PROBLEMS

1. Let G be a finite group and let R be a ring with identity. Assume G acts on R by automorphisms. Let $R^G = \{r \in R \mid \sigma(r) = r \text{ for all } \sigma \in G\}$. Put

$$\mathrm{tr}_G \colon R \to R^G, \quad x \mapsto \sum_{\sigma \in G} \sigma(x).$$

It is easily seen that tr_G is surjective if and only if $1 \in \mathrm{Im}(\mathrm{tr}_G)$. Furthermore, if tr_G is surjective, then $\mathrm{tr}_H : R \to R^H$ is surjective for any subgroup H of G. A result of Aljadeff and Ginosard (based on Chouinard's Theorem) states that if tr_E is surjective onto R^E for all elementary abelian p-subgroups of G (p any prime), then tr_G is surjective onto R^G. Under these assumptions, let $r_E \in R$ so that $\mathrm{tr}_E(r_E) = 1$. By the previous, there exists an $r_G \in R$ so that $\mathrm{tr}_G(r_G) = 1$. By an observation of Shelah, it is known that there exists a formula (a polynomial in variables $g(r_E)$, $g \in G$ and E elementary abelian) to calculate r_G. Find such a formula. There are formulas for abelian groups G. Also for some non-abelian groups a formula is known (e.g., dihedral and quaternion groups). In general, it is enough to deal with p-groups, and one can even restrict to extra special and almost extra special groups G. Recall that if G is such group then it contains a normal cyclic subgroup C of order p such that G/C is an elementary abelian p-group.

E. Aljadeff

2. A ring R is said to have IBN (the invariant basis number property) if $R^n \cong R^m$ (an isomorphism of R-modules) implies that $n = m$. It is known that if a ring R has a ring epimorphic image (mapping the identity to the identity) that has IBN then so has R. Hence, for example, a group algebra KG of a group G over a field K has IBN. Let $t : G \to \mathrm{Aut}(K)$ be a group automorphism of a group G into the automorphism group $\mathrm{Aut}(K)$ of a field K. Does the skew group algebra K_tG has IBN? In case G has subexponential growth the answer is positive.

E. Aljadeff

3. Let G be a torsion free nilpotent group of finite cohomological dimension, say $\mathrm{cd}(G) = n$. Then $\mathrm{gl.dim}(\mathbb{C}G) = n$ and, for $\alpha \in \mathrm{H}^2(G, \mathbb{C}^\times)$, $1 \leq \mathrm{gl.dim}(\mathbb{C}^\alpha G) \leq n$. Also, if H is a subgroup of G with $\mathrm{cd}(H) = r$ and $\alpha_{|H} = 1$, then $\mathbb{C}H \subseteq \mathbb{C}^\alpha G$ and thus $\mathrm{gl.dim}(\mathbb{C}^\alpha G) \geq r$.

Question/Conjecture. If $\mathrm{gl.dim}(\mathbb{C}^\alpha G) = r$ then there exists a subgroup H of G so that $\alpha_{|H} = 1$ and $\mathrm{gl.dim}(\mathbb{C}H) = r$. The case $r = n$ has been answered positively.

E. Aljadeff

4. Let $\mathrm{t}(G)$ be the set of elements of finite order in a group G. Let $\mathrm{V}_\mathrm{t}(\mathbb{Z}G)$ be the set of torsion units in the normalized unit group $\mathrm{V}(\mathbb{Z}G)$ of the integral group ring $\mathbb{Z}G$. For an element $g \in \mathrm{t}(G)$, of order n, define $\widehat{g} = \sum_{i=0}^{n-1} g^i$, and let $\mathrm{B}_g = \mathrm{LB}_g \cup \mathrm{RB}_g$, where $\mathrm{LB}_g = \{g + (g-1)w\widehat{g} \mid g \in \mathrm{t}(G), w \in \mathbb{Z}G\}$ and $\mathrm{RB}_g = \{g + \widehat{g}w(g-1) \mid g \in \mathrm{t}(G), w \in \mathbb{Z}G\}$. Clearly, $\mathrm{B}_g \subseteq \mathrm{V}_\mathrm{t}(\mathbb{Z}G)$. Furthermore, set $\mathrm{BV}(\mathbb{Z}G) = \{x^{-1}yx \mid x \in \mathrm{B}_g \text{and } y \in \mathrm{B}_h, \text{for } g, h \in \mathrm{t}(G)\} \subseteq \mathrm{V}_\mathrm{t}(\mathbb{Z}G)$. For which groups G does $\mathrm{BV}(\mathbb{Z}G) = \mathrm{V}_\mathrm{t}(\mathbb{Z}G)$ hold?

