Graduiertenkolleg

Experimentelle und konstruktive Algebra

Experimental and constructive algebra

Kolloquiumsvortrag

Dienstag, 15. Januar 2019, 14:15 Uhr, Hörsaal IV

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On the Structure of G-Primitive Picard-Vessiot Extensions for the Classical Groups

In classical Galois theory there is the well-known construction of the general equation with Galois group the symmetric group S_n. One starts with n indeterminates $\boldsymbol{T} = (T_1, \ldots, T_n)$ and considers the rational function field $\mathbb{Q}(\boldsymbol{T})$. The group S_n acts on $\mathbb{Q}(\boldsymbol{T})$ by permuting the indeterminates $T_1, \ldots, T_n$. One can show then that $\mathbb{Q}(\boldsymbol{T})$ is a Galois extension of the fixed field $\mathbb{Q}(\boldsymbol{T})^{S_n}$ for a polynomial equation of degree n whose coefficients are the elementary symmetric polynomials $s_1(\boldsymbol{T}), \ldots, s_n(\boldsymbol{T})$ in $\boldsymbol{T}$. Moreover the fixed field is generated by these polynomials and they are algebraically independent over $\mathbb{Q}$.

In this talk we do a similar construction in differential Galois theory for the classical Lie groups. Let G be one of these groups and denote by l its Lie rank. We start our construction with a differential field $C\langle \boldsymbol{\eta} \rangle$ which is differentially generated by l differential indeterminates $\boldsymbol{\eta} = (\eta_1, \ldots, \eta_l)$ over the constants C. We use this purely differential transcendental extension to build our final general extension field $E \supset C\langle \boldsymbol{\eta} \rangle$ by taking into account the structure of G-primitive Picard-Vessiot extensions. These are Picard-Vessiot extensions with differential Galois group G whose fundamental solution matrices satisfy the algebraic relations defining the group G. The structural information of these extensions is obtained by connecting results from the theory of Lie groups with differential Galois theory. As above we define a group action of G on E and show that E is a Picard-Vessiot extension of the fixed field E^G with differential Galois group G. The fixed field is then generated by l invariants $\boldsymbol{h} = (h_1, \ldots, h_l)$ which are differentially algebraically independent over the constants and the coefficients of the linear differential equation defining the extension are differential polynomials in these invariants.

Two matrix differential equations $\partial(\boldsymbol{y}) = A_1\boldsymbol{y}$ and $\partial(\boldsymbol{y}) = A_2\boldsymbol{y}$ are called gauge equivalent if there exists $B \in \mathrm{GL}_n$ such that

$$BA_1B^{-1} + \partial(B)B^{-1} = A_2.$$

Gauge equivalent differential equations define differentially isomorphic Picard-Vessiot extensions. Using the Lie structure of the group we show that every equation defining a G-primitive extension of a specific type is gauge equivalent to a specialization of the above constructed equation. Our construction yields so sections of isomorphism classes of G-primitive Picard-Vessiot extensions and we use the obtained structural information for a classification of all these extensions.

Wir laden alle Interessierten herzlich ein.