

Computing Homology Groups in Maple

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homalg

- homological algebra in Maple:
compute with sequences of modules
- depends on a “ring package” that implements the
ring-specific arithmetics
- compute
 - presentations of modules
 - homology = defect of exactness
 - connecting homomorphism of long exact
sequence
 - ...

Integers

- essentially a sample “ring package” for homalg
- main functionality: computation of Hermite / Smith normal form (elementary divisors)
- application to algebraic topology: computation of (co)homology groups of simplicial complexes

Cohomology

Let G be an abelian group (e.g. \mathbb{Z}),
 (C, ∂) a chain complex.

$$\dots \longrightarrow \text{Hom}(C_{q-1}, G) \xrightarrow{\delta_{q-1}} \text{Hom}(C_q, G) \xrightarrow{\delta_q} \text{Hom}(C_{q+1}, G) \longrightarrow \dots$$

- $\text{Hom}(C_q, G)$: *group of q -cochains with coeff. in G*
- δ : *coboundary operator*
- $\delta(\varphi)$: *coboundary of φ*
- $\delta(\varphi) = 0 \iff \varphi$ *is cocycle*
- $H^q(C, G) := \ker \delta_q / \text{im } \delta_{q-1}$

q -th cohomology group of C with coeff. in G