1
	(4^3)	462	1
	$(5, 1^7)$	330	2
	$(5, 2, 1^5)$	1728	1
	$(5, 2^2, 1^3)$	3696	(3)
	$(5, 2^3, 1)$	3520	1
	$(5, 3, 1^4)$	3564	(2)
	$(5, 3, 2, 1^2)$	7700	?
	$(5, 3, 2^2)$	4455	?
	$(5, 3^2, 1)$	4185	?
	$(5, 4, 1^3)$	3520	1
	$(5, 4, 2, 1)$	5775	?
	$(5, 4, 3)$	2112	3
	$(5^2, 1^2)$	1485	14
	$(5^2, 2)$	1320	2
	$(6, 1^6)$	462	1
	$(6, 2, 1^4)$	2100	2
	$(6, 2^2, 1^2)$	3564	1

n	λ	k	$e.d.$
12	$(6, 2^3)$	1925	1
	$(6, 3, 1^3)$	3696	1
	$(6, 3, 2, 1)$	5632	?
	$(6, 3^2)$	1650	1
	$(6, 4, 1^2)$	3080	2
	$(6, 4, 2)$	2673	4
	$(6, 5, 1)$	1155	2
	(6^2)	132	1
	$(7, 1^5)$	462	2
	$(7, 2, 1^3)$	1728	1
	$(7, 2^2, 1)$	2079	4
	$(7, 3, 1^2)$	2376	2, 2
	$(7, 3, 2)$	1925	6
	$(7, 4, 1)$	1408	3
	$(7, 5)$	297	10
	$(8, 1^4)$	330	3
	$(8, 2, 1^2)$	945	2
	$(8, 2^2)$	616	1
	$(8, 3, 1)$	891	5
	$(8, 4)$	275	2
	$(9, 1^3)$	165	2
	$(9, 2, 1)$	320	11
	$(9, 3)$	154	6
	$(10, 1^2)$	55	6
	$(10, 2)$	54	2
	$(11, 1)$	11	2
	(12)	1	2
13	(1^{13})	1	1
	$(2, 1^{11})$	12	1
	$(2^2, 1^9)$	65	2
	$(2^3, 1^7)$	208	1
	$(2^4, 1^5)$	429	1
	$(2^5, 1^3)$	572	3
	$(2^6, 1)$	429	1
	$(3, 1^{10})$	66	2
	$(3, 2, 1^8)$	429	1
	$(3, 2^2, 1^6)$	1365	2
	$(3, 2^3, 1^4)$	2574	1
	$(3, 2^4, 1^2)$	2860	1
	$(3, 2^5)$	1287	1
	$(3^2, 1^7)$	936	2, 2
	$(3^2, 2, 1^5)$	3432	1
	...		
	$(3^3, 1^4)$	3575	(2)
	...		
	$(3^3, 2^2)$	3432	1
	...		
	$(3^4, 1)$	2574	2
	$(4, 1^9)$	220	1
	$(4, 2, 1^7)$	1430	2
	...		

n	λ	k	$e.d.$
13	$(4, 3, 1^6)$	3640	1
	...		
	$(4, 3^3)$	3432	1
	...		
	$(4^3, 1)$	3432	1
	$(5, 1^8)$	495	2
	$(5, 2, 1^6)$	3003	1
	...		
	$(5, 4^2)$	2574	1
	...		
	$(5^2, 3)$	3432	(2, 3)
	$(6, 1^7)$	792	1
	...		
	$(6^2, 1)$	1287	2
	$(7, 1^6)$	924	2
	...		
	$(7, 3^2)$	3575	(2, 3)
	...		
	$(7, 5, 1)$	2860	2, 6
	$(7, 6)$	429	2
	$(8, 1^5)$	792	1
	$(8, 2, 1^3)$	3003	6
	$(8, 2^2, 1)$	3640	1
	...		
	$(8, 3, 2)$	3432	1
	...		
	$(8, 5)$	572	5
	$(9, 1^4)$	495	2
	$(9, 2, 1^2)$	1430	1
	$(9, 2^2)$	936	1
	$(9, 3, 1)$	1365	22
	$(9, 4)$	429	2, 2
	$(10, 1^3)$	220	3
	$(10, 2, 1)$	429	4
	$(10, 3)$	208	3
	$(11, 1^2)$	66	26
	$(11, 2)$	65	2
	$(12, 1)$	12	1
	(13)	1	2
14	(1^{14})	1	1
	$(2, 1^{12})$	13	1
	$(2^2, 1^{10})$	77	2
	$(2^3, 1^8)$	273	1
	$(2^4, 1^6)$	637	1
	$(2^5, 1^4)$	1001	3
	$(2^6, 1^2)$	1001	1
	(2^7)	429	1
	$(3, 1^{11})$	78	2
	$(3, 2, 1^9)$	560	1
	$(3, 2^2, 1^7)$	2002	2
	...		

