
```
GAP4, Version: 4.3fix5 of June, 2003

gap> tbl:=CharacterTable("A5");
CharacterTable( "A5" )

gap> Display(tbl);
A5

     2  2  2  .  .  .
     3  1  .  1  .  .
     5  1  .  .  1  1

       1a 2a 3a 5a 5b
    2P 1a 1a 3a 5b 5a
    3P 1a 2a 1a 5b 5a
    5P 1a 2a 3a 1a 1a

X.1     1  1  1  1  1
X.2     3 -1  .  A *A
X.3     3 -1  . *A  A
X.4     4  .  1 -1 -1
X.5     5  1 -1  .  .

A = -E(5)-E(5)^4
  = (1-ER(5))/2 = -b5
```

```
gap> Irr(tbl)[2];

Character( CharacterTable( "A5" ),
           [ 3, -1, 0, -E(5)-E(5)^4, -E(5)^2-E(5)^3 ] )


gap> hs:=MolienSeries(tbl,Irr(tbl)[2]);

( 1-z^2-z^3+z^6+z^7-z^9 ) / ( (1-z^5)*(1-z^3)*(1-z^2)^2 )


gap> MolienSeriesWithGivenDenominator(hs,[2,6,10]);

( 1+z^15 ) / ( (1-z^10)*(1-z^6)*(1-z^2) )
```

```
gap> z:=E(5)+E(5)^4;;
gap> rep:=
> # representation of A5 over Z[z], on (2,3,5)-triple
> [ [ [ 0, 1, 0 ], [ 1, 0, 0 ], [ 0, 0,-1 ] ],
>   [ [ 0, 0, 1 ], [ z, 1,-1 ], [-1, 0,-1 ] ],
>   [ [ z, 1,-1 ], [ 0, 0, 1 ], [ 1, 0, 1 ] ] ];;
gap> h:=Group(rep);
<matrix group with 3 generators>
gap> Display(h.1);
[ [  0,  1,  0 ],
  [  1,  0,  0 ],
  [  0,  0, -1 ] ]
gap> Display(h.2);
[ [           0,           0,           1 ],
  [ E(5)+E(5)^4,           1,          -1 ],
  [          -1,           0,          -1 ] ]
gap> Display(h.3);
[ [ E(5)+E(5)^4,           1,          -1 ],
  [           0,           0,           1 ],
  [           1,           0,           1 ] ]
gap> List(GeneratorsOfGroup(h),Order);
[ 2, 3, 5 ]
gap> Size(h);
60
```

```
gap> polring:=PolynomialRing(Cyclotomics,
>                            ["X_1","X_2","X_3"]);
PolynomialRing(..., [ X_1, X_2, X_3 ])

gap> indets:=IndeterminatesOfPolynomialRing(polring);
[ X_1, X_2, X_3 ]

gap> x1:=indets[1];;
gap> x2:=indets[2];;
gap> x3:=indets[3];;

gap> x:=[x1,x2,x3];
[ X_1, X_2, X_3 ]
```

```
gap> f:= # the A5-invariant scalar product
> [ [ 1,          1/2*(-1-z), -1/2 ],
>   [ 1/2*(-1-z),          1,  1/2 ],
>   [       -1/2,        1/2,    1 ] ];;

gap> Display(f);
[ [                    1, 1/2*E(5)^2+1/2*E(5)^3, -1/2 ],
