LMS Short Course on Computational Group Theory

Lab session 3

Using group libraries and conducting searches

1. How many non-abelian groups of order 24 are there? (Of course, with this formulation we always mean "up to isomorphism".)
   Hint : You want to use the **small groups library**, → `?NumberSmallGroups`, → `?AllSmallGroups`, → `?IsAbelian`
2. How many non-abelian groups of order 128 are there?
   Compute the average of the sizes of their centres.
   Hint 1: To automate the counting you can use → `?List` and → `?Collected` or use → `?Filtered` to get them all
   Hint 2: For the centres the → `?arrow notation` could be convenient
3. What ID does the group generated by the three permutations
   $$(2,4,6,8,10), (1,9)(2,8)(3,7)(4,6) \text{ and } (1,6)(2,7)(3,8)(4,9)(5,10)$$
   have?
   Hint : → `?IdGroup`, → `?SmallGroup`
4. Find all composition series of all non-solvable groups of order 120.
   Hint 1: You can avoid making all small groups of order 120 and then `Filtered` by using the more sophisticated syntax of → `?AllSmallGroups`
   Hint 2: Simply compute all normal subgroups of them using → `?NormalSubgroups`
5. How many elements of order 3 do all groups of order 48 have together? (Of course, we mean to take one group of each isomorphism type.)
   Hint 1: Fetch them all using `AllSmallGroups`, for each of them, ask for all elements (→ `?Elements`) and let GAP count.
   Hint 2: If you have a list `L` of groups, you can use a → `?for` loop to run through all of them like this:
   ```
   for g in L do
       do stuff with g
   od;
   ```
   Hint 3: The same technique can be used to run through a list of elements. Use → `?if` to decide, whether or not an element has order 3 as in
   ```
   if Order(x) = 3 then
       increase a counter
   fi;
   ```
6. Show that the Sylow 2-subgroups of the Mathieu group M_{24} and the sporadic simple Held group He are isomorphic.
   Hint 1: → `?MathieuGroup`
   Hint 2: Use the AtlasRep package and → `?AtlasGroup` to fetch generators of He from the internet.
   Hint 3: Use → `?SmallerDegreePermutationRepresentation` for the Sylow 2-subgroup of He
   Hint 4: Use → `?IsomorphismGroups`
7. Find the group with the fewest elements that is non-abelian, has trivial center and contains an element a of order 2 and an element b of order 3 such that ab has order 5.