The computation of the 3-modular characters of the Fischer Group Fi_{23} *

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Abstract

We determine the 35 irreducible 3-modular characters of the Fischer group Fi_{23}. This completes the calculation of all modular character tables of this group.

1 Introduction

We complete the determination of the modular character tables of Fischer's second sporadic simple group Fi_{23} by constructing the 3-modular character table using computational methods. For the other modular character tables of Fi_{23} see [Hiss and Lux, 1989],[Hiss and Lux, 1994], and [Hiss et al., 2006]. Our result is a contribution to the overall goal of computing the modular character tables of all sporadic simple groups and more generally the modular character tables of the groups given in the ATLAS, [Wilson et al., 2016], see also [Breuer et al., 2016].

For the convenience of the reader we summarize the major steps in the proof:

1) It follows from the ordinary character table of Fi_{23} that there are three blocks of Fi_{23} in characteristic 3. All but the principal block are dealt with easily, and so we can focus mainly on the principal block and its 32 modular irreducible characters. The methods we apply are a combination of computing with modular characters and of computing with explicit matrix representations.

2) A basic approach to finding all the modular irreducible characters consists of constructing all the irreducible representations (in the principal block) of Fi_{23} by determining the composition factors of tensor products of irreducible representations of Fi_{23} recursively. More precisely, we take tensor products of known nontrivial irreducible representations, determine the composition factors, use these composition factors to form new tensor products etc. until all 32 irreducible representations have been found and their modular characters have been determined.

   By the straightforward generalization of the Burnside-Brauer theorem to modular characters, see [Isaacs, 1994, page 59] and [Lux and Pahlings, 2010, Exercise 4.3.4 page 320] this approach is guaranteed to succeed. However, computationally, taking tensor products works well as long as the representations are not too big for being analyzed by the MeatAxe,

*Dedicated to Professor Bernd Fischer on the occasion of his 80th birthday see [Parker, 1984] and [Ringe, 1994]. So, when analyzing the tensor products is getting infeasible, we will avoid finding the composition factors directly and instead apply the condensation method, see [Lux and Pahlings, 2010], [Lux and Wiegelmann, 1998], and [Lux et al., 2012], to these tensor products.

3) As the starting point of our investigation we take two representations given in R.A. Wilson's online Atlas (via its GAP interface): an irreducible representation, $253a$, of degree 253 over the field with three elements, $\mathbb{F}_3$, and the transitive permutation representation on 31671 points of Fi_{23} on the cosets of the largest maximal subgroup $2.\mathrm{Fi}_{22}$. By analyzing the tensor product of $253a$ with itself and the permutation representation (over $\mathbb{F}_3$) directly, we get the following 8 irreducible representations $1a, 253a, 528a, 2806a, 4830a, 13122a, 13122b, 27048a$. We then proceed by analyzing the tensor product of $253a$ with $528a$ and obtain a ninth irreducible representation $20470a$ as a composition factor.

4) We compute the modular characters of the nine irreducible representations constructed so far.

5) Using character theory we check that the nine irreducible modular characters together with the modular characters of 23 chosen tensor products of the nine irreducible modular characters form a basis of the space of all rational linear combinations of the modular characters in the principal block. The decomposition of this basis into the modular irreducible characters, that means the base change matrix from the basis to the irreducible modular characters, determines the modular characters of the principal block.

6) Finally, we determine the decomposition of the tensor products using the condensation method. More precisely, we show that condensation with a chosen subgroup gives a Morita equivalence. To verify the Morita equivalence we show that no irreducible representation of Fi_{23} vanishes when condensed. This is easy to show for the nonprincipal blocks. For the principal block we prove this by taking a generating set for the condensation algebra described by Noeske's criterion, see [Noeske, 2005], and exhibiting 32 irreducible condensed representations. The established Morita equivalence allows us to determine the decomposition matrix from 5) by determining instead the decomposition matrix for the condensed tensor products by applying the MeatAxe to the condensed tensor products.

Our computations were performed with the help of the computer algebra system GAP and various implementations of the MeatAxe, in particular the C-MeatAxe, see [Ringe, 1994], and the GAP-package chop developed by F. Noeske and M. Neunhöffer, see [Neunhöffer and Noeske, 2016a]. Furthermore, we have made substantial use of the condensation methods developed in [Noeske, 2005] by F. Noeske, and the GAP-package cond, see [Neunhöffer and Noeske, 2016b], developed by F. Noeske and M. Neunhöffer. One of the authors has also implemented an adaptive approach to chopping, for details see Remark 1.

The degrees of the irreducible 3-modular characters in increasing order are displayed in Table 1. The first 32 characters belong to the principal block, followed by the two of the block of defect 1 and the last character has defect 0. The decomposition matrix of the block of cyclic defect is given in Table 3 and the decomposition matrix of the principal block is given in the Appendix.

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Table 1: The degrees of the irreducible 3-modular characters of Fi_{23}

φ_3	φ_4	φ_5	φ_6	φ_1	φ_8	φ_9	φ_7	φ_2	φ_{18}
1	253	528	2806	4 830	13,122	13,122	20 470	27,048	79,718
φ_{10}	φ_{14}	φ_{11}	φ_{15}	φ_{20}	φ_{12}	φ_{13}	φ_{16}	φ_{17}	φ_{19}
86,273	134,298	253,230	362,342	538,407	725,374	725,374	818,972	818,972	1,252,120
φ_{21}	φ_{23}	φ_{24}	φ_{22}	φ_{26}	φ_{25}	φ_{27}	φ_{28}	φ_{31}	φ_{32}
2,317,180	2,541,706	2,587,707	4,372,622	7,116,660	7,260,778	10,241,165	14,540,255	25,951,520	27,425,385
φ_{29}	φ_{30}	φ_{33}	φ_{34}	φ_{35}					
29,713,355	34,753,159	207,793,431	289,103,904	476,702,577					

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2 The block structure

Using the GAP-interface [Breuer, 2012] to the Atlas character tables, see [Conway et al., 1985], we access the ordinary character table of Fi_{23} and compute some of the invariants of the 3-blocks as printed in Table 2.

