The principal block of $\mathbb{Z}_p S_{2p}$.

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1 Introduction

Let p be an odd prime. This note describes the principal block $\mathcal{B}$ of the group ring of the symmetric group S_{2p} over the p-adic integers $\mathbb{Z}_p$.

The principal block $\overline{\mathcal{B}}$ of $\mathbb{F}_p S_{2p}$ has been considered in several papers. [Mar1] determines the Ext-quiver of $\overline{\mathcal{B}}$, [Mar2] the Loewy series of the projective indecomposable $\overline{\mathcal{B}}$-modules and finally [ErM] gives a presentation for the basic algebra that is Morita equivalent to $\overline{\mathcal{B}}$, where some constants are not fully determined.

The present note does not depend on these results. The most important starting point used here is the decomposition matrix of $\mathcal{B}$, which is calculated with the Jantzen-Schaper formula [Schap], [Jan]. Then the description of $\mathcal{B}$ is a very easy application of the general theory developed by Plesken [Ple1], [Ple2] and further in [Neb].

J. Scopes [Sco] shows that many properties of $\overline{\mathcal{B}}$ also hold for all other symmetric group blocks of defect 2. I am confident that the same methods used here can also be applied to describe these blocks up to Morita equivalence.

2 Notation.

Recall that the iirreducible ordinary respectively p-modular characters of the symmetric group S_n are indexed by the partitions respectively the p-regular partitions of n (see e.g. [Jam]).

To denote the partitions of $2p$ we use the $\langle 2^p \rangle$-notation as explained in [Sco], [ErM], [Mar2]: If $1 \leq j \leq p$ then $\langle j \rangle$ is the $\langle 2^p \rangle$-notation for the partition $(p+j, 1^{p-j})$ of $2p$ and for $1 \leq j < k \leq p$, the symbol $\langle k, j \rangle$ stands for $(k, j+1, 2^{p-k}, 1^{k-j-1})$ and $\langle k, k \rangle$ denotes $(k, 1^{2p-k})$. To distinguish between ordinary and modular characters in $\mathcal{B}$, the ordinary characters are denoted by $\langle\langle j \rangle\rangle$, $\langle\langle k, j \rangle\rangle$ $(1 \leq j \leq k \leq p)$, whereas $\langle j \rangle$, $(1 \leq j \leq p)$ and $\langle k, j \rangle$ $(1 \leq j < k \leq p)$ are the modular characters in $\mathcal{B}$.

3 The decomposition matrix of $\mathcal{B}$

The decomposition matrix of $\mathcal{B}$ is given in [Mar2, Fig 8] (where a 1 is missing in row $\langle\langle k, k-1 \rangle\rangle$ column $\langle k, k-1 \rangle$ and k runs from 4 (or even 3) to $p-1$). It is easily calculated with the Jantzen-Schaper formula.

The decomposition matrix of $\mathcal{B}$ has a very regular shape: Let

$$X_k := \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 1 & 1 & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 1 & 1 \\ 0 & \dots & \dots & 0 & 1 \end{pmatrix} \in \mathbb{Z}^{(k+1)\times k}$$

and $X'_p \in \mathbb{Z}^{p\times p}$ obtained from X_p by deleting the last row. Let

$$Y_k := \begin{pmatrix} 0 & 1 & 0 & \dots & \dots & 0 \\ 1 & 1 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & 0 & 1 & 1 \\ 0 & \dots & \dots & \dots & 0 & 1 \end{pmatrix} \in \mathbb{Z}^{k\times k}$$

and

$$Z_k := \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix} \in \mathbb{Z}^{(k-1)\times k}.$$

Theorem 1 *If we order the partitions of $2p$ that belong to $\mathcal{B}$ lexicographically then the decomposition matrix of $\mathcal{B}$ has the following form:*

$$\begin{matrix} X'_p & 0 & \dots & \dots & \dots & 0 \\ Y_p & X_{p-1} & 0 & \dots & \dots & 0 \\ Z_p & Y_{p-1} & X_{p-2} & 0 & \dots & \vdots \\ 0 & Z_{p-1} & Y_{p-2} & X_{p-3} & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & Z_4 & Y_3 & X_2 \\ 0 & \dots & \dots & 0 & Z_3 & Y_2 \\ 0 & \dots & \dots & \dots & 0 & Z_2 \end{matrix}$$

The ordinary characters that belong to the rows of X'_p are $\langle\langle p\rangle\rangle$, $\langle\langle p-1\rangle\rangle$, $\dots$ $\langle\langle 1\rangle\rangle$ and and the modular characters belonging to the columne of X'_p are $\langle p\rangle$, $\langle p-1\rangle$, $\dots$ $\langle 1\rangle$. The ordinary characters that correspond to the rows of X_k are $\langle\langle k+1,k\rangle\rangle,\dots,\langle\langle k+1,1\rangle\rangle,\langle\langle k+1,k+1\rangle\rangle$, and the modular characters $\langle k+1,k\rangle,\dots,\langle k+1,1\rangle,\langle k+1,k\rangle$, belong to the columns of X_k ($k=0,\dots,p-1$).

[Mar2] uses Scopes result, that the decomposition numbers in $\mathcal{B}$ are ≤ 1 to get the decomposition matrix from the Jantzen-Schaper formula [Schap], [Jan]. In general this formula only gives the positions of the non zero entries in the decomposition matrix and upper bounds for these entries. For $\mathcal{B}$, the formula says that some of the 1 on the diagonal of Y_k might be 2 and the 1 in Z_k might also be 2. In these cases the assumptions of the following proposition which is only formulated and proved for the special case needed here, but holds in more general situations, are satisfied. This shows that one gets the decomposition matrix of $\mathcal{B}$ directly from the Jantzen-Schaper formula.