n	λ	k	$e.d.$
14	$(3^2, 1^8)$	1365	2, 2
	...		
	$(4, 1^{10})$	286	1
	$(4, 2, 1^8)$	2079	2
	...		
	$(5, 1^9)$	715	2
	...		
	$(6, 1^8)$	1287	1
	...		
	$(7, 1^7)$	1716	2
	...		
	(7^2)	429	14
	$(8, 1^6)$	1716	1
	...		
	$(8, 6)$	1001	1
	$(9, 1^5)$	1287	2
	...		
	$(9, 5)$	1001	2, 2
	$(10, 1^4)$	715	1
	$(10, 2, 1^2)$	2079	4
	$(10, 2^2)$	1365	1
	$(10, 3, 1)$	2002	3
	$(10, 4)$	637	2
	$(11, 1^3)$	286	6
	$(11, 2, 1)$	560	13
	$(11, 3)$	273	2
	$(12, 1^2)$	78	7
	$(12, 2)$	77	2
	$(13, 1)$	13	2
	(14)	1	2
15	(1^{15})	1	1
	$(2, 1^{13})$	14	1
	$(2^2, 1^{11})$	90	2
	$(2^3, 1^9)$	350	1
	$(2^4, 1^7)$	910	1
	$(2^5, 1^5)$	1638	1
	$(2^6, 1^3)$	2002	3
	$(2^7, 1)$	1430	1
	$(3, 1^{12})$	91	2
	$(3, 2, 1^{10})$	715	1
	$(3, 2^2, 1^8)$	2835	2
	...		
	$(3^2, 1^9)$	1925	2, 2
	...		
	$(4, 1^{11})$	364	1
	$(4, 2, 1^9)$	2925	2
	...		
	$(5, 1^{10})$	1001	2
	...		
	$(6, 1^9)$	2002	1
	...		

n	λ	k	$e.d.$
15	$(7, 1^8)$	3003	2
	...		
	$(8, 1^7)$	3432	1
	...		
	$(8, 7)$	1430	7
	$(9, 1^6)$	3003	2
	...		
	$(9, 6)$	2002	2
	$(10, 1^5)$	2002	1
	...		
	$(10, 5)$	1638	5
	$(11, 1^4)$	1001	6
	$(11, 2, 1^2)$	2925	2
	$(11, 2^2)$	1925	1
	$(11, 3, 1)$	2835	26
	$(11, 4)$	910	1
	$(12, 1^3)$	364	1
	$(12, 2, 1)$	715	14
	$(12, 3)$	350	3
	$(13, 1^2)$	91	30
	$(13, 2)$	90	2
	$(14, 1)$	14	1
	(15)	1	2
16	(1^{16})	1	1
	$(2, 1^{14})$	15	1
	$(2^2, 1^{12})$	104	2
	$(2^3, 1^{10})$	440	1
	$(2^4, 1^8)$	1260	1
	$(2^5, 1^6)$	2548	1
	...		
	$(2^7, 1^2)$	3432	1
	(2^8)	1430	1
	$(3, 1^{13})$	105	2
	$(3, 2, 1^{11})$	896	1
	...		
	$(3^2, 1^{10})$	2640	2, 2
	...		
	$(4, 1^{12})$	455	1
	...		
	$(5, 1^{11})$	1365	2
	...		
	$(6, 1^{10})$	3003	1
	...		
	(8^2)	1430	2
	...		
	$(9, 7)$	3432	(2, 7)
	...		
	$(11, 1^5)$	3003	2
	...		
	$(11, 5)$	2548	1