  [ 1/2*E(5)^2+1/2*E(5)^3,                    1,  1/2 ],
  [                 -1/2,                  1/2,    1 ] ]

gap> f1:=x*f^(-1)*x; # invariant of degree 2

 3*X_1^2
+(2*E(5)-2*E(5)^2-2*E(5)^3+2*E(5)^4)*X_1*X_2
+(-4*E(5)-2*E(5)^2-2*E(5)^3-4*E(5)^4)*X_1*X_3
+3*X_2^2
+(4*E(5)+2*E(5)^2+2*E(5)^3+4*E(5)^4)*X_2*X_3
+(-3*E(5)-2*E(5)^2-2*E(5)^3-3*E(5)^4)*X_3^2
```

```
gap> v:=NullspaceMat(rep[3]-rep[3]^0)[1];
[ -E(5)-2*E(5)^2-2*E(5)^3-E(5)^4,
  -E(5)-2*E(5)^2-2*E(5)^3-E(5)^4, 1 ]

gap> vorb:=Orbit(h,v);
[ [ -E(5)-2*E(5)^2-2*E(5)^3-E(5)^4,
    -E(5)-2*E(5)^2-2*E(5)^3-E(5)^4, 1 ],
  [ -E(5)-2*E(5)^2-2*E(5)^3-E(5)^4,
    -E(5)-2*E(5)^2-2*E(5)^3-E(5)^4, -1 ],
  [ E(5)+E(5)^4, -E(5)-2*E(5)^2-2*E(5)^3-E(5)^4, -1 ],
  [ -E(5)-2*E(5)^2-2*E(5)^3-E(5)^4, E(5)+E(5)^4, 1 ],
  [ -E(5)-E(5)^4, E(5)+E(5)^4, 1 ],
  [ E(5)+E(5)^4, -E(5)-E(5)^4, -1 ],
  [ -E(5)-E(5)^4, E(5)+E(5)^4,
    -E(5)+E(5)^2+E(5)^3-E(5)^4 ],
  [ E(5)+E(5)^4, -E(5)-E(5)^4,
    E(5)-E(5)^2-E(5)^3+E(5)^4 ],
  [ E(5)+2*E(5)^2+2*E(5)^3+E(5)^4, -E(5)-E(5)^4, -1 ],
  [ -E(5)-E(5)^4, E(5)+2*E(5)^2+2*E(5)^3+E(5)^4, 1 ],
  [ E(5)+2*E(5)^2+2*E(5)^3+E(5)^4,
    E(5)+2*E(5)^2+2*E(5)^3+E(5)^4, 1 ],
  [ E(5)+2*E(5)^2+2*E(5)^3+E(5)^4,
    E(5)+2*E(5)^2+2*E(5)^3+E(5)^4, -1 ] ]
```

```
gap> PermList(List(vorb,x->Position(vorb,-x)));
(1,12)(2,11)(3,10)(4,9)(5,6)(7,8)

gap> vvecs:=vorb{[1,2,3,4,5,7]};;

gap> f2:=Product(List(vvecs,i->x*i));
> # invariant of degree 6

...
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```
 (-E(5)-3*E(5)^2-3*E(5)^3-E(5)^4)*X_1^6
+(-6*E(5)-14*E(5)^2-14*E(5)^3-6*E(5)^4)*X_1^5*X_2
+(-2*E(5)^2-2*E(5)^3)*X_1^5*X_3
+(E(5)+3*E(5)^2+3*E(5)^3+E(5)^4)*X_1^4*X_2^2
+(-16*E(5)-38*E(5)^2-38*E(5)^3-16*E(5)^4)*X_1^4*X_2*X_3
+(11*E(5)+28*E(5)^2+28*E(5)^3+11*E(5)^4)*X_1^4*X_3^2
+(12*E(5)+28*E(5)^2+28*E(5)^3+12*E(5)^4)*X_1^3*X_2^3
+(-16*E(5)-36*E(5)^2-36*E(5)^3-16*E(5)^4)*X_1^3*X_2^2*X_3
+(52*E(5)+120*E(5)^2+120*E(5)^3+52*E(5)^4)*X_1^3*X_2*X_3^2
+(-12*E(5)-20*E(5)^2-20*E(5)^3-12*E(5)^4)*X_1^3*X_3^3
