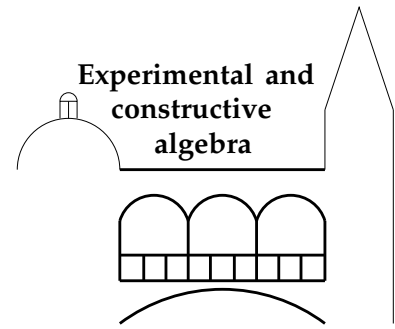


Graduiertenkolleg

Experimentelle und konstruktive Algebra



Kolloquiumsvortrag

Dienstag, 15. Januar 2019, 14:15 Uhr, Hörsaal IV

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On the Structure of G -Primitive Picard-Vessiot Extensions for the Classical Groups

In classical Galois theory there is the well-known construction of the general equation with Galois group the symmetric group S_n . One starts with n indeterminates $\mathbf{T} = (T_1, \dots, T_n)$ and considers the rational function field $\mathbb{Q}(\mathbf{T})$. The group S_n acts on $\mathbb{Q}(\mathbf{T})$ by permuting the indeterminates T_1, \dots, T_n . One can show then that $\mathbb{Q}(\mathbf{T})$ is a Galois extension of the fixed field $\mathbb{Q}(\mathbf{T})^{S_n}$ for a polynomial equation of degree n whose coefficients are the elementary symmetric polynomials $s_1(\mathbf{T}), \dots, s_n(\mathbf{T})$ in \mathbf{T} . Moreover the fixed field is generated by these polynomials and they are algebraically independent over \mathbb{Q} .

In this talk we do a similar construction in differential Galois theory for the classical Lie groups. Let G be one of these groups and denote by l its Lie rank. We start our construction with a differential field $C\langle\boldsymbol{\eta}\rangle$ which is differentially generated by l differential indeterminates $\boldsymbol{\eta} = (\eta_1, \dots, \eta_l)$ over the constants C . We use this purely differential transcendental extension to build our final general extension field $E \supset C\langle\boldsymbol{\eta}\rangle$ by taking into account the structure of G -primitive Picard-Vessiot extensions. These are Picard-Vessiot extensions with differential Galois group G whose fundamental solution matrices satisfy the algebraic relations defining the group G . The structural information of these extensions is obtained by connecting results from the theory of Lie groups with differential Galois theory. As above we define a group action of G on E and show that E is a Picard-Vessiot extension of the fixed field E^G with differential Galois group G . The fixed field is then generated by l invariants $\mathbf{h} = (h_1, \dots, h_l)$ which are differentially algebraically independent over the constants and the coefficients of the linear differential equation defining the extension are differential polynomials in these invariants.

Two matrix differential equations $\partial(\mathbf{y}) = A_1\mathbf{y}$ and $\partial(\mathbf{y}) = A_2\mathbf{y}$ are called gauge equivalent if there exists $B \in \text{GL}_n$ such that

$$BA_1B^{-1} + \partial(B)B^{-1} = A_2.$$

Gauge equivalent differential equations define differentially isomorphic Picard-Vessiot extensions. Using the Lie structure of the group we show that every equation defining a G -primitive extension of a specific type is gauge equivalent to a specialization of the above constructed

equation. Our construction yields so sections of isomorphism classes of G -primitive Picard-Vessiot extensions and we use the obtained structural information for a classification of all these extensions.

Wir laden alle Interessierten herzlich ein.