Proposition 2 *Let $p > 2$ be an odd prime, $n \in \mathbb{N}$ and λ be a p-regular partition of n, χ the irreducible ordinary character and φ the p-modular character of S_n that belongs to λ. Let L be the Specht lattice with head φ. Let φ' be an other modular constituent of χ such that χ is the only irreducible character that has both φ and φ' as a p-modular constituent. Let μ be the multiplicity of φ' in $L^{\#}/L$ calculated by the Jantzen-Schaper formula and $\nu := \nu_p(n!)/\nu_p(\chi(1))$. Then the multiplicity of φ' in L/pL is*

$$d_{\chi,\varphi'} = \frac{\mu}{\nu}.$$

Proof. The group ring $\mathbb{Z}_p S_n$ has a canonical involution $\ ^\circ$ inverting the group elements. Since p is odd, one can find an involution invariant idempotent $i = i^\circ = i^2 \in \mathbb{Z}_p S_n$ such that $i\mathbb{Z}_p S_n i =: \Lambda$ is a basic order ([Neb, Bemerkung 4.3.13]). Now χ is real, so $\Gamma := \Lambda(\chi)$, the projection of Λ into the simple component of $\mathbb{Q}_p\Lambda$ that belongs to χ, is also invariant under $\ ^\circ$. The restriction of $\ ^\circ$ to Γ is an involution on a central simple algebra, it is of the form $x \mapsto F^{-1}x^{tr}F$ for some symmetric $F \in \mathbb{Q}_p\Gamma$.

The lattice L corresponds to the projective Γ-lattice $P = P_\varphi$ with head φ, so $P = \Gamma e$ for some primitive idempotent $e = e^\circ = e^2 \in \Gamma$. Since the multiplicity of φ in χ is 1, e is the lift of the central primitive idempotent in $\Gamma/J(\Gamma)$ that corresponds to φ. Let $f = f^\circ = f^2 \in \Gamma$ be a lift of the central primitive idempotent in $\Gamma/J(\Gamma)$ that corresponds to φ' and choose it orthogonal to e, i.e. $ef = 0 = fe$. Since the other modular constituents of χ do not play any role, we may assume that $e+f = 1$, by replacing Γ by $(e+f)\Gamma(e+f)$. Again since $p \neq 2$, we can choose a basis of P which is orthogonal with respect to F, such that the first basis vector spans $eP \subset P$, write everything with respect to this basis, and renormalize F such that this vector has length 1. The dual lattice of P with respect to F is $P^{\#} = F^{-1}P$ and $\mu = \nu_p(\det(F))$.

On the other hand since χ is the only irreducible character that has both φ and φ' as a p-modular constituent and Λ is a symmetric order, the dual of $e\Gamma f$ with respect to $\chi(1)/(n!)\cdot$trace is $(e\Gamma f)^{\#} = f\Gamma e$ (see [Thé, Proposition (1.6.2)]). Therefore one calculates

$$(e\Gamma f)^{\#} = f\Gamma e = (e\Gamma f)^\circ = F^{-1}(e\Gamma f)^{tr}F.$$

Since F is diagonal with first entry 1, one gets that

$$\mu = \nu_p(\det(F)) = d_{\chi,\varphi'}\nu.$$

$\square$

4 The degrees of the ordinary irreducible characters in $\mathcal{B}$

The next easy lemma (which might be well known) shows that the degrees of the ordinary irreducible characters in $\mathcal{B}$ are ± 1 or $\pm 2 \pmod p$.

Lemma 3 *The degrees of the ordinary irreducible characters that belong to $\mathcal{B}$ are:*
$\deg(\langle\langle k\rangle\rangle) \equiv \deg(\langle\langle k,k\rangle\rangle) \equiv (-1)^{k-1} \pmod p$ *for* $1 \leq k \leq p$,
$\deg(\langle\langle k,j\rangle\rangle) \equiv 2\cdot(-1)^{j+k} \pmod p$ *for* $1 \leq j < k \leq p$.

Proof. This is an easy application of the hook-formula (cf. [Jam, Theorem 20.1]), which says that for a partition λ of n, the dimension $\dim(S^\lambda)$ is $n!$ divided by the product of all hook lengths in λ. Note that $(2p)!/(p^2) \equiv (p-1)!^2 \pmod p$.
The notation $\langle\langle k\rangle\rangle$ stands for the partition $(p+k, 1^{p-k})$. Applying the hook formula, one calculates $\deg(\langle\langle k\rangle\rangle) =$

$$\frac{(2p)!}{(p-k)!(2p)(p+k-1)!} \equiv \frac{(p-1)!^2}{(p-k)!(p+k-1)\cdots(p+1)(p-1)!} \equiv (-1)^{k-1} \pmod p.$$

$\langle\langle k,k\rangle\rangle$ stands for the hook $(k, 1^{2p-k})$. There the product of the hook lengths is $(2p-k)!(2p)(k-1)! \equiv 2p^2(p-1)!^2(-1)^{k-1} \pmod{p^3}$.
If $k > j$ then $\langle\langle k,j\rangle\rangle$ belongs to the partition $(k, j+1, 2^{p-k}, 1^{k-j-1})$. The product of all hook lengths in this partition is $(k-j-1)!(k-j+1)\cdots(p-j)(p-k)!(j-1)!(p+j-k)p(k-j-1)!(k-j+1)\cdots(k-1)p(p+k-j)$. Since $p+k-j \equiv k-j \pmod p$ this is $p^2(p-j)!(p-k)!(j-1)!(-1)(k-1)! \pmod{p^3}$. Now $(j-1)!$ is $(p-j+1)...(p-1)(-1)^{j-1}$ modulo p, therefore one gets $p^2(p-1)!^2(-1)^{k+j}$ modulo p^3. □

5 The exponent matrices.

If χ is an ordinary irreducible character in $\mathcal{B}$, then let $\mathcal{B}(\chi)$ denote the projection of $\mathcal{B}$ into the simple summand of $\mathbb{Q}_p\mathcal{B}$ that corresponds to χ. Since the decomposition numbers of $\mathcal{B}$ are 0 or 1, the order $\mathcal{B}(\chi)$ can be described using the language of exponent matrices as developed in [Ple2]. If $\varphi_1, \ldots, \varphi_h$ are the p-modular constituents of χ, of degree $n_i := \varphi_i(1)$, then there is a matrix $M(\chi) = (m_{ij}(\chi)) \in \mathbb{Z}^{h\times h}$, such that $\mathcal{B}(\chi)$ is conjugate to

$$\Lambda(\varphi_1, \ldots, \varphi_l, M(\chi)) :=$$
$$\{X = (x_{ij})_{i,j=1,\ldots,h} \in \mathbb{Z}_p^{n\times n} \mid x_{ij} \in (p^{m_{ij}(\chi)}\mathbb{Z}_p)^{n_i\times n_j} \text{ for all } 1 \leq i,j \leq h\}.$$

Here $n = n_1 + \ldots + n_h = \chi(1)$ is the degree of χ.