Table 2: The blocks of Fi_{23}

	Principal block	Block 2	Block 3
Defect	13	1	0
Number of ordinary irreducible characters	94	3	1
Number of irreducible Brauer characters	32	2	1

The decomposition matrix of the second block can be found in Table 3 and follows from the theory of blocks of cyclic defect, see [Hiss and Lux, 1989].

Table 3: The decomposition matrix of Block 2

	φ_{33}	φ_{34}
207,793,431	1	.
289,103,904	.	1
496,897,335	1	1

Of the 35 conjugacy classes of Fi_{23} with order co-prime to 3, exactly three pairs are not real: $16A/B$, $22A/B$ and $23A/B$. By Brauer's permutation lemma, see [Lux and Pahlings, 2010, Theorem 2.2.13], we conclude that there are exactly three pairs of complex conjugate irreducible modular characters. Since the irreducible modular characters not in the principal block are all real valued, we conclude that the three pairs of complex conjugate characters all lie in the principal block.

Furthermore, the field $\mathbb{F}_3$ of three elements is a splitting field of all the irreducible representations of Fi_{23} in characteristic 3, since any 3-regular element is conjugate to its 3rd power, as can be checked from the 3-powermap of the ordinary character table.

3 Some modular characters

From now on let $G := \mathrm{Fi}_{23}$ and $F := \mathbb{F}_3$. In the following an irreducible F-representation of G will be labeled by its degree followed by the letters a, b, etc. and its modular character will be labeled in the same way. Further we denote by a, b the standard group generators of G that are defined by the GAP-package AtlasRep, see [Wilson et al., 2011].

As a starting point we consider the permutation representation of G of degree 31671 on the cosets of $2.\mathrm{Fi}_{22}$, which is included in the AtlasRep package. Applying the program chop of M. Ringe's C-Meataxe [Ringe, 1994] in version 2.4.3 we obtain the following composition factors and their multiplicities over F:

$$3 \times 1a,\ 2 \times 253a,\ 4 \times 528a,\ 1 \times 2806a,\ 1 \times 13122a,\ 1 \times 13122b. \tag{1}$$

Analogously, we obtain the composition factors and their multiplicities of the tensor product $253a \otimes_F 253a$:

$$1a,\ 2 \times 528a,\ 2 \times 4830a,\ 1 \times 13122a,\ 1 \times 13122b,\ 1 \times 27048a. \tag{2}$$

Finally, an application of the program chop to the tensor product $253a \otimes_F 528a$ reveals the composition factor $20470a$. Since we do not care for a full analysis of this tensor product, we abort the program after it has constructed the $20470a$. At this stage, we have nine irreducible representations to work with.

Next, we compute the corresponding modular characters of the nine irreducible representations by using the GAP-function BrauerCharacterValue. This function implements the definition of a modular character value as described in [Jansen et al., 1995, pages xvii-xviii], and we apply it to the nine representations. All 3-regular G-conjugacy classes of elements but $13A/B, 16A/B, 22B/C, 23A/B, 26A/B$ are rational and the character fields for these latter classes are as follows.

$13A/B$	$16A/B$	$22B/C$	$23A/B$	$26A/B$
$\mathbb{Q}(\frac{-1+\sqrt{13}}{2})$	$\mathbb{Q}(\sqrt{-2})$	$\mathbb{Q}(\frac{-1+\sqrt{-11}}{2})$	$\mathbb{Q}(\frac{-1+\sqrt{-23}}{2})$	$\mathbb{Q}(\frac{-1+\sqrt{13}}{2})$

It follows that there are automorphisms of the ordinary character table of G interchanging the classes $16A/B, 22B/C, 23A/B, 26A/B$ independently.

The GAP-package AtlasRep does not include a straight line program that produces representatives for the conjugacy classes of G but it offers a straight line program that constructs generators for representatives of the conjugacy classes of maximal cyclic subgroups. So, we get representatives for all rational 3-regular conjugacy classes of elements straightforwardly. For each of the pairs of classes $16A/B, 22B/C, 23A/B, 26A/B$ a generator of the representative of the corresponding conjugacy class of maximal subgroups can lie in either class of the pair and we may choose this class in view of the available table automorphisms of the ordinary table of G. Our choices are as follows.

First, we define the generator of the representative of the conjugacy class of maximal cyclic subgroups containing elements of $26A/26B$ to lie in the conjugacy class $26B$. Note that the character value of this generator (as computed in GAP) in the representation $253a$ is $\frac{-3-\sqrt{13}}{2}$, and hence we can use the representation $253a$ to identify elements in $26B$ resp. $26A$. Furthermore, we take the square of the generator as a representative of the conjugacy class $13A$ using the 2-powermap of the ordinary character table of G.

Next, for the representations $13122a, 13122b$ and the conjugacy classes $16A/B, 22B/C, 23A/B$ we proceed as follows:

1) The computed modular character values of $13122a$ and $13122b$ on the generator of the maximal cyclic subgroup of order 16 as produced by the straight line program shows that their modular character values on $16A/B$ are not real. Hence the modular characters of $13122a$ and $13122b$ are complex conjugate.

2) We determine the sum of the modular characters of $13122a$ and $13122b$ from the modular character of the permutation representation on 31671 points. Since their characters are complex conjugate, we get their values on all but the pairs of classes $16A/B, 22B/C, 23A/B$.

3) We define the representation $13122a$ to be the irreducible representation of degree 13122 in the socle of the tensor product $253a \otimes_F 253a$, a fact which we will use in Section 7.

4) We define the representatives of the classes $16A, 22A, 23A$ as the generators of the representatives of the maximal cyclic subgroups containing elements in $16A/B$,$22B/C$, and $23A/B$ (as produced by the straight line program). The modular character values of $13122a$ on these elements are $2\sqrt{-2}$, $\sqrt{-11}$, $\frac{1+\sqrt{-23}}{2}$. We note that we can use $13122a$ to identify the classes.

Finally, the modular character of $27048a$ can be derived from the character of the tensor product $253a$ with itself, since we already know the characters of the other composition factors.

