ON THE LIE ALGEBRA OF SKEW-SYMMETRIC ELEMENTS OF AN ENVELOPING ALGEBRA

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Let A be an algebra with involution $*$. Denote by

$$A^+ := \{x \in A | x^* = x\}$$

the set of *symmetric elements* of A under $*$, and by

$$A^- := \{x \in A | x^* = -x\}$$

the set of *skew-symmetric elements* of A under $*$.

Theorem (Amitsur, 1968)

Let A be an algebra with involution. If A^+ or A^- satisfies a polynomial identity, then so does A.

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Note that A^- has a structure of a Lie algebra induced by the Lie product of A (defined by $[x, y] := xy - yx$ for every $x, y \in A$).

Let F be a field and G a group.

We consider the group algebra $\mathbb{F}G$ endowed with the *canonical involution* (given by linear extension of the inversion map $g \mapsto g^{-1}$ of G).

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Let p be a prime. We say that group G is *p-abelian* if G' is a finite p-group. By a 0-abelian group we mean simply an abelian group.

Theorem (Giambruno-Sehgal, 1993, 2006)

Let $\mathbb{F}$ be a field of characteristic $p \neq 2$, and let G be a group. Then $\mathbb{F}G^-$ is nilpotent if and only if one the following conditions holds:

1) *G has a nilpotent p-abelian normal subgroup H with $(G \backslash H)^2 = \{1\}$;*
2) *G has an elementary abelian 2-subgroup of index 2;*
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Theorem (Lee, 2000)

Let $\mathbb{F}$ be a field of characteristic $p \neq 2$, and let G be a group with no 2-elements. Then the following conditions are equivalent:

1) $\mathbb{F}G^-$ *is n-Engel for some n;*
2) $\mathbb{F}G$ *is Lie m-Engel for some m;*
3) *either*
   - (i) $p = 0$ *and* G *is abelian or*
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Theorem (Lee-Sehgal-Spinelli, 2009)

Let $\mathbb{F}G$ be the group algebra of a group G over a field $\mathbb{F}$ of characteristic $p \neq 2$. If the set P of the p-elements of G is finite, then FG^- is solvable if and only if P is a normal subgroup of G and one of the following conditions occurs:

1) *G/P is abelian;*
2) *$G/P = A \rtimes \langle x \rangle$, where A is abelian, $o(x) = 2$, and x acts dihedrally upon A;*
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Theorem (Lee-Sehgal-Spinelli, 2009)

Let $\mathbb{F}$ be a field of characteristic $p \neq 2$, and G a group. If the set P of p-elements of G contains an infinite subgroup of bounded exponent, and G contains no nontrivial elements of order dividing $p^2 - 1$, then the following conditions are equivalent:

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For a restricted Lie algebra L over a field $\mathbb{F}$ of characteristic $p > 0$, we denote by $u(L)$ the restricted enveloping algebra of L.

Theorem (Riley-Shalev, 1993)

Let L be a restricted Lie algebra over a field $\mathbb{F}$ of characteristic $p > 0$.

1) *$u(L)$ is Lie nilpotent if and only if L is nilpotent and L' is finite-dimensional and p-nilpotent;*
2) *$u(L)$ is Lie n-Engel for some n if and only if L is nilpotent, L' is p-nilpotent, and L contains a restricted ideal I such that L/I and I' are finite-dimensional;*
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Let L be a restricted Lie algebra over a field $\mathbb{F}$ of characteristic $p > 0$. We consider $u(L)$ endowed with the *principal involution*, that is, the unique $\mathbb{F}$-antiautomorphism $\top$ of $u(L)$ such that $x^\top = -x$ for every x in L.

If $\mathcal{L}$ is an arbitrary Lie algebra, the *principal involution* of the (ordinary) universal enveloping algebra $U(\mathcal{L})$ of $\mathcal{L}$ is defined in a similar way.

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Let L be a restricted Lie algebra over a field $\mathbb{F}$ of characteristic $p > 2$. Then the following conditions are equivalent:

1) $u(L)^-$ *is solvable;*
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Example

Let L be the restricted Lie algebra over a field $\mathbb{F}$ of characteristic 2 with a basis $\{x_1, x_2, y, z\}$ such that $[x_1, x_2] = z$, $[x_1, y] = x_1$, $[x_2, y] = x_2$, $z \in Z(L)$, $x_1^{[2]} = x_2^{[2]} = 0$, $y^{[2]} = y$ and $z^{[p]} = z$. Then $u(L)^-$ is solvable, but $u(L)$ is not Lie solvable.

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1) $u(L)^-$ *is* n*-Engel for some* n*;*
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In odd characteristic, $u(L)^-$ is solvable, nilpotent or n-Engel for some n if and only if so is $u(L)$ as a Lie algebra.

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Now, consider any Lie algebra L over a field $\mathbb{F}$ of characteristic $p \neq 2$ and assume that $U(L)^{-}$ is solvable or n-Engel for some n.

If $p = 0$, as $U(L)$ satisfies a polynomial identity (by Amitsur's Theorem), then L is necessarily abelian (by Latyšev's Theorem).

Suppose then $p > 2$. Put

$$\hat{L} := \sum_{k \geq 0} L^{p^k} \subseteq U(L)$$

where $L^{p^k} = \langle l^{p^k} \mid l \in L \rangle_{\mathbb{F}}$. Then $\hat{L}$ is a restricted Lie algebra with $h^{[p]} = h^p$ for all $h \in \hat{L}$, and $U(L) = u(\hat{L})$. Therefore $\hat{L}'$ is p-nilpotent and then, since $U(L)$ is a domain, we conclude that $L' = 0$.

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If $p = 0$, as $U(L)$ satisfies a polynomial identity (by Amitsur's Theorem), then L is necessarily abelian (by Latyšev's Theorem).

Suppose then $p > 2$. Put

$$\hat{L} := \sum_{k \geq 0} L^{p^k} \subseteq U(L)$$

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Corollary (S., 2010)

Let L be a Lie algebra over a field $\mathbb{F}$ of characteristic $p \neq 2$. Then $U(L)^-$ is solvable or n-Engel for some n if and only if L is abelian.

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