Since $1 \in \mathcal{B}(\chi)$, one clearly has $m_{ii}(\chi) = 0$ for all i. All exponent matrices can be normalized such that the first column consists of zeros only and all entries of M are ≥ 0, by writing the matrices with respect to a suitable basis of the projective $\mathcal{B}(\chi)$-lattice that corresponds to φ_1.

Let c_i denote the set of all ordinary irreducible characters of $\mathcal{B}$, that have φ_i as a p-modular constituent:

$$c_i := \{\chi \in \mathrm{Irr}(\mathcal{B}) \mid d_{\chi,\varphi_i} = 1\}.$$

The next lemma follows from [Ple2, Theorem (III.8)] and [Ple2, Corollary (IV.7)] (applied to the special situation here, that $\mathbb{Q}_p$ is a splitting field and all characters in $\mathcal{B}$ have defect 2).

Lemma 4 *Let* $1 \leq i \neq j \leq l$.
a) $0 < m_{ij}(\chi) + m_{ji}(\chi) \leq 2$ *is bounded by the defect.*
b) If $c_i \cap c_j = \{\chi\}$ *then* $m_{ij}(\chi) + m_{ji}(\chi) = 2$.
c) If $c_i \cap c_j = \{\chi, \psi\}$ *then* $m_{ij}(\chi) + m_{ji}(\chi) = m_{ij}(\psi) + m_{ji}(\psi)$.

The exponent matrices for the hook representations (which are $\langle\langle j \rangle\rangle$ and all non p-regular partitions $\langle\langle k, k \rangle\rangle$ in our particular situation) are given in [Ple2, Theorem (VI.2)]. If χ belongs to a p-regular partition, then $M(\chi)$ is determined by the Jantzen-Schaper formula (see [Schap], [Mar2, Appendix], [MaR, Section 1.2]), as stated in [Ple3, Proposition IV.1]: Order the p-modular constituents of χ such that φ_1 belongs to the same partition as χ. Then the Specht lattice of χ is the projective $\mathcal{B}(\chi)$ lattice L corresponding to φ_1. The Jantzen-Schaper formula gives the multiplicities $m_{1j}(\chi)$ of φ_j in $L^{\#}/L$. It is easy to see that these form the first line of $M(\chi)$. Since all characters in $\mathcal{B}$ are self dual, one has

$$m_{ij}(\chi) = m_{ji}(\chi) + m_{1j}(\chi) - m_{1i}(\chi).$$

Since the $m_{ij}(\chi)$ are nonnegative and $m_{ij}(\chi) + m_{ji}(\chi)$ is either 1 or 2 (for $i \neq j$), this equation has only one solution.

Even without using the additional information on the first rows, the entries in the exponent matrices can be calculated successively by the same strategy as explained in [Ple2, Example (VI.3)] for $p = 5$.

Theorem 5

$$\mathcal{B}(\langle\langle p \rangle\rangle) \cong \Lambda(\langle p \rangle, (0))$$

For $j = p-1, \ldots, 1$*:*

$$\mathcal{B}(\langle\langle j \rangle\rangle) \cong \Lambda(\langle j \rangle, \langle j+1 \rangle, A)$$

$$\mathcal{B}(\langle\langle p, p-1 \rangle\rangle) \cong \Lambda(\langle p, p-1 \rangle, \langle p-1 \rangle, A)$$

$$\mathcal{B}(\langle\langle p, p-2 \rangle\rangle) \cong \Lambda(\langle p, p-2 \rangle, \langle p, p-1 \rangle, \langle p-2 \rangle, \langle p-1 \rangle, \langle p \rangle, B)$$

For $j = p-3, \ldots, 1$*:*

$$\mathcal{B}(\langle\langle p, j \rangle\rangle) \cong \Lambda(\langle p, j \rangle, \langle p, j+1 \rangle, \langle j \rangle, \langle j+1 \rangle, C)$$

$$\mathcal{B}(\langle\langle p, p \rangle\rangle) \cong \Lambda(\langle p, 1 \rangle, \langle 1 \rangle, A)$$

For $k = p-1, \ldots, 3$*:*

$$\mathcal{B}(\langle\langle k, k-1 \rangle\rangle) \cong \Lambda(\langle k, k-1 \rangle, \langle k+1, k-1 \rangle, \langle k+2, k+1 \rangle, D), \ \ \textit{if } k < p-1$$

$$\mathcal{B}(\langle\langle k, k-1 \rangle\rangle) \cong \Lambda(\langle k, k-1 \rangle, \langle k+1, k-1 \rangle, \langle p \rangle, D), \ \ \textit{if } k = p-1$$

$$\mathcal{B}(\langle\langle k, k-2 \rangle\rangle) \cong \Lambda(\langle k, k-2 \rangle, \langle k, k-1 \rangle, \langle k+1, k-2 \rangle, \langle k+1, k-1 \rangle, \langle k+1, k \rangle, B)$$

and for $j = k-3, \ldots, 1$*:*

$$\mathcal{B}(\langle\!\langle k,j \rangle\!\rangle) \cong \Lambda(\langle k,j\rangle, \langle k,j+1\rangle, \langle k+1,j\rangle, \langle k+1,j+1\rangle, C)$$

$$\mathcal{B}(\langle\!\langle k,k \rangle\!\rangle) \cong \Lambda(\langle k,1\rangle, \langle k+1,1\rangle, A)$$