For $w_0 = a \in \mathrm{t}(G)$ and $b \in G$ define inductively $w_{i+1} = w_i + (w_i - 1)b\widehat{w_i}$, and set $S(a, b) = \{w_i \mid i = 0, 1, \ldots\}$. What can be said about the set $S(a, b)$? For example, what is the structure of the group generated by $S(a, b)$?

Comment. Each w_i is of the same order as a. If G is a finite group, then each w_i is conjugate to a by a unit in $\mathbb{Q}G$, since $\chi((a^j - 1)b\hat{a}) = 0$ for each irreducible character χ of G.

V. Bovdi

5. Let A be a finite dimensional K-algebra, K being a field, and let $\mathrm{J}(A)$ be its radical. A vector space basis B of A is said to be *filtered multiplicative* provided that $b_1, b_2 \in B$ implies $b_1b_2 = 0$ or $b_1b_2 \in B$, and $B \cap \mathrm{J}(A)$ is a basis of $\mathrm{J}(A)$. R. Bautista, P. Gabriel, A. V. Roĭter and L. Salmerón (*Invent. Math.* **81** (1985), no. 2, 217–285) proved that if K is algebraically closed and A is of finite representation type, then A has a filtered multiplicative basis.

Suppose G is a finite non-abelian p-group, and $\mathrm{char}(K) = p$. For $p > 2$, is it true that KG does not have a filtered multiplicative basis?

V. Bovdi

6. A discrete group G is *amenable* if it has a finitely-additive left-invariant probability measure. Let G be a group and $\mathrm{V}(\mathbb{Z}G)$ the group of augmentation one units of the integral group ring $\mathbb{Z}G$. Suppose x and y are nontrivial elements in $\mathrm{V}(\mathbb{Z}G)$. When is the group $\langle x, y\rangle$ non-amenable? Note that if a group contains a free (non-abelian) subgroup on two generators then it is non-amenable.

V. Bovdi

7. Let G be a finite group, and x in G an element of order n. For $i > 0$ with $(i, n) = 1$, and a multiple m of $\varphi(n)$, with φ denoting Euler's function, the element

$$u_{i,m}(x) = (1 + x + \cdots + x^{i-1})^m + \frac{1 - i^m}{n}(1 + x + \cdots + x^{n-1})$$

of $\mathbb{Z}G$ is a unit, called a *Bass cyclic unit.* Suppose that $n \notin \{1, 2, 3, 4, 6\}$, and that there is $y \in G$ with $xy \neq yx$. Do their exist natural numbers k, m, r and s so that $\langle u_{k,m}(x), u_{r,s}(yxy^{-1})\rangle \cong \langle u_{k,m}(x)\rangle * \langle u_{r,s}(yxy^{-1})\rangle$, a free product in the unit group of the integral group ring $\mathbb{Z}G$?

J. Gonçalves

8. Let G be a finite p-group and let k be the field with p elements. Suppose that G has a normal complement in the unit group $\mathrm{U}(kG)$ of the group algebra kG. (A *normal complement* for G in $\mathrm{U}(kG)$ is a normal subgroup N of $\mathrm{U}(kG)$ so that $\mathrm{U}(kG) = G \ltimes N$.) Let H be a group such that the k-algebras kG and kH are isomorphic. Are the groups G and H are isomorphic? The proof given by F. Röhl (*Proc. Amer. Math. Soc.* **111** (1991), no. 3, 611–618) contains a mistake. The same question with G of nilpotency class 2 (such a G has a normal complement, for then G is a circle group (R. Sandling, *Math. Z.* **140** (1974), 195–202).

M. Hertweck

9. Let G be a finite group. For an element $\sum_{g\in G} a_g g$ (all a_g in $\mathbb{Z}$) of the integral group ring $\mathbb{Z}G$, and any $x \in G$, set $\varepsilon_x(u) = \sum_{g\in C(x)} a_g$, where $C(x)$ denotes the G-conjugacy class of x. The $\varepsilon_x(u)$ are called the *partial augmentations* of u. The *augmentation* of u is the sum of all coefficients a_g.

