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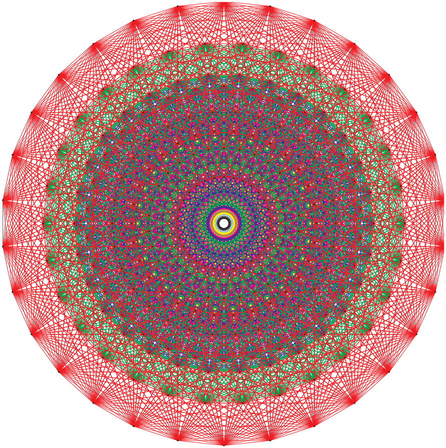
How E_8 made the headlines

Max Neunhöffer



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A news story

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Classification

The unitary representations of the split real form of the simple complex Lie group E_8 have been classified.

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Let $\mathbb{K}$ be either $\mathbb{R}$ or $\mathbb{C}$.

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The easiest example

Let's look a bit closer at

$$\mathrm{SO}_2 = \left\{ \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \middle| 0 \leq \alpha < 2\pi \right\} < \mathrm{GL}_2(\mathbb{R}).$$

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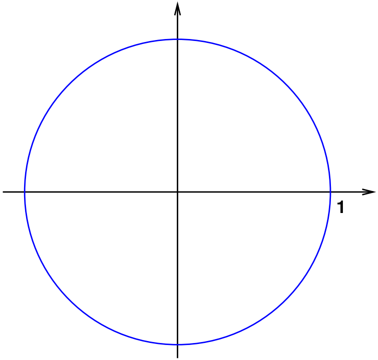

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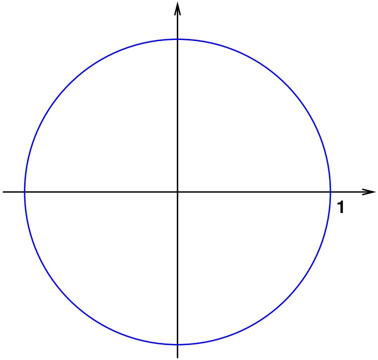

We see that it is smooth and compact (closed+bounded)!

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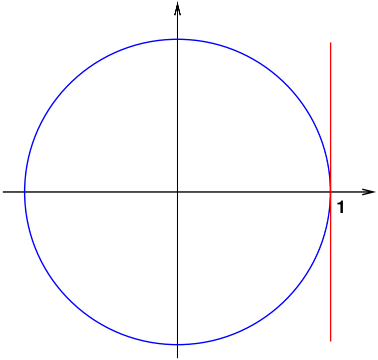

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What does E_8 mean?

Theorem (Lie correspondence)

A connected Lie group is (up to a discrete, central extension) determined by its Lie algebra, which is the tangent space at the identity.

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Simple complex Lie algebras are classified by the Dynkin type A_n, B_n, C_n, D_n, E_6, E_7, E_8, F_4 or G_2 (Killing, 1887).

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This leads to a classification of the connected simple complex Lie groups.

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Classification

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E_8 is simply the largest of the 5 exceptional cases.

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Real and complex

Example: The real and complex worlds are related!

Look at $G := \mathrm{SL}_n(\mathbb{C})$ and two automorphisms of order 2:

Automorphisms:	$c : A \mapsto \overline{A}$	$t : A \mapsto \overline{A^{-T}}$

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Theorem (Cartan, 1890)

A connected, semisimple, complex Lie group has only finitely many real forms.
There is always a split one which is not compact.

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The split real form $E_8{}^8$ of E_8 . . .