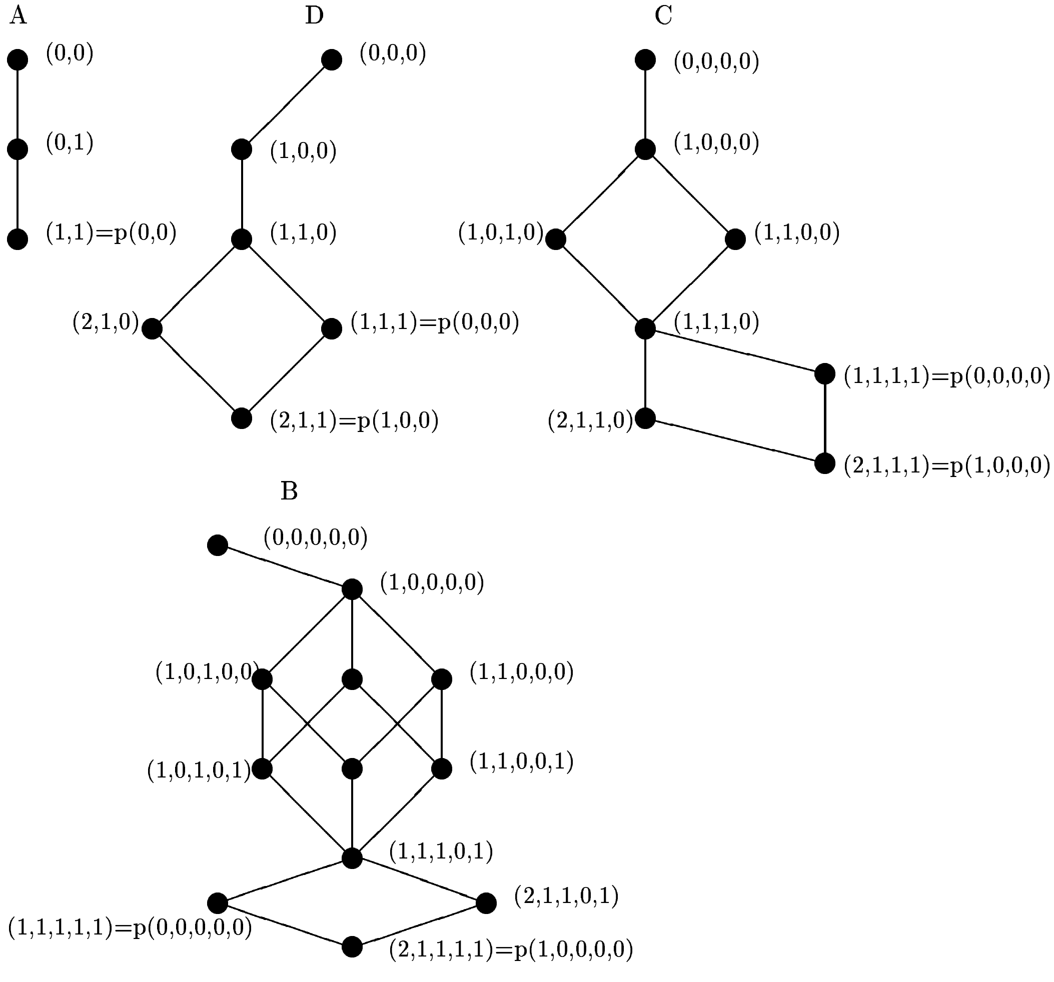
$$\mathcal{B}(\langle\!\langle 2,1 \rangle\!\rangle) \cong \Lambda(\langle 3,1\rangle, \langle 4,3\rangle, A)$$

$$\mathcal{B}(\langle\!\langle 2,2 \rangle\!\rangle) \cong \Lambda(\langle 3,1\rangle, \langle 3,2\rangle, A)$$

$$\mathcal{B}(\langle\!\langle 1,1 \rangle\!\rangle) \cong \Lambda(\langle 3,2\rangle, (0))$$

$$A := \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, B := \begin{pmatrix} 0 & 1 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 \end{pmatrix}, C := \begin{pmatrix} 0 & 1 & 1 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}, D := \begin{pmatrix} 0 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}.$$

From these exponent matrices one can read off the lattice of invariant $\mathbb{Z}_p S_{2p}$-lattices in the corresponding irreducible $\mathbb{Q}_p S_{2p}$-module as explained in [Ple2, Remark II.4]. There are four different pictures, corresponding to the 4 different exponent matrices A, D, C, B:



6 The basic order of $\mathcal{B}$

This section describes the basic order Λ of $\mathcal{B}$. Let $P_1, \ldots, P_h$ represent the isomorphism classes of projective indecomposable $\mathcal{B}$ right modules.

Then $\mathcal{B}$ is Morita equivalent to

$$\Lambda := \mathrm{End}_{\mathcal{B}}(P_1 \oplus \ldots \oplus P_h) = \bigoplus_{i,j=1}^{h} \mathrm{Hom}_{\mathcal{B}}(P_i, P_j)$$

and Λ is a basic order in the sense that the simple Λ-modules are one dimensional vector spaces over $\mathbb{F}_p$.

Inspired by the way to give generators for the basic algebra $\Lambda/p\Lambda$ using the Ext-quiver of $\overline{\mathcal{B}} := \mathcal{B}/p\mathcal{B}$, we give lifts of those generators in Λ, which clearly generate the $\mathbb{Z}_p$-order Λ.

Since the decomposition numbers of $\mathcal{B}$ are ≤ 1, the endomorphism rings $\mathrm{End}_{\mathcal{B}}(P_i)$ are commutative $(1 \leq i \leq h)$.

Let

$$V := \bigoplus_{\chi} V_\chi$$

be the sum over a system of representatives of the isomorphism classes of simple $\mathbb{Q}_p\mathcal{B}$-modules and

$$E := \mathrm{End}_{\mathbb{Q}_p\mathcal{B}}(V) \cong Z(\mathbb{Q}_p\mathcal{B}).$$

For all $1 \leq j \leq h$ choose an embedding

$$\varphi_j : P_j \hookrightarrow V.$$

Let Q_j be the unique $\mathbb{Q}_p\mathcal{B}$-invariant complement of $\mathbb{Q}_p\varphi_j(P_j)$ in V,

$$V = \mathbb{Q}_p\varphi_j(P_j) \oplus Q_j.$$

Then the $\mathcal{B}$-homomorphisms $\varphi \in \mathrm{Hom}_{\mathcal{B}}(P_j, P_i)$ for $1 \leq i, j \leq h$ are considered as elements of E by letting

$$\varphi_{|Q_j} = 0.$$

Definition 6 *For $i = 1, \ldots, h$ let $\varphi_i^{-1} : V \to \mathbb{Q}_p P_i$ be the right inverse of φ_i with $\varphi_i^{-1}(Q_i) = 0$. Then for $1 \leq i, j \leq h$ there are embeddings*

$$\mathrm{Hom}_{\mathcal{B}}(P_i, P_j) \hookrightarrow E, \ \varphi \mapsto \varphi_i^{-1}\varphi\varphi_j.$$

Via these embeddings $\mathrm{Hom}_{\mathcal{B}}(P_i, P_j)$ is viewed as a subset of E:

$$\Lambda_{ij} := (\varphi_i^{-1})\mathrm{Hom}_{\mathcal{B}}(P_i, P_j)\varphi_j \subset E.$$

Note that Λ is determined, when one knows the Λ_{ij} for a fixed system of embeddings $\varphi_i : P_i \hookrightarrow V$.