We can now check that the nine irreducible modular characters of the irreducible representations constructed so far, $1a, 253a, 528a, 2806a, 4830a, 13122a, 13122b, 20470a, 27048a$, (abbreviated by $N1, \dots, N9$ in Table 7) together with the modular characters of 23 selected tensor products of these representations form a basis B of the $\mathbb{Q}$-span of the modular characters in the principal block. The 23 selected tensor products are

a) $253a$ tensored with $528a, 2806a, 4830a, 13122a, 20470a, 27048a$, abbreviated $N10$ to $N15$,

b) $528a$ tensored with $528a, 4830a, 13122a, 20470a, 27048a$, abbreviated $N16$ to $N20$,

c) $2806a$ tensored with $2806a, 4830a, 13122a, 20470a, 27048a$, abbreviated $N21$ to $N25$,

d) $4830a$ tensored with $4830a, 13122a, 20470a, 27048a$, abbreviated $N26$ to $N29$,

e) $13122a$ tensored with $13122a, 20470a, 27048a$, abbreviated $N30$ to $N32$.

4 Choice of a condensation subgroup

We denote by M the 10th maximal subgroup of G of isomorphic to $(2^2 \times 2^{1+8}).(3 \times U_4(2)).2$, see [Conway et al., 1985]. Furthermore let K be the largest normal 2-subgroup of M, so $K \cong 2^2 \times 2^{1+8}$. We take K as the condensation subgroup with corresponding fix idempotent

$$e := \frac{1}{|K|} \sum_{x \in K} x.$$

We seek to find the composition factors and their multiplicities of selected tensor products of the irreducible representations determined in Section 3 by regarding their images under the exact *condensation functor*

$$\Phi_e \colon \text{mod-}FG \to \text{mod-}eFGe,$$

which maps an FG-module V to $\Phi_e(V) := Ve$. Given the modular character φ of an FG-module V, the dimension of the condensed module Ve is also called the *condensed degree* φ^c of φ and may be computed as the scalar product

$$\varphi^c(1) := (\varphi_K, 1_K) = \dim Ve. \tag{3}$$

It follows that the condensed degrees of the three irreducible representations not in the principal block with modular characters $\varphi_{33}, \varphi_{34}$, and φ_{35}, see Section 2, are $100683, 145152, 226476$ and hence these irreducible representations do not map to 0 when condensed.

5 Generators of the condensed algebra

To perform the condensations, we need a set of algebra generators for $eFGe$. According to [Noeske, 2005], one such generating set is given by $\{ege \,|\, g \in S\}$ where the set $S \subseteq G$ is the union of a set of group generators for M and a complete set of non-trivial M-M- double coset representatives.

We make use of the GAP package AtlasRep version 1.6 (experimental), which can be obtained from the developers on request, since it contains a straight line program for the maximal subgroup M not yet contained in the released version 1.5 of AtlasRep. We are left with the task of calculating the double coset representatives. By Mackey's formula we can compute the number of double cosets as the norm of the corresponding permutation character 1_M^G, which is 303. Hence $|S| = 2 + 303 - 1$ (with the trivial double coset omitted). Note that compared to the situation for the 2-modular case, see [Hiss et al., 2006], the number of double cosets we have to consider is about tenfold.

Following the approach in [Hiss et al., 2006], we first consider the action of G on the orbit xG of a non-trivial M-fixpoint x in the irreducible matrix representation $528a$ constructed in Section 3. The set xG is isomorphic as a G-set to the set of right M-cosets in G, thus a set of double coset representatives consists of elements $g_1, \ldots, g_{303} \in G$ such that the M-orbits xg_jM are pairwise distinct for $1 \leq j \leq 303$. A rough estimate for the amount of storage memory needed for xG gives $3 \cdot 528 \cdot [G : M]\text{Bit} \approx 1.54\text{TB}$, and hence calls for a different method than the standard orbit algorithm.

To this end, we employ the orbit-by-suborbit-algorithm described in [Müller et al., 2007], which is part of the GAP-package orb, see [Müller et al., 2014]. Instead of fully enumerating the M-suborbits of xG, only specific points in each M-orbit are stored. Two helper groups $U_1 \leq U_2 \leq M$ of sizes 384 and 7,680 are used to define those specific points called U_2-minimal points.

With this approach, we find 302 M-orbits in xG after enumerating 1.4 million points of xG (see Table 5 for their lengths). The last and very small orbit of length 180 only appears after considering more than 101 million points, of which only 2 million are held in memory at each time.

The required memory is less than 8 GB. Straight line programs for g_1 to g_{303} are available from words in the generators of G. The one for g_{303} has too many lines to be of practical use, so we construct a different straight line program of length 293 instead of 2487 by precomputing frequently-occurring subwords.

6 Applying condensation

We first want to show that e is a faithful idempotent, i.e. $Se \neq 0$ for all simple FG-modules S, which we only have to verify for the simple FG-modules in the principal block. This then implies that FG and $eFGe$ are Morita equivalent.

In order to achieve this goal efficiently, we will work with a subalgebra A of $eFGe$ generated by two elements x_1, x_2. These two generators are defined as $x_1 = e(y_1 y_2)e, x_2 = e(ab)^2 e$, where a, b are the standard generators of G, see Section 3, and y_1, y_2 are the generators produced by the straight line program for the maximal subgroup M of Fi_{23}.

We proceed as follows: we exhibit 32 pairs of A-submodules, $(V_i, W_i), i = 1, \ldots, 32$, with $V_i \leq W_i$, of the condensed $eFGe$-modules of the tensor products N27, N28, N31, N32, (whose composition factors are all in the principal block). We verify that they are even $eFGe$-submodules by showing invariance under all 304 generators of $eFGe$. Furthermore, we show that their quotients W_i/V_i considered as A-modules and as $eFGe$-modules are all simple and pairwise nonisomorphic. Since there are 32 simple FG-modules in the principal block, and we have exhibited 32 pairwise nonisomorphic simple $eFGe$-modules, we have shown that no simple FG-module S in the principal block vanishes when condensed. So we conclude that $eFGe$ and FG are Morita equivalent. Moreover, we see that restriction from $eFGe$ to A gives a bijection of the simple $eFGe$-modules up to isomorphism (in the principal block) onto the corresponding simple A-modules up to isomorphism.