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E_8 is combinatorially described by its root system in $\mathbb{R}^8$:

$$\Phi := \{a \in \{\pm\tfrac{1}{2}\}^8 \mid \sum a_i \text{ even}\} \cup \{a \in \{0, \pm 1\}^8 \mid \sum (a_i)^2 = 2\} \subseteq \mathbb{R}^8$$

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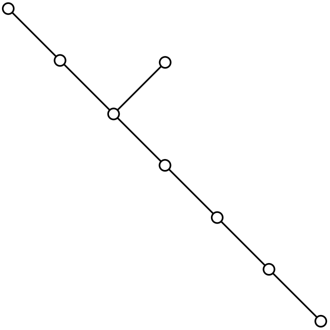

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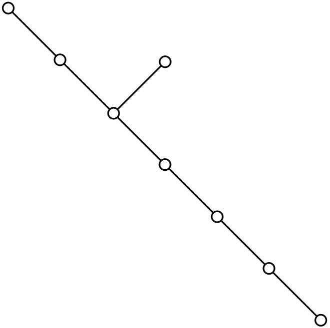
$\langle \Phi \rangle_{\mathbb{Z}}$ is the E_8-lattice: $\{a \in \mathbb{Z}^8 \cup (\mathbb{Z} + \frac{1}{2})^8 \mid \sum a_i \text{ even}\}$



The split real form $E_8{}^8$ of E_8 ...

... is a 248-dimensional real non-compact Lie group.

E_8 is combinatorially described by its root system in $\mathbb{R}^8$:

$$\Phi := \{a \in \{\pm\tfrac{1}{2}\}^8 \mid \sum a_i \text{ even}\} \cup \{a \in \{0, \pm 1\}^8 \mid \sum (a_i)^2 = 2\} \subseteq \mathbb{R}^8$$

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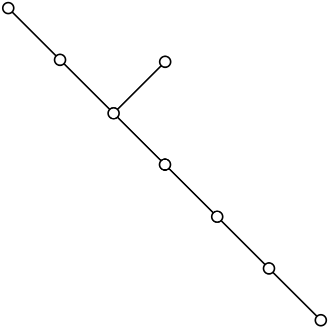

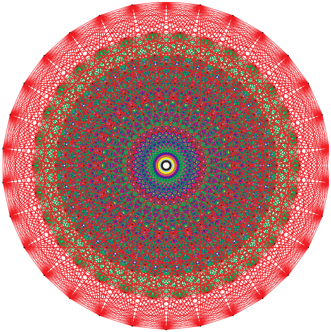

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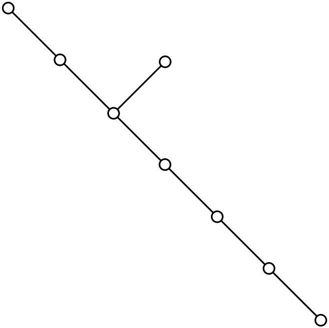

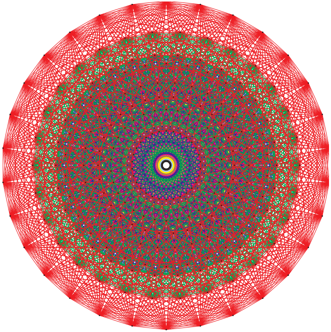

$E_8{}^8$ is a group of symmetries of a 57-dimensional variety.

How E_8 made the headlines

Max Neunhöffer

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Representations

Definition (Representation)

A representation of a Lie group G is a continuous group homomorphism $D : G \to \mathrm{GL}(V)$, (V a $\mathbb{C}$-vector space).

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$$\chi_D : G \to \mathbb{C}, \quad g \mapsto \mathrm{Tr}(D(g)).$$

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Def: $\mathrm{Irr}(G) :=$ {characters of irreducible representations}

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The Atlas project

Aims

Collect and make available information on semisimple Lie groups and their (unitary) representations.

http://www.liegroups.org/

People: The core group

Jeffrey Adams	(University of Maryland),
Dan Barbasch	(Cornell),
John Stembridge	(University of Michigan),
Peter Trapa	(University of Utah),
Marc van Leeuwen	(Poitiers),
David Vogan	(MIT), and
Fokko du Cloux	(Lyons, until his death in 2006).

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Questions about character tables

Fundamental questions

1. How can a character be described in a finite way, if G has infinitely many conjugacy classes?

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- Every unitary representation is a direct sum of finite-dimensional irreducible ones (Peter and Weyl).