n	λ	k	$e.d.$
16	$(12, 1^4)$	1365	1
	...		
	$(12, 2^2)$	2640	2
	...		
	$(12, 4)$	1260	2
	$(13, 1^3)$	455	6
	$(13, 2, 1)$	896	5
	$(13, 3)$	440	6
	$(14, 1^2)$	105	8
	$(14, 2)$	104	2
	$(15, 1)$	15	2
	(16)	1	2
17	(1^{17})	1	1
	$(2, 1^{15})$	16	1
	$(2^2, 1^{13})$	119	2
	$(2^3, 1^{11})$	544	1
	$(2^4, 1^9)$	1700	1
	...		
	$(3, 1^{14})$	120	2
	$(3, 2, 1^{12})$	1105	1
	...		
	$(4, 1^{13})$	560	1
	...		
	$(5, 1^{12})$	1820	2
	...		
	$(13, 1^4)$	1820	2
	...		
	$(13, 4)$	1700	2
	$(14, 1^3)$	560	3
	$(14, 2, 1)$	1105	16
	$(14, 3)$	544	1
	$(15, 1^2)$	120	34
	$(15, 2)$	119	2
	$(16, 1)$	16	1
	(17)	1	2
18	(1^{18})	1	1
	$(2, 1^{16})$	17	1
	$(2^2, 1^{14})$	135	2
	$(2^3, 1^{12})$	663	1
	$(2^4, 1^{10})$	2244	1
	...		
	$(3, 1^{15})$	136	2
	$(3, 2, 1^{13})$	1344	1
	...		
	$(4, 1^{14})$	680	1
	...		
	$(5, 1^{13})$	2380	2
	...		