To describe elements in Λ_{ij}, we use the canonical $\mathbb{Q}_p$-basis consisting of the primitive idempotents ϵ_χ of E.

Whereas $\mathrm{End}_{\mathcal{B}}(P_j)$ is canonically (i.e. independent of the choice of φ_j) embedded into E, the embedding $\mathrm{Hom}_{\mathcal{B}}(P_i, P_j) \hookrightarrow E$ for $i \neq j$ depends on the choice of φ_i and φ_j. In each component χ of E, φ_i can be multiplied by a scalar $d_i^{(\chi)}$. Then the χ-component of $\varphi_i^{-1}\varphi\varphi_j \in \Lambda_{ij}$ gets multiplied by $(d_i^{(\chi)})^{-1}d_j^{(\chi)}$.

Since the valuation of the projection of Λ_{ij} into the components of the maximal order of E is already fixed by the exponent matrices, one may assume that the $d_i^{(\chi)}$ are units in $\mathbb{Z}_p$.

So changing the system of embeddings φ_i without changing the exponent matrices means conjugation by integral invertible diagonal matrices in each component.

An inspection of the decomposition matrix shows:

Lemma 7 *The ordinary and modular constituents of the projective $\mathcal{B}$-modules are given in the following tables:*

$\langle p \rangle$:

ord. char.	deg	modular constituents of this ord. char.					
$\langle\langle p \rangle\rangle$	1	$\langle p \rangle$					
$\langle\langle p-1 \rangle\rangle$	-1	$\langle p \rangle$	$\langle p-1 \rangle$				
$\langle\langle p, p-2 \rangle\rangle$	2	$\langle p \rangle$	$\langle p-1 \rangle$	$\langle p-2 \rangle$	$\langle p, p-1 \rangle$	$\langle p, p-2 \rangle$	
$\langle\langle p-1, p-2 \rangle\rangle$	-2	$\langle p \rangle$				$\langle p, p-2 \rangle$	$\langle p-1, p-2 \rangle$

$\langle p-1 \rangle$:

ord. char.	deg	modular constituents of this ord. char.				
$\langle\langle p-1 \rangle\rangle$	-1	$\langle p \rangle$	$\langle p-1 \rangle$			
$\langle\langle p-2 \rangle\rangle$	1		$\langle p-1 \rangle$	$\langle p-2 \rangle$		
$\langle\langle p, p-1 \rangle\rangle$	-2		$\langle p-1 \rangle$		$\langle p, p-1 \rangle$	
$\langle\langle p, p-2 \rangle\rangle$	2	$\langle p \rangle$	$\langle p-1 \rangle$	$\langle p-2 \rangle$	$\langle p, p-1 \rangle$	$\langle p, p-2 \rangle$

For $j = 2, \ldots, p-2$:

$\langle j \rangle$:

ord. char.	deg	modular constituents of this ord. char.					
$\langle\langle j \rangle\rangle$	$(-1)^{j-1}$	$\langle j+1 \rangle$	$\langle j \rangle$				
$\langle\langle j-1 \rangle\rangle$	$(-1)^{j}$		$\langle j \rangle$	$\langle j-1 \rangle$			
$\langle\langle p, j \rangle\rangle$	$2(-1)^{j-1}$	$\langle j+1 \rangle$	$\langle j \rangle$		$\langle p, j+1 \rangle$	$\langle p, j \rangle$	
$\langle\langle p, j-1 \rangle\rangle$	$2(-1)^{j}$		$\langle j \rangle$	$\langle j-1 \rangle$		$\langle p, j \rangle$	$\langle p, j-1 \rangle$

$\langle 1 \rangle$:

ord. char.	deg	modular constituents of this ord. char.				
$\langle\langle 1 \rangle\rangle$	1	$\langle 2 \rangle$	$\langle 1 \rangle$			
$\langle\langle p, 1 \rangle\rangle$	2	$\langle 2 \rangle$	$\langle 1 \rangle$	$\langle p, 2 \rangle$	$\langle p, 1 \rangle$	
$\langle\langle p, p \rangle\rangle$	1		$\langle 1 \rangle$		$\langle p, 1 \rangle$	$\langle p, j-1 \rangle$

For $k = 4, \dots, p$:
$\langle k, k-1 \rangle$:

ord. char.	deg	modular constituents of this ord. char.					
$\langle\langle k, k\text{-}1 \rangle\rangle$	-2	$\langle k+1, k\text{-}1 \rangle$	$\langle k, k\text{-}1 \rangle$			$\langle k+2, k+1 \rangle$	
$\langle\langle k, k\text{-}2 \rangle\rangle$	2	$\langle k+1, k\text{-}1 \rangle$	$\langle k, k\text{-}1 \rangle$	$\langle k, k\text{-}2 \rangle$		$\langle k+1, k \rangle$	$\langle k+1, k\text{-}2 \rangle$
$\langle\langle k\text{-}1, k\text{-}3 \rangle\rangle$	2		$\langle k, k\text{-}1 \rangle$	$\langle k, k\text{-}2 \rangle$	$\langle k\text{-}1, k\text{-}3 \rangle$	$\langle k, k\text{-}3 \rangle$	$\langle k\text{-}1, k\text{-}2 \rangle$
$\langle\langle k\text{-}2, k\text{-}3 \rangle\rangle$	-2		$\langle k, k\text{-}1 \rangle$		$\langle k\text{-}1, k\text{-}3 \rangle$	$\langle k\text{-}2, k\text{-}3 \rangle$	

where $\langle p+1, j \rangle$ has to be read as $\langle j \rangle$ and $\langle p+2, p+1 \rangle$ as $\langle p \rangle$.

