Exam 4 (April 16, 2003)

MAC2312 Analytic Geometry and Calculus II (F. Lübeck)

You can use the criteria we have discussed in the lecture. In your solutions always say which criterion you use and how. If a criterion only applies under certain assumptions, don't forget to describe these and to check if they are fulfilled.

(1) (*5 points*) Determine the $k \in \mathbb{R}$ for which the series

$$\sum_{n=2}^{\infty} \frac{2}{(3n)\ln(n)^k}$$

converges.

(2) (*5 points*) Determine for each of the following series if it converges. Compute the values of the convergent series. Which of the series are absolutely convergent?

$$\sum_{n=1}^{\infty} \frac{n}{3+n\sqrt{n}}, \qquad \sum_{n=1}^{\infty} \frac{3^{n+2}}{5\cdot 4^n}, \qquad \sum_{n=1}^{\infty} \frac{(-1)^n n}{35\ln(n)}$$

(3) (*5 points*) Find the convergence radius and the interval of convergence of the following power series:

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{\pi(2n)!}, \qquad \sum_{n=0}^{\infty} 2^n \frac{1\cdot 3\cdot 5\cdots(2n+1)}{2\cdot 4\cdot 6\cdots 2n} x^n, \qquad \sum_{n=0}^{\infty} (2n)! x^n$$

(4) (*5 points*) Determine the Taylor series of $f(x)=\cos(x)$ at $a=\pi/2$. What is the Taylor series of $g(x)=\sin(x)$ at $a=\pi/2$ (hint: you can use $g'(x)=f(x)$)?