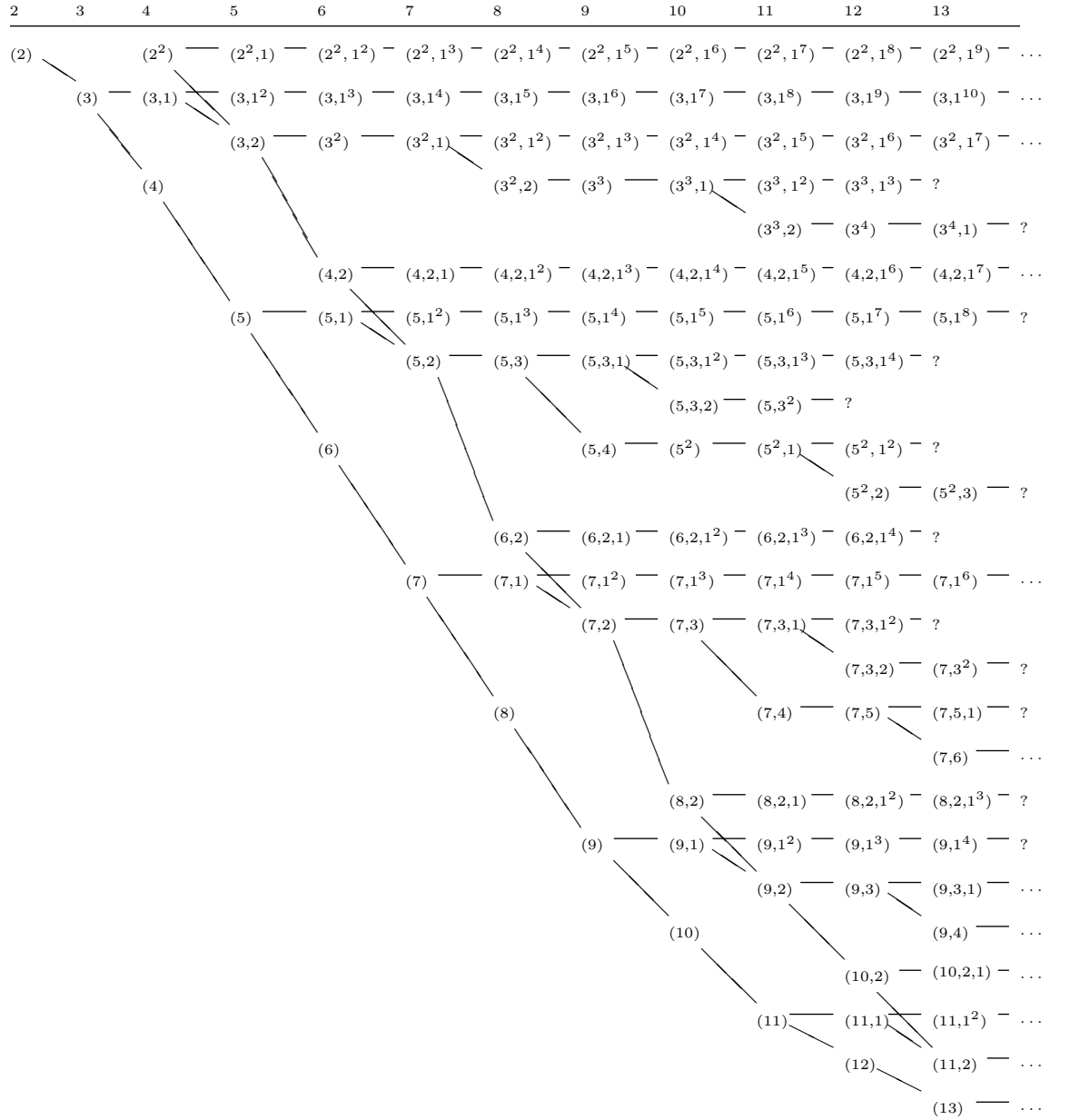
n	λ	k	$e.d.$
18	$(14, 1^4)$	2380	3
	...		
	$(14, 4)$	2244	2
	$(15, 1^3)$	680	2
	$(15, 2, 1)$	1344	17
	$(15, 3)$	663	6
	$(16, 1^2)$	136	9
	$(16, 2)$	135	2
	$(17, 1)$	17	2
	(18)	1	2
19	(1^{19})	1	1
	$(2, 1^{17})$	18	1
	$(2^2, 1^{15})$	152	2
	$(2^3, 1^{13})$	798	1
	$(2^4, 1^{11})$	2907	1
	...		
	$(3, 1^{16})$	153	2
	$(3, 2, 1^{14})$	1615	1
	...		
	$(4, 1^{15})$	816	1
	...		
	$(15, 4)$	2907	2
	$(16, 1^3)$	816	3
	$(16, 2, 1)$	1615	6
	$(16, 3)$	798	3
	$(17, 1^2)$	153	38
	$(17, 2)$	152	2
	$(18, 1)$	18	1
	(19)	1	2
20	(1^{20})	1	1
	$(2, 1^{18})$	19	1
	$(2^2, 1^{16})$	170	2
	$(2^3, 1^{14})$	950	1
	...		
	$(3, 1^{17})$	171	2
	$(3, 2, 1^{15})$	1920	1
	...		
	$(4, 1^{16})$	969	1
	...		
	$(17, 1^3)$	969	6
	$(17, 2, 1)$	1920	19
	$(17, 3)$	950	2
	$(18, 1^2)$	171	10
	$(18, 2)$	170	2
	$(19, 1)$	19	2
	(20)	1	2