Now let $G = \mathrm{GL}_n(k)$, the general linear group of degree n over a finite field k, and let $\mathrm{M}_n(k)$ be the $n \times n$ matrix ring over k. Let $\mathrm{V}(\mathbb{Z}G)$ denote the group of augmentation one units in the integral group ring $\mathbb{Z}G$. The embedding $G \subseteq \mathrm{M}_n(k)$ yields a ring homomorphism $\mathbb{Z}G \to \mathrm{M}_n(k)$ and thus we obtain a surjective group homomorphism $\pi\colon \mathrm{V}(\mathbb{Z}G) \to G$. For a torsion unit u in $\mathrm{V}(\mathbb{Z}G)$, is it true that $\varepsilon_{\pi(u)}(u)$ is nonzero?

M. Hertweck

10. Let G be a finite group, and ζ a primitive $|G|$-th complex root of unity. For each prime divisor p of $|G|$ and each central primitive idempotent e in the group ring $\mathbb{Z}_{(p)}[\zeta]G$ (here $\mathbb{Z}_{(p)}$ denotes localisation of $\mathbb{Z}$ at p), let $g_{p,e}$ be an element of G. Set $u_p = \sum_e eg_{p,e}$ (where the sum runs over all the primitive central idempotents of $\mathbb{Z}_{(p)}[\zeta]G$). Each u_p is a torsion unit in $\mathbb{Z}_{(p)}[\zeta]G$. Suppose that u_p and u_q are conjugate by a unit of $\mathbb{C}G$, for all p, q. Is some u_p (and hence all) conjugate to an element of G by a unit of $\mathbb{C}G$?

M. Hertweck

11. (Well-known problem). Let G and H be finite groups. Assume that $\mathrm{Z}(\mathbb{Z}H) \cong \mathrm{Z}(\mathbb{Z}G)$, an isomorphism of centers of integral group rings. Does this imply that G and H have equivalent character tables? In other words, under the isomorphism, are class sums mapped to class sums? For finite nilpotent groups, the answer is yes (M. Hertweck, *to appear in Proc. Amer. Math. Soc.*).

M. Hertweck

12. Denote by $\mathrm{M}_n(R)$ the $n \times n$ matrix ring over a ring R. Let G and H be finite groups. If $\mathrm{M}_n(\mathbb{Z}G)$ and $\mathrm{M}_n(\mathbb{Z}H)$ are isomorphic rings, does it follow that $\mathbb{Z}G$ and $\mathbb{Z}H$ are isomorphic rings?

E. Jespers

13. Let $G = \langle x, y \mid x^{-1}y^2x = y^{-2},\ y^{-1}x^2y = x^{-2}\rangle$. The group G is free abelian-by-finite, torsion free and supersolvable, see (Chapter 13 in D. S. Passman, *The algebraic structure of group rings*, Wiley-Interscience, New York, 1977). Also, G is not a unique product group (S. D. Promislow, *Bull. London Math. Soc.* **20** (1988), no. 4, 302–304).

Let K be a field. For any proper subgroup H of G, the unit group $\mathrm{U}(KH)$ is trivial, that is, the units of KH are of the from λh, with $h \in H$ and $0 \neq \lambda \in K$. Is it true that $\mathrm{U}(KG)$ is trivial? (The old question whether KG can have nontrivial units if G is torsion free is due to Kaplansky.)

E. Jespers

14. Let Q_8 be the quaternion group of order 8 and C_7 the cyclic group of order 7. Find finitely many units in the integral group ring $\mathbb{Z}[Q_8 \times C_7]$ that generate a subgroup of finite index in the unit group of $\mathbb{Z}[Q_8 \times C_7]$.

The classical quaternion algebra $\mathrm{H}(\mathbb{Q}(\xi_7))$ (where ξ_7 is a primitive 7-th root of unity) is the only noncommutative simple epimorphic image of the rational group algebra $\mathbb{Q}[Q_8 \times C_7]$. The problem hence can be reduced to constructing finitely many units in $\mathrm{H}(\mathbb{Z}[\xi_7])$ that generate a subgroup of finite index in the unit group of $\mathrm{H}(\mathbb{Z}[\xi_7])$. Cf. (C. Corrales, E. Jespers, G. Leal and A. del Río, *Adv. Math.* **186** (2004), no. 2, 498–524).

E. Jespers

15. The *normalizer problem* (NP) for a group G asks whether the normalizer of G in the unit group of $\mathbb{Z}G$ consists of the "obvious" units only, that is, whether the normalizer is generated by G and the group of central units in $\mathbb{Z}G$. If this is true, we say that (NP) holds for G.