$\langle 3, 2 \rangle$:

ord. char.	deg	modular constituents of this ord. char.					
$\langle\langle 3, 2 \rangle\rangle$	-2	$\langle 5, 4 \rangle$		$\langle 4, 2 \rangle$		$\langle 3, 2 \rangle$	
$\langle\langle 3, 1 \rangle\rangle$	2		$\langle 4, 3 \rangle$	$\langle 4, 2 \rangle$	$\langle 4, 1 \rangle$	$\langle 3, 2 \rangle$	$\langle 3, 1 \rangle$
$\langle\langle 2, 2 \rangle\rangle$	-1					$\langle 3, 2 \rangle$	$\langle 3, 1 \rangle$
$\langle\langle 1, 1 \rangle\rangle$	1					$\langle 3, 2 \rangle$	

For $k = 4, \dots, p$:
$\langle k, k-2 \rangle$:

ord. char.	deg	modular constituents of this ord. char.						
$\langle\langle k, k\text{-}2 \rangle\rangle$	2	$\langle k+1, k \rangle$	$\langle k+1, k\text{-}2 \rangle$	$\langle k, k\text{-}1 \rangle$	$\langle k, k\text{-}2 \rangle$			$\langle k+1, k\text{-}1 \rangle$
$\langle\langle k, k\text{-}3 \rangle\rangle$	-2		$\langle k+1, k\text{-}2 \rangle$		$\langle k, k\text{-}2 \rangle$	$\langle k, k\text{-}3 \rangle$		$\langle k+1, k\text{-}3 \rangle$
$\langle\langle k\text{-}1, k\text{-}2 \rangle\rangle$	-2	$\langle k+1, k \rangle$			$\langle k, k\text{-}2 \rangle$		$\langle k\text{-}1, k\text{-}2 \rangle$	
$\langle\langle k\text{-}1, k\text{-}3 \rangle\rangle$	2			$\langle k, k\text{-}1 \rangle$	$\langle k, k\text{-}2 \rangle$	$\langle k, k\text{-}3 \rangle$	$\langle k\text{-}1, k\text{-}2 \rangle$	$\langle k\text{-}1, k\text{-}3 \rangle$

$\langle 3, 1 \rangle$:

ord. char.	deg	modular constituents of this ord. char.				
$\langle\langle 3, 1 \rangle\rangle$	2	$\langle 4, 3 \rangle$	$\langle 4, 2 \rangle$	$\langle 4, 1 \rangle$	$\langle 3, 2 \rangle$	$\langle 3, 1 \rangle$
$\langle\langle 3, 3 \rangle\rangle$	1			$\langle 4, 1 \rangle$		$\langle 3, 1 \rangle$
$\langle\langle 2, 1 \rangle\rangle$	-2					$\langle 3, 1 \rangle$
$\langle\langle 2, 2 \rangle\rangle$	-1				$\langle 3, 2 \rangle$	$\langle 3, 1 \rangle$

For $k = 4, \dots, p$ and $j = 2, \dots, k-3$:
$\langle k, j \rangle$:

ord. char.	deg	modular constituents of this ord. char.						
$\langle\langle k, j \rangle\rangle$	$2(\text{-}1)^{k+j}$	$\langle k+1, j \rangle$	$\langle k, j+1 \rangle$	$\langle k, j \rangle$			$\langle k+1, j+1 \rangle$	
$\langle\langle k, j\text{-}1 \rangle\rangle$	$2(\text{-}1)^{k+j-1}$	$\langle k+1, j \rangle$		$\langle k, j \rangle$	$\langle k, j\text{-}1 \rangle$		$\langle k+1, j\text{-}1 \rangle$	
$\langle\langle k\text{-}1, j \rangle\rangle$	$2(\text{-}1)^{k+j-1}$		$\langle k, j+1 \rangle$	$\langle k, j \rangle$		$\langle k\text{-}1, j \rangle$	$\langle k\text{-}1, j+1 \rangle$	$\langle k, k\text{-}1 \rangle^*$
$\langle\langle k\text{-}1, j\text{-}1 \rangle\rangle$	$2(\text{-}1)^{k+j}$			$\langle k, j \rangle$	$\langle k, j\text{-}1 \rangle$	$\langle k\text{-}1, j \rangle$	$\langle k\text{-}1, j\text{-}1 \rangle$	

where $\langle k, k\text{-}1 \rangle^*$ only occurs for $j = k-3$.

For $k = 4, \ldots, p$:
$\langle k,1 \rangle$:

ord. char.	deg	modular constituents of this ord. char.					
$\langle\langle k,1 \rangle\rangle$	$2(-1)^{k+1}$	$\langle k+1,2 \rangle$	$\langle k+1,1 \rangle$	$\langle k,2 \rangle$	$\langle k,1 \rangle$		
$\langle\langle k,k \rangle\rangle$	$(-1)^{k-1}$		$\langle k+1,1 \rangle$		$\langle k,1 \rangle$		
$\langle\langle k-1,1 \rangle\rangle$	$2(-1)^{k}$			$\langle k,2 \rangle$	$\langle k,1 \rangle$	$\langle k-1,2 \rangle$	$\langle k-1,1 \rangle$
$\langle\langle k-1,k-1 \rangle\rangle$	$(-1)^{k}$				$\langle k,1 \rangle$		$\langle k-1,1 \rangle$

The tables are organized as follows: The head denotes the projective indecomposable $\mathcal{B}$-lattice L. The first column lists the irreducible characters that occur in $\mathbb{Q}_p L$, then the degree of these characters modulo p is given and the last columns give the $\mathcal{B}$-composition factors of L/pL.

Theorem 8 *There are embeddings $\varphi_i : P_i \hookrightarrow V$, such that Λ is generated by the idempotents* $\mathrm{id}_{|P_{\langle p \rangle}}, \ldots, \mathrm{id}_{|P_{\langle 3,1 \rangle}}$ *and the following elements $\varphi_{x,y} \in \Lambda_{x,y}$:*