The second goal is to find the composition factors and their multiplicities of the 23 tensor product FG-modules from Section 3, which are all in the principal block.

We again apply condensation with the subalgebra A. More precisely, we determine the A-composition factors and their multiplicities in the condensed tensor products. Since we have shown that restriction from $eFGe$ to A gives a bijection of simple $eFGe$- and simple A-modules in the principal block, the composition factors and their multiplicities of the condensed tensor products as $eFGe$-modules follow from this explicit bijection.

The multiplicities of the A-composition factors in the two largest condensed modules, namely the condensed modules of N31 with dimension 128358 and N32 with dimension 184644 are not found by the program chop of the GAP-package Chop directly. First note that the dimension of $N32$ is about 10 times bigger than the dimension of the largest condensed tensor product that had to be considered in the 2-modular case, see [Hiss et al., 2006]. Instead we guide the analysis by the program chop in the following way: we first determine a cyclic A-submodule in the condensed module of N31, and an ascending chain of five cyclic A-submodules in the condensed module of N32 and feed the smaller subquotients to the program chop.

The cyclic submodules are found as follows: we apply the C-MeatAxe program pwkond to determine a peakword w for the simple A-module $1a$ with respect to 28 already known simple A-modules from smaller condensed tensor products. This means that w has nullity one when evaluated in the simple A-module $1a$ and is invertible when evaluated in any of the other 27 simple A-modules. The important property of the peakword w is that vectors in its stable null space when evaluated in a given A-module V with composition factors amongst the 28 simple A-modules generate A-submodules of a very special type: modulo their radical they are a direct sum of the simple A-module $1a$, for details see [Lux et al., 1994]. The resulting peakword w computed by pwkond is:

$$w := x_1^4 x_2 x_1^2 + x_2 x_1^2 x_2 x_1 x_2^2 + x_2 x_1^2 x_2^3 x_1 + x_1 x_2 x_1^2 x_2^2 x_1 + x_2 x_1 x_2 x_1 x_2 x_1 + id.$$

Next, we compute a variant of the minimal polynomial of T, the matrix of w in the condensed module of $N32$. We choose a vector v in $N32$ and compute the monic polynomial p in the polynomial ring $\mathbb{F}[X]$ of least degree such that $p(T)$ is the zero matrix. It turns out that the polynomial p of the chosen vector v has lowest term x^6, and hence p can be factored as $p = x^6 \cdot q$ for some polynomial q co-prime to x. The vectors $v \cdot q(T)T^{5-i}$, $i = 0, \ldots, 5$ are all in the stable null space of T and we use the spinning procedure of the GAP-package Chop to compute the ascending chain of A-submodules they generate. The resulting dimensions are:

$$5911, \quad 9621, \quad 42949, \quad 65070, \quad 99842, \quad 145292.$$

The subquotients in this ascending chain of A-submodules are of dimension less than 60.000 and therefore are more amenable to be dealt with by the chop program of the GAP-package Chop.

In the case of the condensed module of $N31$, where we are aiming at a single subspace, it turns out that the A-submodule generated by the whole null space of the 4th power of the peakword w gives a convenient A-submodule of dimension 52924.

Remark 1 *The algebra A was initially found by trial and error. One of the authors has written an extension to the program chop of the GAP-package Chop: it takes a small subset of the generators of $eFGe$ described by Noeske's criterion and tries to verify that a composition series found for the algebra generated by the subset is invariant under all generators of $eFGe$. In case it is not, the program adds the first element found for which the series is not invariant and recomputes a composition series etc. In this way, we determined a subset of size 8, and worked with the corresponding algebra generated by those 8 elements. However, for the last two condensed modules, the condensed modules of $N31, N32$, we decided to look for an even smaller subset. The result of our successful search are the elements x_1 and x_2 from the beginning of this section.*

Remark 2 *The computational challenges involve restricting the matrix representations of G to the condensation subgroup K, performing precomputations for the condensation algorithm and finally evaluating and condensing specific elements of G. The demands on memory and computation time are foremost dependent on the degrees of the matrix representations, the number of algebra generators and the length of the straight line programs used. See Table 6 for an overview.*

The overall process greatly benefits from a parallel run of the invariance tests during the adaptive approach described above. However for groups larger than Fi_{23} the matrix representation degrees may easily become the single prohibiting factor.

Table 7 gives the composition factors and their multiplicities of the condensed tensor products.

7 Matching simple FG-modules and their condensations

Finally, we match each of the nine simple FG-modules S from Section 3 with the corresponding condensed simple $eFGe$-module Se. Table 4 summarizes the correspondence.

Table 4: Simple FG-modules and their condensations

$1a$	$253a$	$528a$	$2806a$	$4830a$	$13122a$	$13122b$	$20470a$	$27048a$
$1a$	$5a$	$17a$	$15a$	$10b$	$54a$	$54b$	$45a$	$10a$

The matchings for $253a$, $528a$ and $2806a$ can already be inferred from their respective condensed degrees which in turn are easily computed using their modular characters and Formula 3.

We settle the correct matching for $13122a/b$ by looking at the tensor product module $253a \otimes_F 253a$ and the corresponding condensed tensor product. Recall from Section 3 that we have defined $13122a$ to be in the socle of the tensor product. Since FG and $eFGe$ are Morita equivalent, and

$54a$ is in the socle of the condensed tensor product, it follows that $13122a$ corresponds to $54a$ and hence $13122b$ corresponds to $54b$. Note that $54a$ labels the 9th row of the multiplicity matrix of the condensed tensor products and $54b$ the 8th row. This implies that the modular character φ_9 is the modular character of $13122a$ and φ_8 is the modular character of $13122b$. The correspondence for $4830a$ and $27048a$ also follows by comparing the composition factors and their multiplicities in $253a \otimes_F 253a$ (see Equation 2) and its condensation.

