**Exam 2**(February 26, 2003)

**MAS4105 Linear Algebra I** (F. Lübeck)

**(1)** Mark as true or false. (*One point* for each correct answer.)

Let $A = \begin{pmatrix} 1 & -1 & 3 & 0 \\ 1 & -2 & 4 & 1 \\ 0 & -1 & 1 & 1 \end{pmatrix} \in M_{3\times 4}(\mathbb{R})$, $B = \begin{pmatrix} 1 & -1 & 3 & 0 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \in M_{3\times 4}(\mathbb{R})$

and $C = \begin{pmatrix} 1 & -1 & 3 & 0 \\ 3 & 1 & -1 & -1 \\ 0 & 2 & -3 & 1 \\ 2 & 0 & -3 & 4 \end{pmatrix} \in M_{4\times 4}(\mathbb{R})$.

(a) The matrix product $D = AB$ is defined and $D_{2,3} = 5$. true: ☐ false: ☐

(b) The matrix product $F = BC$ is defined and $F_{1,3} = -5$. true: ☐ false: ☐

(c) The matrix $B$ is in reduced row echelon form. true: ☐ false: ☐

(d) The matrix $B$ can be obtained from $A$ by two elementary row operations.
true: ☐ false: ☐

(e) Each elementary row operation on $B$ can be described by multiplication with an invertible matrix $E \in M_{3\times 3}$ from the left.
true: ☐ false: ☐

(f) Elementary row operations preserve the column space of a matrix. true: ☐ false: ☐

(g) The rank of $A$ is 2. true: ☐ false: ☐

(h) The rank of a matrix is the maximal number of linearly independent columns it has.
true: ☐ false: ☐

(i) If the rank of an $n \times n$-matrix is $n$ then its reduced echelon form is the identity matrix $I_n$.
true: ☐ false: ☐

(j) The nullspace of $A$ is a subspace of $\mathbb{R}^{4\times 1}$ and has dimension 1. true: ☐ false: ☐

**(2)** (*three points*) Let $C = \begin{pmatrix} 1 & 2 & 2 \\ -1 & -1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \in M_{3\times 3}(\mathbb{R})$ and $I_3 \in M_{3\times 3}(\mathbb{R})$ the identity matrix. Compute the matrix $AAAA - I_3$.

**(3)** (*four points*) Compute the reduced row echelon form of the matrix

$$A = \begin{pmatrix} 1 & 2 & 3 & 1 & 2 & 3 \\ 2 & 4 & 6 & 1 & 2 & 5 \\ 3 & 6 & 9 & 4 & 8 & 3 \end{pmatrix} \in M_{3\times 6}(\mathbb{R}).$$

Determine a basis of the nullspace $N(A)$.

**(4)** (*three points*) Compute the inverse of the matrix $\begin{pmatrix} 0 & -1 & 0 \\ -3 & 1 & -2 \\ -8 & 0 & -5 \end{pmatrix} \in M_{3\times 3}(\mathbb{R})$.