$\varphi_{\langle p \rangle,\langle p-1 \rangle} = \epsilon_{\langle\langle p-1 \rangle\rangle} + p\epsilon_{\langle\langle p,p-2 \rangle\rangle}$, $\qquad \varphi_{\langle p-1 \rangle,\langle p \rangle} = p\epsilon_{\langle\langle p-1 \rangle\rangle} + \frac{1}{2}\epsilon_{\langle\langle p,p-2 \rangle\rangle}$,
$\varphi_{\langle p \rangle,\langle p,p-2 \rangle} = \epsilon_{\langle\langle p,p-2 \rangle\rangle} + \epsilon_{\langle\langle p-1,p-2 \rangle\rangle}$, $\qquad \varphi_{\langle p,p-2 \rangle,\langle p \rangle} = p\epsilon_{\langle\langle p,p-2 \rangle\rangle} + p\epsilon_{\langle\langle p-1,p-2 \rangle\rangle}$,
$\varphi_{\langle p-1 \rangle,\langle p-2 \rangle} = \epsilon_{\langle\langle p-2 \rangle\rangle} + \epsilon_{\langle\langle p,p-2 \rangle\rangle}$, $\qquad \varphi_{\langle p-2 \rangle,\langle p-1 \rangle} = p\epsilon_{\langle\langle p-2 \rangle\rangle} - \frac{p}{2}\epsilon_{\langle\langle p,p-2 \rangle\rangle}$,
$\varphi_{\langle p-1 \rangle,\langle p,p-1 \rangle} = \epsilon_{\langle\langle p,p-1 \rangle\rangle} + \epsilon_{\langle\langle p,p-2 \rangle\rangle}$, $\qquad \varphi_{\langle p,p-1 \rangle,\langle p-1 \rangle} = p\epsilon_{\langle\langle p,p-1 \rangle\rangle} + p\epsilon_{\langle\langle p,p-2 \rangle\rangle}$.
For $j = 2, \ldots, p-2$
$\varphi_{\langle j \rangle,\langle j-1 \rangle} = \epsilon_{\langle\langle j-1 \rangle\rangle} + \epsilon_{\langle\langle p,j-1 \rangle\rangle}$, $\qquad \varphi_{\langle j-1 \rangle,\langle j \rangle} = p\epsilon_{\langle\langle j-1 \rangle\rangle} - \frac{p}{2}\epsilon_{\langle\langle p,j-1 \rangle\rangle}$,
$\varphi_{\langle j \rangle,\langle p,j \rangle} = \epsilon_{\langle\langle p,j \rangle\rangle} + \epsilon_{\langle\langle p,j-1 \rangle\rangle}$, $\qquad \varphi_{\langle p,j \rangle,\langle j \rangle} = p\epsilon_{\langle\langle p,j \rangle\rangle} + p\epsilon_{\langle\langle p,j-1 \rangle\rangle}$,
$\varphi_{\langle 1 \rangle,\langle p,1 \rangle} = \epsilon_{\langle\langle p,1 \rangle\rangle} + \epsilon_{\langle\langle p,p \rangle\rangle}$, $\qquad \varphi_{\langle p,1 \rangle,\langle 1 \rangle} = p\epsilon_{\langle\langle p,j \rangle\rangle} - 2p\epsilon_{\langle\langle p,p \rangle\rangle}$.
For $k = 4, \ldots, p$
$\varphi_{\langle k,k-1 \rangle,\langle k,k-2 \rangle} = \epsilon_{\langle\langle k,k-2 \rangle\rangle} + \epsilon_{\langle\langle k-1,k-3 \rangle\rangle}$, $\qquad \varphi_{\langle k,k-2 \rangle,\langle k,k-1 \rangle} = p\epsilon_{\langle\langle k,k-2 \rangle\rangle} - p\epsilon_{\langle\langle k-1,k-3 \rangle\rangle}$,
$\varphi_{\langle k,k-1 \rangle,\langle k-1,k-3 \rangle} = \epsilon_{\langle\langle k-1,k-3 \rangle\rangle} + \epsilon_{\langle\langle k-2,k-3 \rangle\rangle}$, $\qquad \varphi_{\langle k-1,k-3 \rangle,\langle k,k-1 \rangle} = p\epsilon_{\langle\langle k-1,k-3 \rangle\rangle} + p\epsilon_{\langle\langle k-2,k-3 \rangle\rangle}$,
$\varphi_{\langle k,k-2 \rangle,\langle k,k-3 \rangle} = \epsilon_{\langle\langle k,k-3 \rangle\rangle} + \epsilon_{\langle\langle k-1,k-3 \rangle\rangle}$, $\qquad \varphi_{\langle k,k-3 \rangle,\langle k,k-2 \rangle} = p\epsilon_{\langle\langle k,k-3 \rangle\rangle} + p\epsilon_{\langle\langle k-1,k-3 \rangle\rangle}$,
$\varphi_{\langle k,k-2 \rangle,\langle k-1,k-2 \rangle} = \epsilon_{\langle\langle k-1,k-2 \rangle\rangle} + \epsilon_{\langle\langle k-1,k-3 \rangle\rangle}$, $\qquad \varphi_{\langle k-1,k-2 \rangle,\langle k,k-2 \rangle} = p\epsilon_{\langle\langle k-1,k-2 \rangle\rangle} + p\epsilon_{\langle\langle k-1,k-3 \rangle\rangle}$,
and for $j = 2, \ldots, k-3$
$\varphi_{\langle k,j \rangle,\langle k-1,j \rangle} = \epsilon_{\langle\langle k-1,j \rangle\rangle} + \epsilon_{\langle\langle k-1,j-1 \rangle\rangle}$, $\qquad \varphi_{\langle k-1,j \rangle,\langle k,j \rangle} = p\epsilon_{\langle\langle k-1,j \rangle\rangle} + p\epsilon_{\langle\langle k-1,j-1 \rangle\rangle}$,
$\varphi_{\langle k,j \rangle,\langle k,j-1 \rangle} = \epsilon_{\langle\langle k,j-1 \rangle\rangle} + \epsilon_{\langle\langle k-1,j-1 \rangle\rangle}$, $\qquad \varphi_{\langle k,j-1 \rangle,\langle k,j \rangle} = p\epsilon_{\langle\langle k,j-1 \rangle\rangle} + p\epsilon_{\langle\langle k-1,j-1 \rangle\rangle}$,
$\varphi_{\langle k,1 \rangle,\langle k-1,1 \rangle} = \epsilon_{\langle\langle k-1,1 \rangle\rangle} + \epsilon_{\langle\langle k-1,k-1 \rangle\rangle}$, $\qquad \varphi_{\langle k-1,1 \rangle,\langle k,1 \rangle} = p\epsilon_{\langle\langle k-1,1 \rangle\rangle} - 2p\epsilon_{\langle\langle k-1,k-1 \rangle\rangle}$
and
$\varphi_{\langle 3,2 \rangle,\langle 3,1 \rangle} = \epsilon_{\langle\langle 3,1 \rangle\rangle} + \epsilon_{\langle\langle 2,2 \rangle\rangle}$, $\qquad \varphi_{\langle 3,1 \rangle,\langle 3,2 \rangle} = p\epsilon_{\langle\langle 3,1 \rangle\rangle} + \frac{1}{2}p\epsilon_{\langle\langle 2,2 \rangle\rangle}$.