8 Computing the irreducible modular characters

The irreducible modular characters in the principal block can now be computed by multiplying the inverse of the multiplicity matrix (transposed) with the basis B from Section 3. The resulting decomposition matrix can be found in the Appendix.

Tables

Table 5: Length of the M-orbits in Section 5

1×1	1×180	1×540	1×810
1×1536	1×3456	1×4320	3×12960
1×23040	2×69120	1×73728	1×81920
8×103680	1×110592	1×122880	1×138240
2×207360	1×276480	1×368640	8×414720
3×829440	1×983040	3×1105920	10×1244160
1×1327104	1×1474560	5×1658880	5×2211840
1×2654208	1×2949120	23×3317760	1×4423680
9×6635520	3×8847360	21×9953280	19×13271040
2×17694720	21×19906560	20×26542080	34×39813120
5×53084160	42×79626240	29×159252480	5×318504960

Table 6: Computation time

Representation	Restriction to K		Precomputation	Adaptive Chop
	left factor	right factor		
$253a \otimes_F 253a$	4s			32s (full run)
$253a \otimes_F 528a$	23s			76s (full run)
$253a \otimes_F 4830a$	2,6h			4h (full run)
$4830a \otimes_F 4830a$	7,1h			20h (full run)
$253a \otimes_F 20470a$	1s	1-2d per matrix	13h	up to 2h per matrix
$4830a \otimes_F 27048a$	3h	2-3d per matrix	36h	up to 13h per matrix

Table 7: Composition factors and multiplicities of the condensed tensor products

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
$10b$	1	.	.	.	.	.	.	.	.	.	2	.	2	2	2	2	5	1	6	10	7	5	7	30	9	14	18	19	41	58	44	99
$10a$	.	1	.	.	.	.	.	.	.	.	.	.	2	2	2	.	3	1	4	6	5	4	6	16	8	6	11	15	16	24	31	50
$1a$	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	2	1	.	1	2	4	1	.	10	2	6	4	3	14	18	8	28
$5a$	.	.	.	1	.	.	.	.	.	3	1	5	2	4	10	2	2	2	14	4	8	14	16	27	32	10	14	44	32	57	84	87
$17a$	.	.	.	.	1	.	.	.	.	.	2	.	.	2	.	4	3	1	2	5	7	2	2	19	5	12	8	8	28	37	18	52
$15a$	.	.	.	.	.	1	.	.	.	2	.	4	1	1	4	.	.	1	6	2	2	8	7	11	15	2	7	19	10	20	34	33
$45a$	.	.	.	.	.	.	1	.	.	2	.	4	3	.	9	.	.	1	12	.	2	12	12	12	23	2	7	34	12	29	52	45
$54b$	.	.	.	.	.	.	.	1	.	.	1	.	.	2	.	2	2	2	1	4	5	1	2	17	4	9	8	6	24	30	15	42
$54a$	.	.	.	.	.	.	.	.	1	.	1	.	.	2	.	2	2	2	1	4	5	1	1	17	4	9	7	6	24	30	16	43
$90a$	.	.	.	.	.	.	.	.	.	1	.	3	4	.	8	.	.	.	11	.	.	11	11	7	22	.	5	33	6	20	49	34
$150a$	.	.	.	.	.	.	.	.	.	.	.	2	2	.	4	.	.	1	5	.	1	5	5	5	10	.	4	14	4	11	19	16
$170a$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	.	.	1	.	.	1	2	1	2	.	2	5	2	6	9	10
$170b$	.	.	.	.	.	.	.	.	.	.	.	.	1	.	2	.	.	.	1	.	.	1	1	1	2	.	1	5	2	6	8	11
$156a$	.	.	.	.	.	.	.	.	.	.	1	.	.	1	.	1	1	.	.	1	1	.	1	2	.	2	1	1	3	3	3	4
$145a$	.	.	.	.	.	.	.	.	.	.	.	1	1	.	2	.	.	.	3	.	.	3	2	2	5	.	1	6	2	6	8	8
$430a$	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	1	.	.	2	2	.	2	6	.	2	5	1	6	9	7	17
$430b$	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	1	1	.	2	2	.	1	6	.	2	4	1	6	8	8	18
$148a$	.	.	.	.	.	.	.	.	.	.	.	.	.	2	.	1	1	1	.	2	3	.	.	11	.	5	5	.	14	18	4	25
$705a$	.	.	.	.	.	.	.	.	.	.	.	.	1	.	1	.	.	.	1	.	.	1	2	.	4	.	.	6	.	1	7	2
$600a$	.	.	.	.	.	.	.	.	.	.	1	.	.	1	.	.	1	1	.	1	2	.	.	3	.	3	2	.	5	3	2	8
$885a$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	1	.	.	3	5	1	3
$2038a$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	2	.	2	.	.	3	.	1	6	.
$1758a$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	1	.	.	3	.	2	1	.	4	6	.	4
$1872a$	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	1	.	1	.	.	.	3	.	1	2	.	4	3	.	4
$3315a$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	3	.	1	2	.	4	5	2	12
$3570a$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	.	1	.	.	2	.	1	5	2
$4392a$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	1	.	.	1	.	.	4	1
$5475a$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1
$13343a$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.
$14598a$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	1	.	.	2	.
$12126a$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	1	.	2
$14343a$	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	1	1	.	3