+(E(5)+3*E(5)^2+3*E(5)^3+E(5)^4)*X_1^2*X_2^4
+(16*E(5)+36*E(5)^2+36*E(5)^3+16*E(5)^4)*X_1^2*X_2^3*X_3
+(50*E(5)+168*E(5)^2+168*E(5)^3+50*E(5)^4)*X_1^2*X_2^2*X_3^2
+(20*E(5)-4*E(5)^2-4*E(5)^3+20*E(5)^4)*X_1^2*X_2*X_3^3
+(-16*E(5)-19*E(5)^2-19*E(5)^3-16*E(5)^4)*X_1^2*X_3^4
+(-6*E(5)-14*E(5)^2-14*E(5)^3-6*E(5)^4)*X_1*X_2^5
+(16*E(5)+38*E(5)^2+38*E(5)^3+16*E(5)^4)*X_1*X_2^4*X_3
+(52*E(5)+120*E(5)^2+120*E(5)^3+52*E(5)^4)*X_1*X_2^3*X_3^2
+(-20*E(5)+4*E(5)^2+4*E(5)^3-20*E(5)^4)*X_1*X_2^2*X_3^3
+(12*E(5)-22*E(5)^2-22*E(5)^3+12*E(5)^4)*X_1*X_2*X_3^4
+(-6*E(5)-6*E(5)^4)*X_1*X_3^5
+(-E(5)-3*E(5)^2-3*E(5)^3-E(5)^4)*X_2^6
+(2*E(5)^2+2*E(5)^3)*X_2^5*X_3
+(11*E(5)+28*E(5)^2+28*E(5)^3+11*E(5)^4)*X_2^4*X_3^2
+(12*E(5)+20*E(5)^2+20*E(5)^3+12*E(5)^4)*X_2^3*X_3^3
+(-16*E(5)-19*E(5)^2-19*E(5)^3-16*E(5)^4)*X_2^2*X_3^4
+(6*E(5)+6*E(5)^4)*X_2*X_3^5
+(-E(5)+E(5)^2+E(5)^3-E(5)^4)*X_3^6
```

```
gap> w:=NullspaceMat(rep[2]-rep[2]^0)[1];
[ E(5)+4*E(5)^2+4*E(5)^3+E(5)^4,
  3*E(5)+6*E(5)^2+6*E(5)^3+3*E(5)^4, 1 ]

gap> worb:=Orbit(h,w);;

gap> PermList(List(worb,x->Position(worb,-x)));
(1,20)(2,19)(3,17)(4,14)(5,18)
      (6,15)(7,16)(8,11)(9,12)(10,13)

gap> wvecs:=worb{[1,2,3,4,5,6,7,8,9,10]};;

gap> f3:=Product(List(wvecs,i->x*i));
> # invariant of degree 10

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```
(-140235*E(5)-367140*E(5)^2-367140*E(5)^3-140235*E(5)^4)*X_1^10+(-1045250*E(5)\
-2736500*E(5)^2-2736500*E(5)^3-1045250*E(5)^4)*X_1^9*X_2+(-178550*E(5)-467450*\
E(5)^2-467450*E(5)^3-178550*E(5)^4)*X_1^9*X_3+(-2720559*E(5)-7122516*E(5)^2-71\
22516*E(5)^3-2720559*E(5)^4)*X_1^8*X_2^2+(-1983066*E(5)-5191734*E(5)^2-5191734\
*E(5)^3-1983066*E(5)^4)*X_1^8*X_2*X_3+(683016*E(5)+1788159*E(5)^2+1788159*E(5)\
^3+683016*E(5)^4)*X_1^8*X_3^2+(-2107224*E(5)-5516784*E(5)^2-5516784*E(5)^3-210\
7224*E(5)^4)*X_1^7*X_2^3+(-7721400*E(5)-20214888*E(5)^2-20214888*E(5)^3-772140\
0*E(5)^4)*X_1^7*X_2^2*X_3+(3005232*E(5)+7867800*E(5)^2+7867800*E(5)^3+3005232*\
E(5)^4)*X_1^7*X_2*X_3^2+(943560*E(5)+2470272*E(5)^2+2470272*E(5)^3+943560*E(5)\
