UNIVERSITY OF ST ANDREWS
MT5827 Lie Algebras
Tutorial Sheet 3: Chapters 3 and 4

1. Check for the action of $\mathrm{sl}_2(\mathbb{C})$ on the module V_d given in Proposition 7.1 of the course that the relations $[e, f] = h$ and $[h, e] = 2e$ and $[h, f] = -2f$ hold on all basis vectors, that is

$$(X^a Y^b)h = ((X^a Y^b)e)f - ((X^a Y^b)f)e$$

for all $a, b \in \mathbb{N} \cup \{0\}$ with $a + b = d$, and the same for the other two relations.

2. Use the classification of $\mathrm{sl}_2(\mathbb{C})$-modules in the course to write down all isomorphism types of 5-dimensional representations of $\mathrm{sl}_2(\mathbb{C})$.

3. Let V be a finite-dimensional vector space over $\mathbb{F}$ and let $S, T \in \mathrm{End}(V)$ be two endomorphisms that commute, that is, $ST = TS$. Let $0 < W \leq V$ be the eigenspace for the eigenvalue λ of S. Show that W is invariant under T, that is, $WT \leq W$.
Conclude from this that if both S and T are diagonalisable, then there is a basis $(v_1, \ldots, v_n)$ of V such that both S and T correspond to diagonal matrices with respect to this basis.