3-modular decomposition matrix of the principal block of Fi_{23}

	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8	φ_9	φ_{10}	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	φ_{16}	φ_{17}	φ_{18}	φ_{19}	φ_{20}	φ_{21}	φ_{22}	φ_{23}	φ_{24}	φ_{25}	φ_{26}	φ_{27}	φ_{28}	φ_{29}	φ_{30}	φ_{31}	φ_{32}
1	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
782	.	.	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
3588	.	.	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
5083	1	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
25806	1	.	.	2	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
30888	.	.	1	1	3	1	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
60996	2	.	1	3	2	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
106743	.	.	.	1	1	.	.	1	1	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
111826	1	.	.	1	.	.	1	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
274482	2	.	1	4	3	1	2	2	2	1	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
279565	1	1	.	2	1	2	1	.	.	1	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
752675	.	1	.	2	.	1	1	.	.	1	1	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
789360	3	1	1	4	3	4	3	1	1	3	1	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
812889	2	.	1	4	3	1	2	2	2	1	.	.	.	.	.	.	.	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.
837200	1	.	.	1	.	.	1	.	.	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
837200	1	.	.	1	.	.	1	.	.	1	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
850850	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
850850	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
1677390	1	1	1	2	2	2	1	1	1	1	1	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.
1951872	3	2	1	8	4	4	5	2	2	4	1	.	.	1	1	.	.	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.
2236520	3	3	.	1	.	3	1	.	.	1	1	.	.	1	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
2322540	3	2	.	7	1	3	5	1	1	4	2	.	.	.	2	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.
3913910	6	4	1	8	3	5	5	2	2	4	2	.	.	1	1	1	1	1	.	1	.	.	.	.	.	.	.	.	.	.	.	.
5533110	4	2	2	9	6	5	6	4	4	5	2	.	.	1	1	.	.	2	.	2	.	.	1	.	.	.	.	.	.	.	.	.
6709560	3	3	1	5	1	3	3	1	1	2	1	.	.	.	.	1	1	1	.	.	.	1	.	.	.	.	.	.	.	.	.	.
7468032	3	1	2	5	4	3	3	3	3	2	2	.	.	1	.	.	.	1	1	.	.	.	1	1	.	.	.	.	.	.	.	.
8783424	4	3	1	5	2	2	3	2	2	2	2	.	.	1	1	1	1	1	.	1	.	.	1	1	.	.	.	.	.	.	.	.
9108736	4	2	1	2	2	1	1	1	2	1	.	.	1	.	.	.	1	1	.	.	.	.	.	.	1	.	.	.	.	.	.	.
9108736	4	2	1	2	2	1	1	2	1	1	.	1	.	.	.	1	.	1	.	.	.	.	.	.	1	.	.	.	.	.	.	.
10567557	8	5	1	12	4	6	9	3	3	8	4	1	1	1	2	1	1	1	1	1	.	.	.	1	.	.	.	.	.	.	.	.
10674300	6	4	2	7	4	3	3	3	3	2	1	.	.	.	1	1	1	2	.	1	.	.	.	.	1	.	.	.	.	.	.	.
12077208	4	.	2	6	4	2	4	4	4	2	2	.	.	1	1	.	.	2	.	1	1	.	2	1	.	.	.	.	.	.	.	.
15096510	8	5	1	12	4	6	9	3	3	8	4	1	1	1	2	1	1	1	1	1	.	.	.	.	.	1	.	.	.	.	.	.
17892160	5	4	1	9	2	4	7	2	2	6	3	1	1	.	1	1	1	1	1	.	.	1	.	.	.	1	.	.	.	.	.	.
18812574	12	8	2	17	6	8	12	5	5	10	6	1	1	2	3	2	2	2	1	1	.	.	1	2	.	.	.	.	.	.	.	.
20322225	10	3	3	10	6	4	6	5	5	4	2	1	1	1	1	1	1	3	.	1	1	.	1	1	1	.	.	.	.	.	.	.
21135114	3	3	1	10	3	5	6	2	2	5	2	.	.	1	1	1	1	1	.	1	.	1	1	.	.	.	1	.	.	.	.	.
21348600	11	8	3	23	9	11	16	7	7	13	7	1	1	1	3	1	1	3	2	2	.	1	1	1	.	.	.	.	.	.	.	.
22644765	5	5	.	13	2	8	10	1	1	10	4	1	1	1	2	1	1	.	1	1	.	1	.	.	.	.	1	.	.	.	.	.
26838240	9	5	3	18	7	7	12	7	7	9	6	1	1	1	3	1	1	4	1	2	1	1	2	2	.	.	.	.	.	.	.	.
28464800	11	8	3	17	6	8	12	5	5	10	6	1	1	1	2	2	2	2	2	1	.	1	1	1	.	1	.	.	.	.	.	.
29354325	12	7	5	19	10	9	11	9	9	7	6	.	.	2	2	2	2	5	1	2	1	1	3	2	.	.	.	.	.	.	.	.