^4)*X_1^7*X_3^3+(2860794*E(5)+7489656*E(5)^2+7489656*E(5)^3+2860794*E(5)^4)*X_\
1^6*X_2^4+(-13096872*E(5)-34288056*E(5)^2-34288056*E(5)^3-13096872*E(5)^4)*X_1\
^6*X_2^3*X_3+(1583040*E(5)+4144452*E(5)^2+4144452*E(5)^3+1583040*E(5)^4)*X_1^6\
*X_2^2*X_3^2+(6721416*E(5)+17596896*E(5)^2+17596896*E(5)^3+6721416*E(5)^4)*X_1\
^6*X_2*X_3^3+(-688962*E(5)-1803726*E(5)^2-1803726*E(5)^3-688962*E(5)^4)*X_1^6*\
X_3^4+(6304948*E(5)+16506568*E(5)^2+16506568*E(5)^3+6304948*E(5)^4)*X_1^5*X_2^\
5+(-7179988*E(5)-18797452*E(5)^2-18797452*E(5)^3-7179988*E(5)^4)*X_1^5*X_2^4*X\
_3+(-10759472*E(5)-28168664*E(5)^2-28168664*E(5)^3-10759472*E(5)^4)*X_1^5*X_2^\
3*X_3^2+(15802600*E(5)+41371744*E(5)^2+41371744*E(5)^3+15802600*E(5)^4)*X_1^5*\
X_2^2*X_3^3+(-171052*E(5)-447820*E(5)^2-447820*E(5)^3-171052*E(5)^4)*X_1^5*X_2\
*X_3^4+(-1269296*E(5)-3323060*E(5)^2-3323060*E(5)^3-1269296*E(5)^4)*X_1^5*X_3^\
5+(2860794*E(5)+7489656*E(5)^2+7489656*E(5)^3+2860794*E(5)^4)*X_1^4*X_2^6+(717\
9988*E(5)+18797452*E(5)^2+18797452*E(5)^3+7179988*E(5)^4)*X_1^4*X_2^5*X_3+(-20\
100240*E(5)-52623110*E(5)^2-52623110*E(5)^3-20100240*E(5)^4)*X_1^4*X_2^4*X_3^2\
+(10144040*E(5)+26557440*E(5)^2+26557440*E(5)^3+10144040*E(5)^4)*X_1^4*X_2^3*X\
_3^3+(7782370*E(5)+20374510*E(5)^2+20374510*E(5)^3+7782370*E(5)^4)*X_1^4*X_2^2\
*X_3^4+(-4020432*E(5)-10525628*E(5)^2-10525628*E(5)^3-4020432*E(5)^4)*X_1^4*X_\
2*X_3^5+(-371326*E(5)-972144*E(5)^2-972144*E(5)^3-371326*E(5)^4)*X_1^4*X_3^6+(\
-2107224*E(5)-5516784*E(5)^2-5516784*E(5)^3-2107224*E(5)^4)*X_1^3*X_2^7+(13096\
872*E(5)+34288056*E(5)^2+34288056*E(5)^3+13096872*E(5)^4)*X_1^3*X_2^6*X_3+(-10\
759472*E(5)-28168664*E(5)^2-28168664*E(5)^3-10759472*E(5)^4)*X_1^3*X_2^5*X_3^2\
+(-10144040*E(5)-26557440*E(5)^2-26557440*E(5)^3-10144040*E(5)^4)*X_1^3*X_2^4*\
X_3^3+(14862680*E(5)+38911000*E(5)^2+38911000*E(5)^3+14862680*E(5)^4)*X_1^3*X_\
2^3*X_3^4+(-2834240*E(5)-7420136*E(5)^2-7420136*E(5)^3-2834240*E(5)^4)*X_1^3*X\
_2^2*X_3^5+(-1412168*E(5)-3697104*E(5)^2-3697104*E(5)^3-1412168*E(5)^4)*X_1^3*\
X_2*X_3^6+(60152*E(5)+157480*E(5)^2+157480*E(5)^3+60152*E(5)^4)*X_1^3*X_3^7+(-\