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Maximal Tori

Definition (Torus)

Let $S^1 := \{z \in \mathbb{C} \mid |z| = 1\}$. A torus is a direct product

$$\mathbb{T} := \underbrace{S^1 \times S^1 \times \cdots \times S^1}_{r \text{ factors}}$$

This is a compact, abelian Lie group,

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Every conjugacy class of G meets $\mathbb{T}$.
$x, y \in \mathbb{T}$ *conjugate in* $G \iff x, y$ *conjugate in* $N_G(\mathbb{T})$

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Character tables for compact Lie groups

Theorem (Weyl Character Formula)

G: connected, compact Lie group, $\mathbb{T}$: a maximal torus.

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$$G = SU_2 = \left\{ \begin{bmatrix} z & -\overline{w} \\ w & \overline{z} \end{bmatrix} \in \mathbb{C}^{2\times 2} \;\middle|\; |z|^2 + |w|^2 = 1 \right\}$$

$$\mathbb{T} = S^1 = \left\{ \begin{bmatrix} z & 0 \\ 0 & \overline{z} \end{bmatrix} \in \mathbb{C}^{2\times 2} \;\middle|\; |z|^2 = 1 \right\}$$

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The characters of $\mathbb{T}$ are indexed by $\mathbb{Z}$, the highest weights are $\mathbb{N} \cup \{0\}$, the character χ_n on $\mathbb{T}$ is

$$\chi_n(z) = \frac{z^{n+1} - z^{-(n+1)}}{z - z^{-1}}.$$

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Trouble ahead

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Big questions:

Which of them are unitary and what are their dimensions and character values?

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“Character tables” of non-compact Lie groups

Theorem (Jantzen-Zuckerman “translation principle”)

$$\mathrm{Irr}(G) = \dot{\bigcup}_{i=1}^{N} \mathcal{F}_i \qquad \textit{(“translation families”)}$$

Each $\mathcal{F}_i$ *is parametrised by the set* $\mathrm{IChar}(G)$ *of infinitesimal characters:* $\mathcal{F}_i = \{\chi_i^\lambda \mid \lambda \in \mathrm{IChar}(G)\}$.

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$$\chi_i^\lambda = \sum_{j=1}^{N} P_{i,j}(1) \cdot \Theta_j^\lambda \qquad \text{for } 1 \le i \le N$$

for certain functions $\Theta_1^\lambda, \ldots, \Theta_N^\lambda$ (Harish-Chandra).

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The polynomials $P_{i,j} \in \mathbb{Z}[q, q^{-1}]$ are the famous Kazhdan-Lusztig-Vogan-polynomials.

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"Character tables" of non-compact Lie groups

Theorem (Jantzen-Zuckerman "translation principle")

$$\mathrm{Irr}(G) = \dot{\bigcup}_{i=1}^{N} \mathcal{F}_i \qquad \textit{("translation families")}$$

Each $\mathcal{F}_i$ is parametrised by the set $\mathrm{IChar}(G)$ of infinitesimal characters: $\mathcal{F}_i = \{\chi_i^\lambda \mid \lambda \in \mathrm{IChar}(G)\}$.

$$\chi_i^\lambda = \sum_{j=1}^{N} P_{i,j}(1) \cdot \Theta_j^\lambda \qquad \text{for } 1 \leq i \leq N$$

for certain functions $\Theta_1^\lambda, \ldots, \Theta_N^\lambda$ (Harish-Chandra).

The polynomials $P_{i,j} \in \mathbb{Z}[q, q^{-1}]$ are the famous Kazhdan-Lusztig-Vogan-polynomials.

The $P_{i,j}$ do not depend on λ, and $P_{i,j} = 0$ for $i < j$.

$\rightarrow$ Very deep results about intersection homology by Kazhdan, Lusztig, Beilinson, Bernstein, Vogan, ...

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The ${E_8}^8$-computation

Facts:

- For ${E_8}^8$ there are 453060 translation families.

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- Altogether, the computation now needs about 50 hours and produces 60 GB of output.

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- Extremely valuable: quotes from external experts

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Lesson learned:

With enough effort, it can be done!

Thanks!