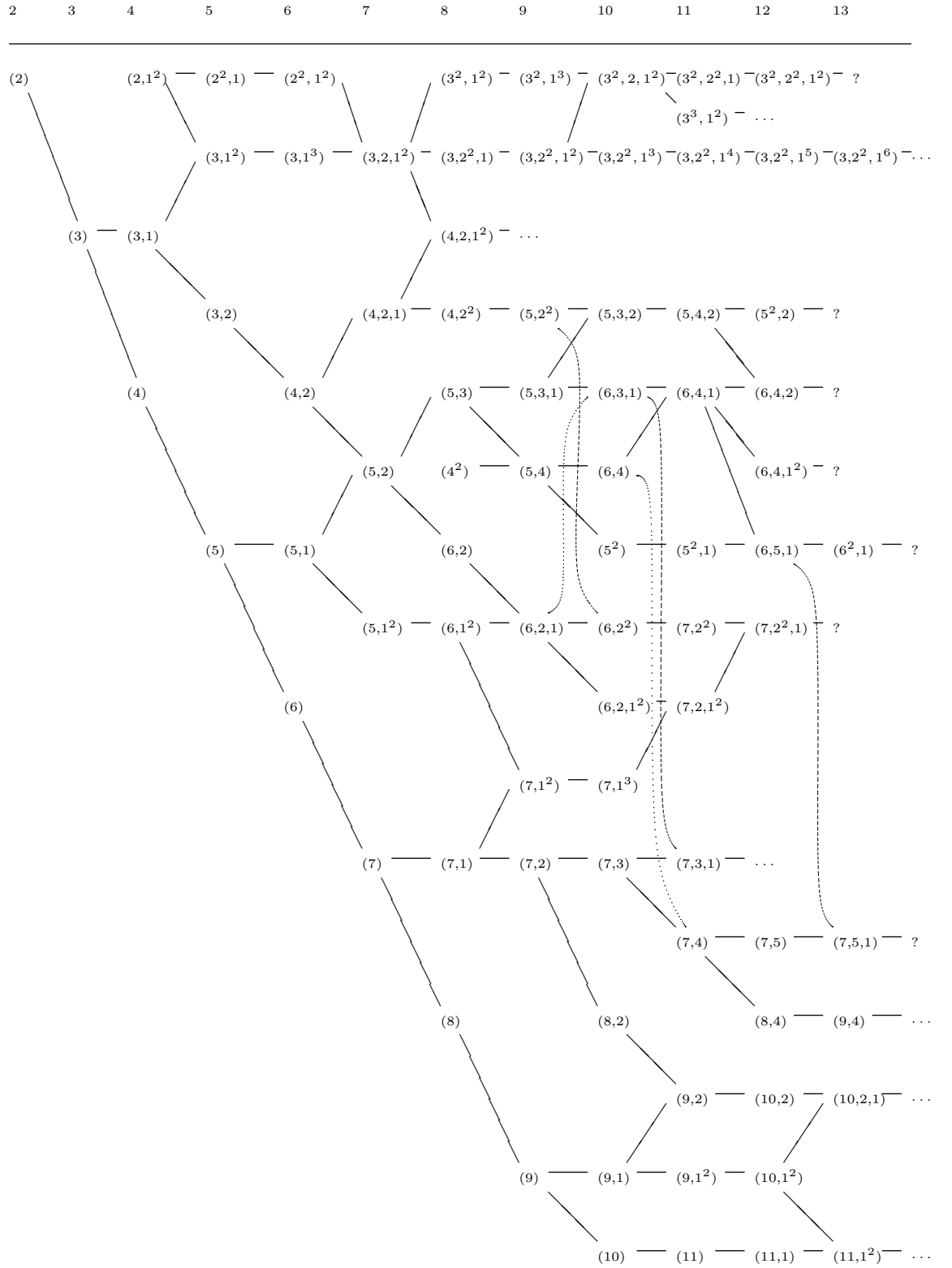
Graphs of type $\mathcal{C}_p^2$

On the basis of the previous tables, we order partitions in graphs of type $\mathcal{C}_p^2$. If for some vertex no successor is computed, this is marked by a question mark. If successors are known but not listed, this is marked by three dots. The edges are not marked by arrows, because the direction is clear.