Proof. Let x, y be two distinct Brauer characters in $\mathcal{B}$ such that the Cartan invariant $c_{x,y}$ is two. Let $\{\chi, \psi\} = c_x \cap c_y$ be the two ordinary irreducibles in $\mathcal{B}$, in which x and y occur both as modular constituents. Then the rank of $\Lambda_{x,y}$ is also two, and, since $m_{x,y}(\chi) + m_{y,x}(\chi) = m_{x,y}(\psi) + m_{y,x}(\psi) = 1$ by Theorem 5, the $\mathbb{Z}_p$-lattice $\Lambda_{x,y}$ is generated by

$$ap^{m_{x,y}(\chi)}\epsilon_\chi + bp^{m_{x,y}(\psi)}\epsilon_\psi \text{ and } p^{m_{x,y}(\psi)+1}\epsilon_\psi$$

with $a, b \in \mathbb{Z}_p^*$. Changing the basis one can always choose $a = 1$ and $b \in \{1, \ldots, p-1\}$. But in fact one can choose the embeddings φ_i, such that

$$(\star) \quad a = b = 1 \text{ if } x < y.$$

Since Λ is a symmetric order, $\Lambda_{y,x}$ is the dual lattice of $\Lambda_{x,y}$. So in the normalization of $(\star)$ one calculates $\Lambda_{y,x}$ for $x < y$ to be generated by

$$p^{1-m_{x,y}(\chi)}\epsilon_\chi + (-\frac{\chi(1)}{\psi(1)})p^{1-m_{x,y}(\psi)}\epsilon_\psi \text{ and } p^{2-m_{x,y}(\psi)}\epsilon_\psi.$$

It remains to prove $(\star)$: Let $x < y$ and $c_x \cap c_y = \{\chi, \psi\}$ with $\chi < \psi$ (say). An inspection of the decomposition matrix shows that then the characters y and ψ belong to the same p-regular partition: $y = \psi$. So starting with the last ordinary character $\chi = \langle 1, 1\rangle$ in $\mathcal{B}$ one may choose the χ-component of the generators of the $\Lambda_{x,y}$ ($x < y \neq \chi$) by changing the basis of $\Lambda_{x,y}$ and adopt the χ-component of the generators of $\Lambda_{x,\chi}$ by conjugation in $\Lambda(\chi)$.

The following decorated exponent matrices show, which entries below the diagonal one may choose by base change (*) and which one by conjugation ($^+$).

$$A := \begin{pmatrix} 0 & 1 \\ 0^+ & 0 \end{pmatrix}, B := \begin{pmatrix} 0 & 1 & 1 & 2 & 1 \\ 0^+ & 0 & 1 & 1 & 1 \\ 0^+ & 1 & 0 & 1 & 1 \\ 0 & 0^* & 0^* & 0 & 0 \\ 0^+ & 1 & 1 & 1^* & 0 \end{pmatrix}, C := \begin{pmatrix} 0 & 1 & 1 & 2 \\ 0^+ & 0 & 1 & 1 \\ 0^+ & 1 & 0 & 1 \\ 0 & 0^* & 0^* & 0 \end{pmatrix}, D := \begin{pmatrix} 0 & 1 & 2 \\ 0^+ & 0 & 1 \\ 0 & 0^* & 0 \end{pmatrix}.$$

□

This investigation of the block $\mathcal{B}$ shows that the projections of $\mathcal{B}$ into the simple components of $\mathbb{Q}_p\mathcal{B}$ are as big as possible which also means that the amalgamations between these projections are maximal:

Corollary 9 *Let φ, ϑ denote two different irreducible Brauer characters in $\mathcal{B}$. Then the Cartan invariant $c_{\varphi,\vartheta}$ is two if and only if the dimension of the* Ext*-group between the two simple $\mathbb{F}_p\mathcal{B}$-modules is one.*

7 Quiver and relations for $\overline{\mathcal{B}}$

Relations for the Ext-quiver for $\overline{\mathcal{B}} = \mathcal{B}/p\mathcal{B}$ are given in [ErM] where they could not determine all the constants. Taking the explicit generators of the basic order of $\mathcal{B}$ from Theorem 8 instead of the ones given in [ErM], it is easy to compute the exact relations, even for $\mathcal{B}$ and not only modulo p.

Corollary 10 *Using the notation of [ErM], we denote by $\tilde{\epsilon}$ the generator of Λ given in Theorem 8 that maps to a multiple of the generator for $\Lambda/p\Lambda$ denoted by ϵ in [ErM].* id *denotes the identity on the respective projective indecomposable Λ-lattice. Then the following relations hold in Λ:*

1) $\tilde{\epsilon}_i\tilde{\epsilon}_{i+1} = 0 = \tilde{\eta}_i\tilde{\eta}_{i+1}$, $\tilde{\epsilon}_i\tilde{\eta}_i\tilde{\epsilon}_i = p\tilde{\epsilon}_i$, $\tilde{\eta}_i\tilde{\epsilon}_i\tilde{\eta}_i = p\tilde{\eta}_i$, $\tilde{\epsilon}_i\tilde{\eta}_i + \tilde{\eta}_{i+1}\tilde{\epsilon}_{i+1} = p \cdot \mathrm{id}$.

2) *These paths are also 0 in Λ.*

3) *If $e = \langle j\rangle$:*
$\tilde{\gamma}_{i+1}\tilde{\delta}_{i+1} + \tilde{\delta}_i\tilde{\gamma}_i + u_i\tilde{\eta}\tilde{\epsilon} = p \cdot \mathrm{id}$, *where* $u_i = \frac{3}{2}$ *if* $e \neq \langle p-1\rangle$ *and* $u_i = 1$, *if* $e = \langle p-1\rangle$.
If $e = \langle k, 1\rangle$:
$\tilde{\gamma}_{i+1}\tilde{\delta}_{i+1} + \tilde{\delta}_i\tilde{\gamma}_i + 3\tilde{\eta}\tilde{\epsilon} = -2p \cdot \mathrm{id}$.

4) $\varphi_{\langle p,j-1\rangle,\langle p,j\rangle}\varphi_{\langle p,j\rangle,\langle j\rangle} = -2\varphi_{\langle p,j-1\rangle,\langle j-1\rangle}\varphi_{\langle j-1\rangle,\langle j\rangle}$. *All other squares commute.*

5) $\tilde{\lambda}\tilde{\kappa} = \tilde{\eta}\tilde{\epsilon} - \tilde{\rho}_1\tilde{\zeta}_1$.

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