Semigroup Package
—
CommSemi 0.1

by

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Contents

1 Finitely Generated Commutative Semigroups (preliminary)

Given a finitely presented semigroup S, we can turn it into a commutative finitely presented semigroup by factoring the free semigroup underlying S by the relations of S, together with the ones commuting generators. We call the commutative finitely presented semigroup obtained in this way the abelianization of S.

1 ► `Abelianization(` S `)` A

returns the abelianization of the finitely presented semigroup S.

Knuth Bendix procedure is known to terminate for commutative finitely presented semigroups. Furthermore there are some special methods for them, namely to compute whether a commutative finitely presented semigroup is infinite or not, its size and compare Green's classes.

2 ► `IsFinite(` S `)`
► `Size(` S `)`
► `IsGreensLessThanOrEqual(` $C1$`,`$C2$ `)`

One can think of words, in a free commutative semigroup, as vectors with length the number of the generators and entries in the set of natural numbers including zero (if the semigroup has no identity, the zero vector is not allowed). Each entry is sum of the exponents of the corresponding generator in the word. Actually, vectors and words are in a one-one correpondence. Therefore, an element of a finitely presented semigroup can also be represented by a vector (though not uniquely) - the vector of the underlying word of that element.

3 ► `ElementOfFpSemigroupAsVector(` e `)` F

for an element e of a finitely presented semigroup. Returns the vector of exponents of the underlying word.

4 ► `VectorToElementOfCommutativeFpSemigroup(` S, v `)` F

for a commutative semigroup S and a vector v. Returns the element of S which has as underlying word the product of the free generators with the exponents the entries of the vector.

```
gap> f := FreeSemigroup( "a" , "b" );;
gap> a := GeneratorsOfSemigroup( f )[ 1 ];;
gap> b := GeneratorsOfSemigroup( f )[ 2 ];;
gap> h := Abelianization( f / [ [ a^3 , a ],[ b^2 , a*b ] ]);
<fp semigroup on the generators [ a, b ]>
gap> IsFinite( h );
true
```

```
gap> Size( h );
5
gap> Elements( h );
[ a, b, a^2, a*b, a^2*b ]
gap> ha := GreensHClassOfElement(h,Elements(h)[1]);
{a}
gap> hb := GreensHClassOfElement(h,Elements(h)[2]);
{b}
gap> hab := GreensHClassOfElement(h,Elements(h)[3]);
{a^2}
gap> IsGreensLessThanOrEqual(hb,ha);
false
gap> IsGreensLessThanOrEqual(hab,ha);
true
```

2 What is available

1 ► `IsCommutativeSemigroupRws(` *obj* `)` C

This is the category of commutative semigroup rewriting systems.

2 ► `CommutativeSemigroupRws(` S`,` *vlteq* `)` A

returns the commutative rewriting system of the commutative FpSemigroup S with respect to the *vlteq* ordering on vectors.

3 ► `VectorRulesOfCommutativeSemigroupRws(` *commrws* `)` AM

the rules of the commutative rws written as vector rules

4 ► `IsReducedConfluentCommutativeSemigroupRws(` *obj* `)` C

This is the category of reduced confluent commutative semigroup rewriting systems.

5 ► `ReducedConfluentCommutativeSemigroupRws(` S `)` AM

returns a reduced confluent commutative rewriting system for the commutative semigroup S.

6 ► `EpimorphismToLargestSemilatticeHomomorphicImage( s )` A

for a commutative semigroup s, returns the epimorphism to the largest semilattice homomorphic image

7 ► `LargestSemilatticeHomomorphicImage(` S `)` O

returns the largest semilattice homomorphic image

8 ► `ArchimedeanRelation(` S `)` O

returns the ArchimedeanRelation on the semigroup S.

9 ► `StabilizerOfGreensClass(` C `)` A

returns the subsemigroup of the parent of C, which stabilizes C

10 ► `AssocWordToVector(` w `)` F

for an associative word w. Returns the vector of exponents of each generator in *word*.

11 ► `ElementOfFpSemigroupAsVector(` e `)` F

for an element e of a finitely presented semigroup. Returns the vector of exponents of the underlying word.

12 ► `VectorToElementOfCommutativeFpSemigroup(` S`,` v `)` F

for a commutative semigroup S and a vector v. Returns the element of S which has as underlying word the product of the free generators with the exponents the entries of the vector.

13 ► `EpimorphismAbelianization(` S `)` A

returns an epimorphism from an fp semigroup to its abelianization.

14 ► `Abelianization(` S `)` A

returns the abelianization of the finitely presented semigroup S.

15 ► `BasisOfSemigroupIdeal( `*id*` )` A

for an ideal of a commutative finitely presented semigroup S It returns the subset of reduced words of the set of minimal generators of the ideal of the words representing elements in the principal ideal of S generated by e. Notice that this is a decision procedure to decide whether a given word represents an element of the principal ideal of S generated by e.

16 ► `VectorBasisOfSemigroupIdealWithFactors( `*id*` )` A

17 ► `VectorBasisOfSemigroupIdeal( `*id*` )` O

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This index covers only this manual. A page number in *italics* refers to a whole section which is devoted to the indexed subject. Keywords are sorted with case and spaces ignored, e.g., "PermutationCharacter" comes before "permutation group".