Let G be a group so that (NP) holds for all of its finite normal subgroups. Does (NP) holds for G? (this might be already implicit in Hertweck's papers). Is there a satisfactory classification of the groups for which (NP) holds?

S. O. Juriaans

16. The *finite conjugacy center* of a group G, denoted by $\Delta(G)$, is the subgroup of G consisting of the elements that have only finitely many conjugates.

Classify the groups G for which $\Delta(\mathrm{U}(\mathbb{Z}G))$ is non-central in $\mathrm{U}(\mathbb{Z}G)$.

Let A be a finite dimensional algebra over an algebraic number field and let Γ be a $\mathbb{Z}$-order in A. Suppose that the finite conjugacy centre $\Delta(\mathrm{U}(\Gamma))$ of the unit group $\mathrm{U}(\Gamma)$ of Γ is non-central in Γ. What can be said about A? Determine the Wedderburn–Malcev decomposition of A.

S. O. Juriaans

17. The *hypercenter* of a group G, denoted by $\mathrm{Z}_\infty(G)$, is the union of the terms of the upper central series of G.

For a group G, set $\mathcal{U} = \mathrm{U}(\mathbb{Z}G)$. Is $\mathrm{Z}_\infty(\mathcal{U}) \leq \Delta(\mathcal{U})$? Is $\Delta(\mathcal{U}) \leq \mathrm{N}_{\mathcal{U}}(G)$?

Let A be a finite dimensional simple $\mathbb{Q}$-algebra and Γ a $\mathbb{Z}$-order in A. Determine the finite conjugacy center of the unit group $\mathrm{U}(\Gamma)$ of Γ. The same problem for a Wedderburn component A of $\mathbb{Q}G$, where G is the counter-example to (NP) given by M. Hertweck. When Γ contains the projection $\bar{G}$ of G onto A, also determine the normalizer $\mathrm{N}_{\mathrm{U}(\Gamma)}(\bar{G})$.

S. O. Juriaans

18. (1) Classify the groups G for which $\mathrm{U}(\mathbb{Z}G)$ is a hyperbolic group in the sense of Gromov. A contribution to this problem is (S. O. Juriaans, I. B. S. Passi and D. Prasad, *Proc. Amer. Math. Soc.* **133** (2005), no. 2, 415–423).

(2) Determine groups G for which the unit group of $\mathbb{Z}G$ is finitely generated.

(3) Find a Dehn presentation of the group $\mathrm{SL}_1(\mathrm{H}(R))$, where R is the ring $\mathbb{Z}[(1+\sqrt{-7})/2]$, for which a presentation is given in (C. Corrales, E. Jespers, G. Leal and A. del Río, *Adv. Math.* **186** (2004), no. 2, 498–524). Furthermore, study the geometric properties of the compact manifold determined by this group.

S. O. Juriaans

19. Let $\mathcal{S}$ be the class of the finite groups which, whenever they are isomorphic, for some finite group G, to a subgroup of the normalized unit group $\mathrm{V}(\mathbb{Z}G)$, are necessarily isomorphic to a subgroup of G. Find members of $\mathcal{S}$.

Remark. Cyclic groups of prime power order are in $\mathcal{S}$ (J. A. Cohn and D. Livingstone, *Canad. J. Math.* **17** (1965), 583–593); $C_2 \times C_2$ is in $\mathcal{S}$ (W. Kimmerle, 2006, answering a question of Z. Marciniak); $C_p \times C_p$, for an odd prime p, is in $\mathcal{S}$ (M. Hertweck, *to appear in Comm. Algebra*). This suggests to study membership of p-groups, or of abelian groups, to $\mathcal{S}$. Not all groups belong to $\mathcal{S}$, as the counter example to the isomorphism problem shows (M. Hertweck, *Ann. of Math.* (2) **154** (2001), no. 1, 115–138).

W. Kimmerle

20. A *section* (or sub-quotient) of a group is a quotient of some subgroup of it. Let G be a finite group and denote by $\mathrm{V}(\mathbb{Z}G)$ the normalized unit group of its integral group ring $\mathbb{Z}G$. Is it true that each simple section of a finite subgroup of $\mathrm{V}(\mathbb{Z}G)$ is isomorphic to a section of G?

