LMS Short Course on Computational Group Theory

Lab session 4

Finitely presented groups

1. Make in GAP a free group F on two generators a and b. Assign the generators to two variables `a` and `b` and produce a few words in F. See how inverses are cancelled automatically.
   Hint 1: Use `FreeGroup("a","b");`
   Hint 2: Use `GeneratorsOfGroup`.
2. Give the presentation
   $$G := \left\langle a, b \mid a^2, b^3, (ab)^{11}, [a,b]^6, (ababab^{-1})^6 \right\rangle$$
   to GAP. Find the order of G.
   Hint 1: Type in the relations in a list `R` and use the `F/R` operation to form G.
   Hint 2: Simply try the `Size` command.
3. Compute an isomorphism to a permutation group.
   Hint 1: $\rightarrow$ `?IsomorphismPermGroup` and $\rightarrow$ `?Image`
4. Perform a coset enumeration of
   $$H := \left\langle a, b \mid a^2, b^3, abab \right\rangle$$
   on the cosets of the trivial group.
   Hint 1: $\rightarrow$ `?TrivialSubgroup` and $\rightarrow$ `?CosetTable`.
5. Perform a coset enumeration of H on the cosets of the group generated by a. Derive from this a group homomorphism into a symmetric group (without using `FactorCosetAction`).
   Hint 1: $\rightarrow$ `?CosetTable` and $\rightarrow$ `?PermList`
6. Enter the group
   $$K := \left\langle s, t \mid s^3, t^2 \right\rangle$$
   into GAP and determine its size.
   Hint 1: Hit "Ctrl-C" on the keyboard to interrupt GAP.
   Hint 2: Compute the $\rightarrow$ `?AbelianInvariants`.
   Hint 3: Use $\rightarrow$ `?LowIndexSubgroupsFpGroup` and then `AbelianInvariants` for some of the subgroups.
7. Investigate the Fibonacci group
   $$F(5) := \langle a, b, c, d, e \mid ab = c, bc = d, cd = e, de = a, ea = b \rangle$$
8. Investigate the Fibonacci group
   $$F(6) := \langle a, b, c, d, e, f \mid ab = c, bc = d, cd = e, de = f, ef = a, fa = b \rangle$$
9. Use the following program to make an FP group:

```
n:=10; f:=FreeGroup(10); g:=GeneratorsOfGroup(f); rels:=[];
for i in [1..n] do Add(rels,g[i]^2); od;
for i in [1..n-2] do for j in [i+2..n] do
    Add(rels,Comm(g[i],g[j]));
od; od;
for i in [1..n-1] do Add(rels,(g[i]*g[i+1])^3); od;
G := f/rels;
```

   Determine the order of G.
   Hint 1: Try to enumerate the cosets of a subgroup of G.
   Hint 2: Once you have the group homomorphism, compute its $\rightarrow$ `?Kernel`.