

A new
programmer's
interface for
vectors and
matrices

Max Neunhoffer

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A new programmer's interface for vectors and matrices

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What is a vector? What is a matrix?

Up to now in GAP, they are just lists:

```
gap> v := [1,2,3];
[ 1, 2, 3 ]
gap> m := [[0,1],[1,0]];
[ [ 0, 1 ], [ 1, 0 ] ]
```

However, there are different **representations**:

```
gap> m := m*Z(2);;
gap> for r in m do ConvertToVectorRep(r,2); od;
gap> m;
[ <a GF2 vector of length 2>,
  <a GF2 vector of length 2> ]
gap> ConvertToMatrixRep(m,2);;
gap> m;
<a 2x2 matrix over GF2>
```

We can use the method selection only for the last matrix!

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Method selection problems

```
gap> h:=[1..100];;
gap> m:=List([1..100000],i->Z(2)*[1..1000]);;
gap> TypeObj(m);; time;
1908
gap> TypeObj(m);; time;
16
gap> for i in h do Reversed(m); od; time;
24
gap> for i in h do ReversedOp(m); od; time;
2888
gap> ConvertToMatrixRep(m,2);;
gap> TypeObj(m);; time;
0
gap> for i in h do TypeObj(m); od; time;
0
```

Type computation and method selection for
mutable plain lists
can take a significant amount of time!

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New filters

Solution: Wrap 'em up. Define an **interface to them**.

```
DeclareCategory("IsRowVectorObj",  
               IsVector and IsCopyable);
```

```
DeclareCategory("IsMatrixObj",  
               IsVector and IsScalar and IsCopyable);
```

Vectors and matrices are **no longer necessarily lists**.

```
DeclareCategory("IsRowListMatrix",  
               IsMatrixObj);  
DeclareCategory("IsFlatMatrix", IsMatrixObj);
```

These two types of matrices are not only **different representations**, they also **behave differently**.

“Row list” vs. “flat” matrices

A row list matrix

- behaves like a list of row objects and
- has individual GAP objects as rows,
- is like a list that insists on being **dense** and containing **only row objects of the right type and size.**

A flat matrix

- consists of a **single GAP object**,
- the rows are **part of this object**, **not individual objects**,
- **has to copy rows** to exchange or permute them.

All matrices

- know their **base domain**,
- know their **dimensions**, and
- can have **0 rows or 0 columns.**

Operations

Attributes for vectors:

BaseDomain, Length.

Attributes for matrices:

BaseDomain, Length, RowLength, DimensionsMat.

Lots of operations are defined (see below).

Important:

Objects and derived objects keep their representation!

Generic code does not have to worry about this!

```
gap> Display(m);
  1 . 1
  . 1 .
gap> ExtractSubMatrix(m, [2, 1], [1, 3]);
<a 2x2 matrix over GF2>
gap> Display(last);
  . .
  1 1
```

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Constructing new vectors and matrices

```
gap> v := NewRowVector(IsPlistVectorRep,  
                        Rationals, [1, 2, 3]);  
<plist vector over Rationals of length 3>  
gap> m := NewMatrix(IsPlistMatrixRep,  
                    Rationals, 3, [[4, 5, 6]]);  
<1x3-matrix over Rationals>  
gap> Add(m, v);
```

This uses GAP's [constructors](#).

A constructor is an operation, for which the method selection works **differently in the first argument**:
The argument is a filter, and a method must be installed for a subfilter to be taken.

Packages can have constructor methods for new types.

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GAP's constructors explained

```
DeclareCategory ("IsA", IsComponentObjectRep);
DeclareConstructor ("MakeA", [IsA, IsInt]);
tA := NewType (CyclotomicsFamily, IsA);;
InstallMethod (MakeA, [IsA, IsInt],
  function (f, x)
    return Objectify (tA, rec (x := x));
  end);

DeclareCategory ("IsAB", IsA);
tAB := NewType (CyclotomicsFamily, IsAB);;
InstallMethod (MakeA, [IsAB, IsInt],
  function (f, x)
    return Objectify (tAB, rec (x := x));
  end);

gap> a := MakeA (IsA, 17);;
gap> [ IsA(a), IsAB(a) ];
[ true, false ]
gap> b := MakeA (IsAB, 17);;
gap> [ IsA(b), IsAB(b) ];
[ true, true ]
```

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Preserving the representation

```
gap> ConstructingFilter(m);  
<Operation "IsPlistMatrixRep">
```

Derived objects:

ZeroMutable, ShallowCopy, OneImmutable,
MutableCopyMat, ...

New objects in same representation:

```
gap> v := NewRowVector(IsPlistVectorRep,  
                       Rationals, [1, 2, 3]);;  
gap> m := NewMatrix(IsPlistMatrixRep,  
                    Rationals, 3, [[4, 5, 6]]);;  
gap> ZeroVector(10, v);  
<plist vector over Rationals of length 10>  
gap> Vector([6, 7, 8, 9], m);  
<plist vector over Rationals of length 4>  
gap> IdentityMatrix(12, m);  
<12x12-matrix over Rationals>  
gap> n := Matrix([], 3, m);  
<0x3-matrix over Rationals>
```

Flat vs. row list matrices

Objects in the filter `IsRowListMatrix`

- have most **list operations**: Add, Remove, IsBound, Unbind, `[]`, `[] :=`, `{}`, `{} :=`, Append, ShallowCopy, List,
- they simply **insist** on being **dense** and on containing only **vectors of the right length and type**.

Objects in the filter `IsFlatMatrix`

- have `[]`, which **creates a reference**,
- `[] :=`, `{}`, `{} :=`, which **copy data**, and
- **do not support** Add, Remove, IsBound, Unbind, Append.
- ShallowCopy is a **full copy**.

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Creating a companion matrix

```
cm := function(p,mat)
  local bd,one,l,n,ll,i;
  bd := BaseDomain(mat); one := One(bd);
  l := CoefficientsOfUnivariatePolynomial(p);
  n := Length(l)-1;
  l := Vector(-l{[1..n]},mat);
  ll := ListWithIdenticalEntries(n,0);
  ll[n] := l;
  for i in [1..n-1] do
    ll[i] := ZeroMutable(l);
    ll[i][i+1] := one;
  od;
  return Matrix(ll,n,mat);
end;

gap> x:=X(Rationals);;
gap> Display(cm(x^3-2*x^2-5,m));
<3x3-matrix over Rationals:
[[ 0, 1, 0 ]
 [ 0, 0, 1 ]
 [ 5, 0, 2 ]]>
```