Exam 1(January 29, 2003)

MAS4105 Linear Algebra I (F. Lübeck)

(1) Mark as true or false. (*One point* for each correct answer.)
Let F be a field and V be a vector space over F.

(a) V contains exactly one vector x such that $y + x = y$ for all $y \in V$. true: ☐ false: ☐

(b) A matrix in $M_{m\times n}(F)$ has m rows and n columns. true: ☐ false: ☐

(c) The intersection of a subspace of V with any subset of V is again a subspace. true: ☐ false: ☐

(d) Each vector space contains a subspace. true: ☐ false: ☐

(e) Let $S_1, S_2 \subseteq V$ with $S_1 \subseteq S_2$. Then $\mathrm{span}(S_2) \subseteq \mathrm{span}(S_1)$. true: ☐ false: ☐

(f) The span of each subset of V is a subspace of V. true: ☐ false: ☐

(g) A subset $S \subset V$ is linearly independent if each proper subset $S' \subsetneq S$ is linearly independent. true: ☐ false: ☐

(h) A subset $S \subset V$ is linearly dependent if and only if it is not linearly independent. true: ☐ false: ☐

(i) Assume that V has a finite generating set. Then any generating set of V has at most as many elements as a basis of V. true: ☐ false: ☐

(j) If $\dim V = n \in \mathbb{N}$, then any generating set of V with n elements is linearly independent. true: ☐ false: ☐

(2) (*three points*) Let $V = \mathbb{R}^3$ and $S = \{(1, 2, 0), (0, 1, 0)\} \subseteq V$. Why can S be extended to a basis of V? Find a basis β of V that contains S, and explain how you found that basis.

(3) (*four points*) Let $S = \{x^3 - 1, x^2 - 1, x - 1\} \subseteq P(\mathbb{R})$ (polynomials over $\mathbb{R}$). Show that the polynomial $1 \in P(F)$ is not contained in $\mathrm{span}(S)$.

(4) (*three points*) Let V be a vector space and W_1 and W_2 subspaces of V. Prove that $W_1 \cup W_2$ is a subspace of V if and only if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$.