Recognising Matrix Groups

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Lots of others contributed ideas, results, and code.

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$\mathbb{F}_q$ field with q elements

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- Do all this "efficiently".

We call this "solving the word problem in G".

Straight line programs

Example:

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# input:
r:= [ a, b, c ];
# program:
r[4]:= r[1]^2*r[2]*r[1]^-2;
r[5]:= r[4]*r[3]^7;
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- can express long products memory efficiently.

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$\Longrightarrow$ this decision shortened SLPs from 500 000 steps down to 500 in examples

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$\Longrightarrow$ We work “modulo” this problem.

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- Improve algorithms

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With a matrix group $G = \langle M_1, \ldots, M_k \rangle \leq \mathrm{GL}_d(q)$ we can

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- create (pseudo-) random elements
- act with matrices on vectors or on subspaces $\longrightarrow$ gives homomorphisms to permutation groups

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- $\Longrightarrow g^{-1} \cdot g' \in N$
  $\longrightarrow$ this creates a (pseudo-) random element in N

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What does this help for G?

- $|G| = |H| \cdot |N|$
- G has a subgroup N and a factor group H
- we can solve the word problem in G!

Get the recursion going . . .

Choose as "nice generators" $M'_1, \ldots, M'_{k'}$ for G:

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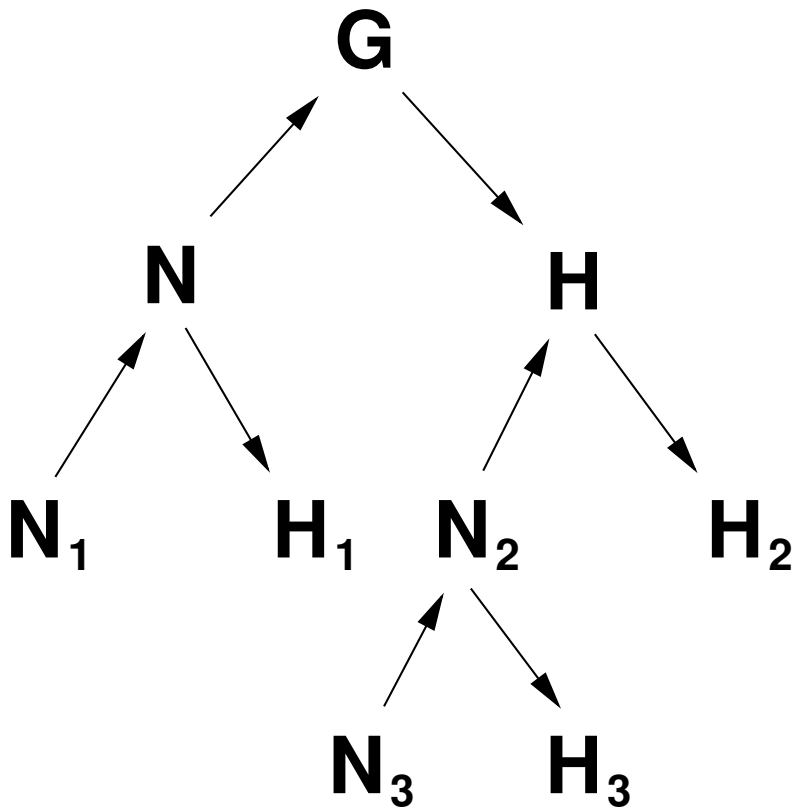
- put together S from S' and S'' plus one multiplication

A Composition Tree



Upward arrows: monomorphisms
Downward arrows: epimorphisms

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Unfortunately, it is not yet analysed to be polynomial-time!

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Theorem (Aschbacher, 1984)

If $G < \mathrm{GL}_d(q)$ then it falls under at least one of:

C1 *G leaves invariant a subspace* $0 < W < \mathbb{F}_q^{1\times d}$

C2 *G preserves a decomposition* $\mathbb{F}_q^{1\times d} \cong V_1 \oplus \cdots \oplus V_j$

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C4 *G preserves a decomposition* $\mathbb{F}_q^{1\times d} \cong V_1 \otimes V_2$

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C8 *G contains a "classical group" like* $\mathrm{SL}_d(q)$ *or* $\mathrm{Sp}_d(q)$

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All classes C1 to C7 are defined "geometrically" and promise some kind of homomorphism or "simplification".

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C2 *G preserves a decomposition* $\mathbb{F}_q^{1\times d} \cong V_1 \oplus \cdots \oplus V_j$

...

C4 *G preserves a decomposition* $\mathbb{F}_q^{1\times d} \cong V_1 \otimes V_2$

...

C8 *G contains a "classical group" like* $\mathrm{SL}_d(q)$ *or* $\mathrm{Sp}_d(q)$

C9 *G is a quasi-simple group*

All classes C1 to C7 are defined "geometrically" and promise some kind of homomorphism or "simplification".

C8 and C9 produce leaves in the composition tree.

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- directly solve the word problem.

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Already there:

- a completely working framework for composition trees
- documentation of it

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- a start of a database of hints for recognised leaves

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Still missing:

- analysis of the low index procedure

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- a whole lot of documentation

Help is appreciated

Everybody is welcome to contribute.

We need ideas, code, and analysis.