Remark. This is true for soluble G, since then finite subgroups of $\mathrm{V}(\mathbb{Z}G)$ are known to be soluble. Also, any group basis of $\mathbb{Z}G$ has the same chief factors as G (W. Kimmerle, R. Lyons, R. Sandling and D. N. Teague, *Proc. London Math. Soc.* (3) **60** (1990), no. 1, 89–122).

W. Kimmerle

21. Let $\Pi(X)$ be the prime graph of a group X (also called the Gruenberg–Kegel graph of X). For a finite group G, is it true that $\Pi(\mathrm{V}(\mathbb{Z}G)) = \Pi(G)$? This means that whenever $\mathrm{V}(\mathbb{Z}G)$ contains an element of order pq, for distinct primes p and q, then also G should contain an element of order pq.

Remark. This holds when G is soluble or a Frobenius group. It has been verified for some simple groups using the Luthar–Passi method. See (W. Kimmerle, *in: Groups, rings and algebras, Contemp. Math., vol. 420*, Amer. Math. Soc., 2006, pp. 215–228). The first Zassenhaus conjecture for G makes a much stronger claim. In particular for simple groups, however, it seems to be reasonable to study the "prime graph question" first.

W. Kimmerle

22. Let H be a finite group. Is there a finite group G containing H such that each torsion unit in $\mathrm{V}(\mathbb{Z}H)$ is conjugate to an element of H by a unit of $\mathbb{Q}G$? If so, is it possible to take the conjugating units from $\mathbb{Q}H$?

Remark. At the meeting, M. Hertweck remarked that if one restricts attention to torsion units of prime order only, the answer to the first question is yes.

W. Kimmerle

23. Let G be a finite group, let p be a prime, let X be a simple $\mathbb{Q}G$-module, and let L be a $\mathbb{Z}_{(p)}G$-lattice in X. Write $E := \mathrm{End}_{\mathbb{Z}_{(p)}G} L$. We have an operation morphism $\mathbb{Z}_{(p)}G \to \mathrm{End}_E L$. Determine the index of its image as an abelian subgroup of $\mathrm{End}_E L$.

Considering the whole Wedderburn embedding instead of, as is done here, its projection to a single factor, the answer is known under some technical conditions; cf. Theorem 2.15 in (M. Künzer, *J. Group Theory* **7** (2004), 197–229).

M. Künzer

24. Let $p \geq 3$ be a prime, and ζ_{p^3} a primitive p^3-th root of unity. Let π be the norm of $\zeta_{p^3} - 1$ with respect to the subgroup $\mathcal{C}_{p-1}$ of $\mathrm{Gal}(\mathbb{Q}(\zeta_{p^3})|\mathbb{Q})$. This yields a purely ramified extension $\mathbb{Z}_{(p)}[\pi] \,|\, \mathbb{Z}_{(p)}$ with $\mathrm{Gal}(\mathbb{Q}(\pi)|\mathbb{Q}) = \mathcal{C}_{p^2}$. We have operation maps $\mathcal{C}_{p^2} \to \mathrm{End}_{\mathbb{Z}_{(p)}} \mathbb{Z}_{(p)}[\pi]$ and $\mathbb{Z}_{(p)}[\pi] \to \mathrm{End}_{\mathbb{Z}_{(p)}} \mathbb{Z}_{(p)}[\pi]$. Let R be the subring generated by the union of their images. Then R is isomorphic to the twisted (aka skew) group ring $\mathbb{Z}_{(p)}[\pi] \wr \mathcal{C}_{p^2}$. Describe R inside $\mathrm{End}_{\mathbb{Z}_{(p)}} \mathbb{Z}_{(p)}[\pi] \cong \mathbb{Z}_{(p)}^{p^2 \times p^2}$ by congruences of matrix entries—without direct reference to the (complicated) matrix entries of the elements of the image of $\mathcal{C}_{p^2}$ under its operation map.

This is known for ζ_{p^2} instead of ζ_{p^3}; cf. Theorem 1.19 and Example 5.4 in (M. Künzer, H. Weber, *Comm. Alg.* **33** (12) (2005), 4415–4455]). For an "upper bound" in the general case, see Lemma 1.5 in loc. cit.

