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                     SINGULAR                                 /
 A Computer Algebra System for Polynomial Computations       /   version 3-1-5
                                                           0<
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FB Mathematik der Universitaet, D-67653 Kaiserslautern        \
> // --- Examples: declaration of coefficient rings
;
poly f = 0;
   ? no ring active
   ? `poly` is undefined
   ? error occurred in or before STDIN line 3: `poly f = 0;`
> ring r1 = 0,x,dp;
poly f = (x-1)*(x+1);
> f;
x2-1
> ring r2 = (0,a,b),(x,y,z),dp;
> r2;
//   characteristic : 0
//   2 parameter    : a b
//   minpoly        : 0
//   number of vars : 3
//        block   1 : ordering dp
//                  : names    x y z
//        block   2 : ordering C
> ring r3 = 32003,(x(1..10)),dp;
> r3;
//   characteristic : 32003
//   number of vars : 10
//        block   1 : ordering dp
//                  : names    x(1) x(2) x(3) x(4) x(5) x(6) x(7) x(8) x(9) x(10)
//        block   2 : ordering C
> ring r4 = (0,i),(x,y,z),dp;
> minpoly = i^2+1;
> r4;
//   characteristic : 0
//   1 parameter    : i
//   minpoly        : (i2+1)
//   number of vars : 3
//        block   1 : ordering dp
//                  : names    x y z
//        block   2 : ordering C
> i^2;
-1
> ring r5a = (7,a),(x,y,z),dp; minpoly = a^2+a+3;
> // a is an algebraic element over Z/7 with minimal polynomial a^2+a+3
. r5a;
//   characteristic : 7
//   1 parameter    : a
//   minpoly        : (a2+a+3)
//   number of vars : 3
//        block   1 : ordering dp
//                  : names    x y z
//        block   2 : ordering C
> ring r5b = (7^2,b),(x,y,z),dp;
> // b is some generator of the cyclic group of units of GF(49)
. r5b;
//   # ground field : 49
//   primitive element : b
//   minpoly        : 1*b^2+6*b^1+3*b^0
//   number of vars : 3
//        block   1 : ordering dp
//                  : names    x y z
//        block   2 : ordering C
> ring r6 = (real,50),(x,y,z),dp;
> r6;
//   characteristic : 0 (real:50 digits, additional 50 digits)
//   number of vars : 3
//        block   1 : ordering dp
//                  : names    x y z
//        block   2 : ordering C
> ring r7 = (complex,10,i),(x,y,z),dp;
> r7;
//   characteristic : 0 (complex:10 digits, additional 10 digits)
//   1 parameter    : i
//   minpoly        : (i^2+1)
//   number of vars : 3
//        block   1 : ordering dp
//                  : names    x y z
//        block   2 : ordering C
> ring r8 = integer,(x,y,z), dp;
> r8;
//   coeff. ring is : Integers
//   number of vars : 3
//        block   1 : ordering dp
//                  : names    x y z
```

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//        block   2 : ordering C
> ring r9 = (integer,100),(x,y,z), dp;
> r9;
//   coeff. ring is : Z/100
//   number of vars : 3
//        block   1 : ordering dp
//                  : names    x y z
//        block   2 : ordering C
> -42;
-42
> number(-42);
58
> 1/42;
   ? '1/42' is undefined
   ? error occurred in or before STDIN line 31: '1/42;'
> number(1)/number(42);
   ? Division not possible, even by cancelling zero divisors.
   ? error occurred in or before STDIN line 32: 'number(1)/number(42);'
> 1/41;
   ? '1/41' is undefined
   ? error occurred in or before STDIN line 33: '1/41;'
> number(1)/number(41);
61
> ring r10 = (integer,6,3),(x,y,z), dp;
> r10;
//   coeff. ring is : Z/6^3
//   number of vars : 3
//        block   1 : ordering dp
//                  : names    x y z
//        block   2 : ordering C
> ring r11 = (integer,6^3),(x,y,z), dp;
> r11;
//   coeff. ring is : Z/216
//   number of vars : 3
//        block   1 : ordering dp
//                  : names    x y z
//        block   2 : ordering C
> // ---
;
>
// --- Examples: Hilbert series
;
> ring RH = 0,(x,y,z,t),dp;
> ideal I = x^31-x^6-x-y, x^8-z, x^10-t;
> hilb(I);
// ** I is no standard basis
//         1 t^0
//        -1 t^8

//         1 t^0
//         1 t^1
//         1 t^2
//         1 t^3
//         1 t^4
//         1 t^5
//         1 t^6
//         1 t^7
// dimension (proj.)  = 2
// degree (proj.)   = 8
> I = std(I);
> hilb(I);
//         1 t^0
//        -1 t^3
//        -1 t^4
//       -10 t^5
//        17 t^6
//        -2 t^7
//        -5 t^8
//         1 t^9

//         1 t^0
//         3 t^1
//         6 t^2
//         9 t^3
//        11 t^4
//         2 t^5
//        -1 t^6
// dimension (proj.)  = 0
// degree (proj.)   = 31
> hilb(I,1);  // WARNING: last entry is not part of Hilbert series
1,0,0,-1,-1,-10,17,-2,-5,1,0
> hilb(I,2);  // WARNING: last entry is not part of Hilbert series
1,3,6,9,11,2,-1,0
> LIB "poly.lib";
// ** loaded /usr/share/Singular/LIB/poly.lib (14852,2012-04-30)
```

```
// ** loaded /usr/share/Singular/LIB/ring.lib (15100,2012-07-10)
// ** loaded /usr/share/Singular/LIB/primdec.lib (14732,2012-03-30)
// ** loaded /usr/share/Singular/LIB/absfact.lib (14191,2011-05-04)
// ** loaded /usr/share/Singular/LIB/triang.lib (13499,2010-10-15)
// ** loaded /usr/share/Singular/LIB/matrix.lib (13658,2010-11-16)
// ** loaded /usr/share/Singular/LIB/nctools.lib (14246,2011-05-26)
// ** loaded /usr/share/Singular/LIB/random.lib (14661,2012-03-05)
// ** loaded /usr/share/Singular/LIB/elim.lib (14661,2012-03-05)
// ** loaded /usr/share/Singular/LIB/inout.lib (13499,2010-10-15)
// ** loaded /usr/share/Singular/LIB/general.lib (14191,2011-05-04)
> hilbPoly(I);
31
> // ---
;
>
// --- Examples: multiplicities
;
> // What is the multiplicity of K[x,y,z] / <x2-y3, x+y+z-1>?
;
> ring RM = 0,(x,y,z),dp;
> ideal I = x2-y3, x+y+z-1;
> mult(std(I));
3
> // What is the multiplicity of K[x,y,z]_<x,y,z> / <x2-y3, x+y+z-1>?
;
> ring RM2 = 0,(x,y,z),ds;
> ideal I = imap(RM,I);
> mult(std(I));
0
> // What is the multiplicity of K[x,y,z]_<x,y,z-1> / <x2-y3, x+y+z-1>?
;
> mult(std(subst(I,z,z+1)));
2
> // What is the multiplicity of K[x,y,z]_<x+1,y-1,z-1> / <x2-y3, x+y+z-1>?
;
> mult(std(subst(I,x,x-1,y,y+1,z,z+1)));
1
>
```