Final Exam (April 30, 2003)

MAC2312 Analytic Geometry and Calculus II (F. Lübeck)

You can use criteria, formulae and rules we have discussed in the course. In your solutions *always say what you use and how.*

(1) (*5 points*) Compute the area enclosed by the functions $\sin(x)$ and $\cos(x)$ in the range $\pi/4 < x < 5\pi/4$. Sketch the function graphs and this area.

(2) (*5 points*) Sketch the area enclosed by the graphs of $y = e^x$, $x = 0$ and $y = 3$. Consider the solid obtained by rotating this area about the x-axis. Compute the volume of the solid by a method of your choice (but don't forget to explain the method you use).

(3) (*5 points*) Compute the integral

$$\int_2^5 x \ln(\sqrt[4]{x})\, dx.$$

(4) (*5 points*) Compute the integral

$$\int \frac{x^3 + x^2 - 1}{x^3 - x^2}\, dx.$$

(5) (*5 points*) Consider the following indefinite integrals.

$$\int_1^\infty e^{-x/2}\, dx \quad \text{and} \quad \int_0^\infty x\cos(x)\, dx$$

For each of them decide if it converges or diverges and give a reason for your answer. If an integral converges then also compute its value.

(6) (*5 points*) Consider the following sequences.

$$\{\frac{\sqrt{n+4}}{n+3}\} \quad \text{and} \quad \{\frac{4^{n+2}}{5^{n+10}}\}$$

For each of them decide if it converges or diverges and give a reason for your answer. If a sequence converges then also compute its limit.

(7) (*5 points*) Determine the $p \in \mathbb{R}$, $p > 0$, for which the series

$$\sum_{n=2}^\infty \frac{1}{n\ln(n)^p}$$

converges.

(8) (*5 points*) Find the Taylor series of $f(x) = \frac{1}{x}$ at $a = 1$. (Here you get 3 points if you compute at least the Taylor polynomial of degree 4 at $a = 1$.)