M. Künzer

25. Let G be a finite group, and let R be the ring of integers in a finite extension field K of the p-adic numbers. Write $RG = \bigoplus_i P_i$ as a direct sum of indecomposable projective left RG-modules. What can be said about the structure of the endomorphism rings $E_i = \mathrm{End}_{RG}(P_i)$? Let $\epsilon_1, \dots, \epsilon_s$ be the central primitive idempotents of KG. Then E_i is a subdirect product of the $\epsilon_j E_i$, which are local R-orders in a simple K-algebra. What can be said about these building blocks $\epsilon_j E_i$? Invariants of their conjugacy classes? Which local R-orders really can occur?

Some results describing p-adic group rings are presented in (G. Nebe, *Resenhas* **5** (2002), no. 4, 329–350, Around group rings (Jasper, AB, 2001)).

G. Nebe

26. Suppose $\mathcal{P}$ is a property of groups. Then a group is called *virtually* $\mathcal{P}$ if it has a subgroup of finite index which has property $\mathcal{P}$.

Let G be a finite group and write $\mathbb{Q}G = \bigoplus_{i=1}^n A_i$, a direct product of simple algebras. Let R_i be an order in A_i. By R_i^1 one denotes the units of reduced norm one. Denote by $\mathrm{vcd}(R_i^1)$ the virtual cohomological dimension of R_i^1. E. Jespers, A. Pita, A. del Río, M. Ruiz and P. Zalesskii (*Adv. Math.* 212 (2007), no. 2, 692–722) proved the following for the unit group $\mathrm{U}(\mathbb{Z}G)$ of the integral group ring $\mathbb{Z}G$.

$$\begin{aligned}
&\mathrm{U}(\mathbb{Z}G) \text{ is either abelian or finite} \\
&\qquad \Leftrightarrow \mathrm{vcd}(R_i^1) = 0 \text{ for all } 1 \le i \le n; \\
&\mathrm{U}(\mathbb{Z}G) \text{ is virtually a direct product of free groups} \\
&\qquad \Leftrightarrow \mathrm{vcd}(R_i^1) \le 1 \text{ for all } 1 \le i \le n; \\
&\mathrm{U}(\mathbb{Z}G) \text{ is virtually a direct product of free-by-free groups} \\
&\qquad \Leftrightarrow \mathrm{vcd}(R_i^1) \le 2 \text{ for all } 1 \le i \le n.
\end{aligned}$$

Can these results be extended to include the case when all $\mathrm{vcd}(R_i^1) \le 3$?

Á. del Río

27. A group G is said to be *subgroup separable* if for every finitely generated subgroup H of G and every $g \in G \setminus H$ there exists a subgroup N of finite index in G containing H but not g. When is the unit group $\mathrm{U}(\mathbb{Z}G)$ of the integral group ring $\mathbb{Z}G$ of a finite group G a subgroup separable group?

Á. del Río

28. Let G be a finite group. If $g, x \in G$ and g has order n, then the element $1 + (1-g)x(1 + g + \cdots + g^{n-1})$ of the integral group ring $\mathbb{Z}G$ is easily seen to be a unit, and is called a *bicyclic unit*. Let $\mathcal{B}$ stand for the set of all bicyclic units in $\mathbb{Z}G$. If $b_1, b_2 \in \mathcal{B}$, then either $\langle b_1, b_2\rangle$ is nilpotent or there exists a positive integer m so that $\langle b_1, b_2^m\rangle$ is a free non-cyclic group, see (J. Gonçalves, Á. del Río, Bicyclic units, Bass cyclic units and free groups, *to appear in J. Group Theory*, `arXiv:math/0612091v2 [math.RA]`). Pairs of bicyclic units of the second type exists if and only if G is non-Hamiltonian (Z. S. Marciniak, S. K. Sehgal, *Proc. Amer. Math. Soc.* **125** (1997), no. 4, 1005–1009). For a non-Hamiltonian group, define

$$m(G) = \min\{m \mid \langle b_1, b_2^m\rangle \text{ free for all } b_1, b_2 \in \mathcal{B} \text{ with } \langle b_1, b_2\rangle \text{ not nilpotent}\}.$$

Is $\{m(G) \mid G \text{ a finite non-Hamiltonian group}\}$ bounded? It is enough to consider symmetric groups. We know that $m(S_3) = 1$ and $m(S_4) = 2$ (J. Gonçalves, Á. del Río, loc.cit.). Related is the following. A nonzero complex number z is said to be a *free point* if the group $\langle \left(\begin{smallmatrix}1&1\\0&1\end{smallmatrix}\right), \left(\begin{smallmatrix}1&0\\z&1\end{smallmatrix}\right)\rangle$ is free. Set $m(z) = \min\{k \in \mathbb{N} \mid kz \text{ is a free point}\}$. Is the set

$$\{m(z) \mid z \text{ an algebraic integer, } z \neq 0\}$$

bounded? If this is true, the first question has a positive answer. (For known results on free points, see (J. Bamberg, *J. London Math. Soc.* (2) **62** (2000), no. 3, 795–801).)