	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8	φ_9	φ_{10}	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	φ_{16}	φ_{17}	φ_{18}	φ_{19}	φ_{20}	φ_{21}	φ_{22}	φ_{23}	φ_{24}	φ_{25}	φ_{26}	φ_{27}	φ_{28}	φ_{29}	φ_{30}	φ_{31}	φ_{32}
35225190	14	9	3	21	7	8	14	6	6	12	6	2	2	.	3	2	2	3	2	1	.	1	.	1	1	1	.	.	.	.	.	.
37573536	5	1	3	8	5	1	4	5	5	2	2	1	1	.	1	.	.	2	1	.	1	.	1	1	.	.	.	.	.	.	1	.
40840800	4	2	2	8	2	1	3	2	2	3	1	1	1	.	1	.	.	.	1	.	1	.	.	.	.	.	.	.	.	1	.	.
42270228	9	6	2	10	5	5	7	4	4	6	4	1	1	1	1	2	2	2	1	2	.	.	1	1	.	.	.	.	.	.	.	1
48034350	15	11	2	26	6	13	19	5	5	17	8	2	2	2	3	3	3	2	2	1	.	2	1	1	.	1	1	.	.	.	.	.
48308832	3	2	.	9	1	3	5	1	1	5	1	1	1	.	.	1	1	.	.	.	.	1	.	.	.	.	1	.	1	.	.	.
55740960	19	11	4	28	10	14	19	8	8	16	8	2	2	3	4	3	3	4	1	2	1	1	2	1	1	1	1	.	.	.	.	.
56360304	25	13	7	28	14	12	17	12	12	13	7	2	2	2	3	4	4	7	1	2	1	1	2	2	2	1	.	.	.	.	.	.
56360304	25	13	7	28	14	12	17	12	12	13	7	2	2	2	3	4	4	7	1	2	1	1	2	2	2	1	.	.	.	.	.	.
57254912	21	12	4	21	8	9	12	6	6	11	3	2	2	1	2	4	4	4	.	1	.	1	.	.	3	1	1	.	.	.	.	.
58708650	8	4	4	18	8	6	10	7	7	7	4	1	1	1	2	1	1	3	1	1	1	1	2	1	.	.	1	.	.	.	1	.
58708650	21	15	4	33	10	16	22	8	8	19	9	2	2	2	4	4	4	4	2	2	.	2	1	1	1	1	1	.	.	.	.	.
65875680	25	11	7	19	12	6	9	10	10	6	1	2	2	.	2	4	4	6	.	1	1	.	.	.	4	1	.	1	.	.	.	.
73531392	12	7	4	12	7	4	6	6	6	5	3	2	2	.	1	2	2	3	1	1	.	.	.	1	1	.	.	.	.	.	1	1
73531392	10	6	3	19	5	6	10	4	4	10	3	2	2	1	1	1	1	1	2	.	.	1	.	.	1	1	1	.	.	1	.	.
78278200	26	15	5	20	10	9	12	8	8	11	5	3	3	1	2	5	5	5	1	2	.	.	.	1	3	1	.	.	.	.	.	1
93933840	17	11	5	33	9	12	20	8	8	18	8	3	3	1	4	2	2	3	3	1	1	2	1	1	1	1	1	.	.	1	.	.
97976320	38	23	9	48	19	24	31	16	16	26	15	3	3	4	6	7	7	8	3	3	1	2	3	3	2	2	1	.	.	.	.	.
133398252	26	18	6	32	12	13	19	10	10	18	10	4	4	.	4	4	4	4	4	4	.	.	.	1	2	.	.	.	.	1	.	2
153014400	45	26	10	60	22	25	37	18	18	32	12	5	5	3	6	8	8	10	2	4	1	3	2	1	4	2	2	.	1	.	.	.
153014400	35	24	8	50	17	21	31	14	14	28	14	5	5	2	5	6	6	6	5	3	.	2	1	2	2	2	1	.	.	1	.	1
166559744	33	21	7	56	15	22	35	13	13	32	13	5	5	2	6	5	5	5	4	2	1	3	1	1	2	2	2	.	1	1	.	.
166559744	49	25	14	51	28	21	28	23	23	22	11	5	5	3	5	8	8	13	2	4	2	1	3	3	5	1	1	.	.	.	1	1
176125950	36	19	11	46	20	16	25	17	17	21	10	5	5	2	5	5	5	8	3	3	2	1	2	2	3	1	1	.	.	1	1	1
176125950	36	19	11	46	20	16	25	17	17	21	10	5	5	2	5	5	5	8	3	3	2	1	2	2	3	1	1	.	.	1	1	1
203802885	40	26	10	56	18	21	33	15	15	32	14	7	7	2	6	6	6	6	6	2	1	2	.	1	3	3	1	.	.	2	.	1
211351140	45	26	12	61	21	22	35	18	18	32	14	7	7	2	7	6	6	8	5	3	2	2	1	2	4	2	1	.	.	2	.	1
216154575	23	8	9	36	14	6	15	12	12	14	3	6	6	.	4	1	1	4	2	1	3	.	.	.	3	.	2	.	.	2	2	1
216770400	52	27	16	68	32	26	39	27	27	30	16	6	6	4	8	8	8	14	3	5	3	2	5	4	4	1	2	.	.	.	2	1
244563462	32	12	12	36	16	8	16	15	15	16	4	7	7	.	2	2	2	5	4	.	4	.	.	.	4	1	.	1	1	3	.	1
263376036	56	27	15	54	28	17	27	23	23	22	6	8	8	.	4	9	9	12	2	2	2	1	.	.	8	2	2	1	1	.	2	1
264188925	47	28	12	71	23	26	42	20	20	39	17	8	8	3	7	7	7	8	6	3	2	3	2	2	3	3	2	.	1	2	.	1
264536064	35	18	11	48	16	13	24	15	15	23	8	7	7	1	4	4	4	4	5	1	3	1	.	.	3	2	1	1	1	3	.	1
264536064	49	22	15	50	27	16	26	22	22	22	8	8	8	1	5	6	6	10	3	3	3	.	1	1	6	1	1	1	.	1	2	2
286274560	47	28	12	74	23	27	42	19	19	39	15	8	8	3	7	7	7	7	5	3	2	3	1	1	3	2	3	.	1	2	1	1
287721720	72	37	19	80	40	30	45	32	32	37	14	9	9	3	8	11	11	16	3	5	2	1	2	2	8	2	2	1	1	.	2	1
289027200	60	34	16	92	33	34	55	28	28	47	20	9	9	4	10	9	9	13	5	6	3	4	4	3	4	2	3	.	1	1	1	1
289027200	62	35	18	86	34	34	50	29	29	43	22	8	8	5	9	9	9	13	7	4	3	3	4	4	4	3	2	.	.	2	1	1
313112800	70	38	20	84	38	31	47	32	32	39	19	9	9	4	9	11	11	16	5	6	3	2	4	4	6	2	2	.	.	1	2	2
313112800	59	31	17	76	31	26	41	26	26	36	13	9	9	2	7	8	8	11	5	3	3	2	1	1	6	2	2	1	1	2	1	1
313112800	59	31	17	76	31	26	41	26	26	36	13	9	9	2	7	8	8	11	5	3	3	2	1	1	6	2	2	1	1	2	1	1
322058880	81	47	22	106	44	43	64	37	37	54	27	10	10	5	12	13	13	19	7	8	3	4	5	5	6	3	2	.	.	1	1	2
336061440	66	36	17	84	32	31	48	27	27	43	16	10	10	2	7	10	10	11	6	3	3	3	1	1	6	3	2	1	2	2	.	1
341577600	55	27	17	77	29	24	41	25	25	37	13	10	10	2	7	6	6	9	6	2	4	2	1	1	5	2	2	1	1	3	1	1