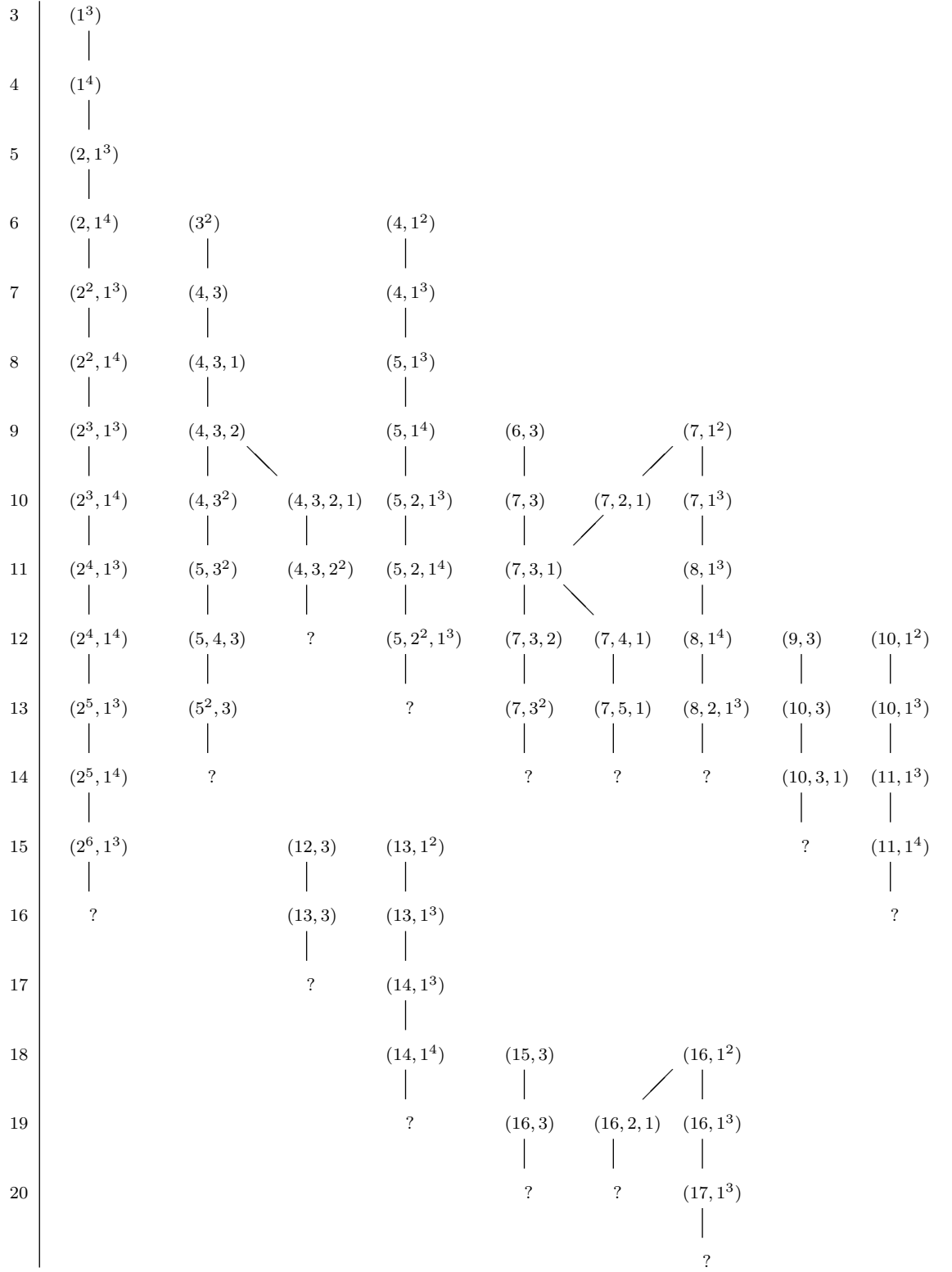
p = 2 :

For purposes of clarity, $\mathcal{C}_2^2$ for $n \leq 13$ is split up into two partial graphs. The first one contains the partitions related to Conjecture 6.9 and those of type $(\lambda_1, \dots, \lambda_m)$ with $\lambda_1, \dots, \lambda_{m-1}$ odd and $\lambda_m \neq 1$. The second one contains the rest, together with some connections to the first. All in all, every known edge of $\mathcal{C}_2^2$ for $n \leq 13$ can be found in at least one of the two partial graphs.





p = 3 :



p = 5 :

5	(3, 1 ²)		
6	(3, 2, 1)		
7	(3 ² , 1)		
8	(3 ² , 1 ²)		
9	(4, 3, 1 ²)		
10	(4, 3, 2, 1)	(5 ²)	(8, 1 ²)
11	(4, 3 ² , 1)	(6, 5)	(8, 2, 1)
12	?	(7, 5)	(8, 3, 1)
13		(8, 5)	?
14		?	
15			(10, 5) (13, 1 ²)
16			? (13, 2, 1)
			?

p = 7 :

7	(5, 1 ²)		
8	(5, 2, 1)		
9	(5, 3, 1)		
10	(5, 4, 1)		
11	(5 ² , 1)		
12	(5 ² , 1 ²)		
13	?		
14		(7 ²)	(12, 1 ²)
15		(8, 7)	(12, 2, 1)
16		(9, 7)	?
		?	