J. Gonçalves, Á. del Río

29. For a finite group G, and a Dedekind domain R of characteristic zero in which no prime divisor of the order of G is invertible, K. W. Roggenkamp (*Arch. Math. (Basel)* **25** (1974), 125–128) proved that RG has no nontrivial idempotent ideals if and only if G is solvable. Determine what happens in case G is an infinite group. For polycyclic-by-finite groups, this has been done by P. A. Linnell, G. Puninski and P. Smith (*J. Algebra* **305** (2006), no. 2, 845–858).

S. K. Sehgal

30. For a group G, and a commutative ring R, denote by $\Delta(G) = \Delta_R(G)$ the augmentation ideal of RG. Set $\delta_1(RG) = [\Delta(G), \Delta(G)]RG$, and define inductively $\delta_{n+1}(RG) = [\delta_n(RG), \delta_n(RG)]RG$ for all $n \in \mathbb{N}$. Let $G_{[n]}$ denote the n-th term of the derived series of G. Assuming G is a free group, is it true that $G \cap (1 + \delta_n(\mathbb{Z}G)) = G_{[n]}$? For free groups G, and a field F of positive characteristic, determine $G \cap (1 + \delta_n(FG))$. The answer will only depend on the characteristic (and not on the chosen field). Finally, investigate these problems for an arbitrary group G.

S. K. Sehgal

31. (M. Wursthorn). In a talk delivered at the workshop "Computational Representation Theory" which took place at IBFI Schloß Dagstuhl, Germany, May 1997, M. Wursthorn asked the following. Can any automorphism of the group algebra kG, where G is a finite p-group and k the field with p-elements, be written as the composition of an automorphism of G and a unipotent automorphism? Recall that an automorphism of kG is called *unipotent* if it induces the identity on $\Delta(G)/\Delta(G)^2$. Here, $\Delta(G)$ denotes the augmentation ideal of the group algebra kG.

Remark. A positive answer to this question would give a solution to the modular isomorphism problem, by Kimmerle's $G \times G$ trick.

M. Soriano

32. Let p be a prime and k the field with p elements. Count (estimate, or give bounds for) the number of isomorphism classes of local symmetric k-algebras of dimension p^n, for $n \in \mathbb{N}$. A finite dimensional k-algebra A is *symmetric* if there is a linear map $\lambda\colon A \to k$ whose kernel contains no left or right ideals different from zero, and satisfies $\lambda(ab) = \lambda(ba)$ for all $a, b \in A$.

M. Soriano

33. For a ring R, let $\mathrm{U}(R)$ be its group of units. If $\mathrm{U}(R)$ is solvable, let $\mathrm{dl}(\mathrm{U}(R))$ be its derived length. If R is Lie solvable, let $\mathrm{dl}_L(R)$ denote its Lie derived length. Let KG be a Lie solvable group algebra of a group G over a field K of odd characteristic. Does there exist a function $f\colon \mathbb{N} \to \mathbb{N}$ such that

$$\mathrm{dl}(\mathrm{U}(KG)) \leq f(\mathrm{dl}_L(KG))?$$

M. B. Smirnov (Vestsī Akad. Navuk BSSR Ser. Fīz.-Mat. Navuk (1983), no. 5, 20–23 (Russian)) proved for an arbitrary Lie solvable ring R without 2-torsion that $\mathrm{U}(R)$ is solvable and that

$$\mathrm{dl}(\mathrm{U}(R)) \leq 4\mathrm{dl}_L(R) + 3$$

if $\mathrm{dl}_L(R) > 2$, and $\mathrm{dl}(\mathrm{U}(R)) \leq 3$ otherwise. So it is really asked whether it is possible to improve Smirnov's bound when $R = KG$.

E. Spinelli

34. For a modular group algebra KG of a non-torsion group G over a field K of positive characteristic with solvable unit group $\mathrm{U}(KG)$, find lower bounds for the derived length $\mathrm{dl}(\mathrm{U}(KG))$.

E. Spinelli

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