	φ_1	φ_2	φ_3	φ_4	φ_5	φ_6	φ_7	φ_8	φ_9	φ_{10}	φ_{11}	φ_{12}	φ_{13}	φ_{14}	φ_{15}	φ_{16}	φ_{17}	φ_{18}	φ_{19}	φ_{20}	φ_{21}	φ_{22}	φ_{23}	φ_{24}	φ_{25}	φ_{26}	φ_{27}	φ_{28}	φ_{29}	φ_{30}	φ_{31}	φ_{32}
343529472	68	33	19	73	36	24	38	29	29	32	10	10	10	2	6	10	10	14	3	4	3	1	1	1	8	2	2	1	1	1	2	2
352251900	83	46	20	88	39	34	50	32	32	44	17	11	11	3	8	14	14	16	5	5	2	2	1	2	9	3	2	1	1	1	1	2
362316240	71	38	18	91	36	33	53	30	30	46	19	11	11	4	9	11	11	14	5	5	3	3	3	3	6	3	3	.	1	1	2	2
504627200	100	51	29	120	52	43	65	44	44	57	23	14	14	5	11	14	14	20	8	5	5	3	3	3	10	4	3	1	1	3	2	2
504627200	88	48	25	116	46	39	64	39	39	57	23	15	15	3	11	12	12	16	9	6	4	3	2	3	8	3	3	1	1	3	2	3
526752072	100	47	29	107	50	33	55	43	43	48	16	16	16	2	9	13	13	19	7	3	6	1	1	2	12	3	2	2	2	3	2	2
528377850	95	47	28	111	50	35	57	42	42	50	16	15	15	2	9	13	13	18	7	4	5	2	1	1	11	3	3	2	2	3	2	2
559458900	83	45	24	117	41	36	61	36	36	56	19	15	15	3	10	12	12	14	8	4	5	4	2	1	8	3	5	1	2	4	2	2

References

[Breuer, 2012] Breuer, T. (2012). The GAP Character Table Library, Version 1.2.1. GAP package.

[Breuer et al., 2016] Breuer, T., Hiss, G., Lübeck, G., Lux, K., Müller, J., Parker, R., and Wilson, R. (2016). The Modular Atlas project. `http://www.math.rwth-aachen.de/~MOC/`.

[Conway et al., 1985] Conway, J., Curtis, R., Norton, S., Parker, R., and Wilson, R. (1985). *Atlas of Finite Groups*. Oxford, England: Clarendon Press.

[Hiss and Lux, 1989] Hiss, G. and Lux, K. (1989). *Brauer trees of sporadic groups*. Oxford Science Publications. The Clarendon Press, Oxford University Press, New York.

[Hiss and Lux, 1994] Hiss, G. and Lux, K. (1994). The 5-modular characters of the sporadic simple Fischer groups Fi_{22} and Fi_{23}. *Comm. Algebra*, 22(9):3563–3590. With an appendix by T. Breuer.

[Hiss et al., 2006] Hiss, G., Neunhöffer, M., and Noeske, F. (2006). The 2-modular characters of the Fischer group Fi_{23}. *J. Algebra*, 300(2):555–570.

[Isaacs, 1994] Isaacs, I. M. (1994). *Character theory of finite groups*. Dover Publications Inc. Corrected reprint of the 1976 original.

[Jansen et al., 1995] Jansen, C., Lux, K., Parker, R., and Wilson, R. (1995). *An atlas of Brauer characters*, volume 11 of *London Mathematical Society Monographs. New Series*. The Clarendon Press, Oxford University Press, New York. Appendix 2 by T. Breuer and S. Norton, Oxford Science Publications.

[Lux et al., 1994] Lux, K., Müller, J., and Ringe, M. (1994). Peakword condensation and submodule lattices: an application of the MEAT-AXE. *J. Symbolic Comput.*, 17(6):529–544.

[Lux et al., 2012] Lux, K., Neunhöffer, M., and Noeske, F. (2012). Condensation of homomorphism spaces. *LMS J. Comput. Math.*, 15:140–157.

[Lux and Pahlings, 2010] Lux, K. and Pahlings, H. (2010). *Representations of groups*, volume 124 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge. A computational approach.

[Lux and Wiegelmann, 1998] Lux, K. and Wiegelmann, M. (1998). Condensing tensor product modules. In *The atlas of finite groups: ten years on (Birmingham, 1995)*, volume 249 of *London Math. Soc. Lecture Note Ser.*, pages 174–190. Cambridge Univ. Press, Cambridge.

[Müller et al., 2014] Müller, J., Neunhöffer, M., and Noeske, F. (2014). orb, a GAP package, Version 4.7.1. `http://neunhoef.github.io/orb/`. GAP package.

[Müller et al., 2007] Müller, J., Neunhöffer, M., and Wilson, R. A. (2007). Enumerating big orbits and an application: B acting on the cosets of Fi_{23}. *J. Algebra*, 314(1):75 – 96.

[Neunhöffer and Noeske, 2016a] Neunhöffer, M. and Noeske, F. (2016a). chop – new and improved. unpublished. GAP package.

[Neunhöffer and Noeske, 2016b] Neunhöffer, M. and Noeske, F. (2016b). cond – a package for computing condensations. unpublished. GAP package.

[Noeske, 2005] Noeske, F. (2005). *Morita-Äquivalenzen in der algorithmischen Darstellungstheorie*. Dissertation, RWTH Aachen University.

[Parker, 1984] Parker, R. (1984). The computer calculation of modular characters. (the meat-axe). *Computational group theory (Durham, 1982)*, pages 267–274.

[Ringe, 1994] Ringe, M. (1994). The C–Meataxe. *Manual RWTH Aachen University*.

[Wilson et al., 2016] Wilson, R., Parker, R., Nickerson, S., and Bray, J. (2016). ATLAS of Finite Group Representations. `http://brauer.maths.qmul.ac.uk/Atlas/`.

[Wilson et al., 2011] Wilson, R., Parker, R., Nickerson, S., Bray, J., and Breuer, T. (2011). AtlasRep, a GAP interface to the atlas of group representations, Version 1.5. Refereed GAP package.

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