2720559*E(5)-7122516*E(5)^2-7122516*E(5)^3-2720559*E(5)^4)*X_1^2*X_2^8+(772140\
0*E(5)+20214888*E(5)^2+20214888*E(5)^3+7721400*E(5)^4)*X_1^2*X_2^7*X_3+(158304\
0*E(5)+4144452*E(5)^2+4144452*E(5)^3+1583040*E(5)^4)*X_1^2*X_2^6*X_3^2+(-15802\
600*E(5)-41371744*E(5)^2-41371744*E(5)^3-15802600*E(5)^4)*X_1^2*X_2^5*X_3^3+(7\
782370*E(5)+20374510*E(5)^2+20374510*E(5)^3+7782370*E(5)^4)*X_1^2*X_2^4*X_3^4+\
(2834240*E(5)+7420136*E(5)^2+7420136*E(5)^3+2834240*E(5)^4)*X_1^2*X_2^3*X_3^5+\
(-2061492*E(5)-5397056*E(5)^2-5397056*E(5)^3-2061492*E(5)^4)*X_1^2*X_2^2*X_3^6\
+(40632*E(5)+106376*E(5)^2+106376*E(5)^3+40632*E(5)^4)*X_1^2*X_2*X_3^7+(26340*\
E(5)+68959*E(5)^2+68959*E(5)^3+26340*E(5)^4)*X_1^2*X_3^8+(-1045250*E(5)-273650\
0*E(5)^2-2736500*E(5)^3-1045250*E(5)^4)*X_1*X_2^9+(1983066*E(5)+5191734*E(5)^2\
+5191734*E(5)^3+1983066*E(5)^4)*X_1*X_2^8*X_3+(3005232*E(5)+7867800*E(5)^2+786\
7800*E(5)^3+3005232*E(5)^4)*X_1*X_2^7*X_3^2+(-6721416*E(5)-17596896*E(5)^2-175\
96896*E(5)^3-6721416*E(5)^4)*X_1*X_2^6*X_3^3+(-171052*E(5)-447820*E(5)^2-44782\
0*E(5)^3-171052*E(5)^4)*X_1*X_2^5*X_3^4+(4020432*E(5)+10525628*E(5)^2+10525628\
*E(5)^3+4020432*E(5)^4)*X_1*X_2^4*X_3^5+(-1412168*E(5)-3697104*E(5)^2-3697104*\
E(5)^3-1412168*E(5)^4)*X_1*X_2^3*X_3^6+(-40632*E(5)-106376*E(5)^2-106376*E(5)^\
3-40632*E(5)^4)*X_1*X_2^2*X_3^7+(47308*E(5)+123854*E(5)^2+123854*E(5)^3+47308*\
E(5)^4)*X_1*X_2*X_3^8+(-1042*E(5)-2728*E(5)^2-2728*E(5)^3-1042*E(5)^4)*X_1*X_3\
^9+(-140235*E(5)-367140*E(5)^2-367140*E(5)^3-140235*E(5)^4)*X_2^10+(178550*E(5\
)+467450*E(5)^2+467450*E(5)^3+178550*E(5)^4)*X_2^9*X_3+(683016*E(5)+1788159*E(\
5)^2+1788159*E(5)^3+683016*E(5)^4)*X_2^8*X_3^2+(-943560*E(5)-2470272*E(5)^2-24\
70272*E(5)^3-943560*E(5)^4)*X_2^7*X_3^3+(-688962*E(5)-1803726*E(5)^2-1803726*E\
(5)^3-688962*E(5)^4)*X_2^6*X_3^4+(1269296*E(5)+3323060*E(5)^2+3323060*E(5)^3+1\
269296*E(5)^4)*X_2^5*X_3^5+(-371326*E(5)-972144*E(5)^2-972144*E(5)^3-371326*E(\
5)^4)*X_2^4*X_3^6+(-60152*E(5)-157480*E(5)^2-157480*E(5)^3-60152*E(5)^4)*X_2^3\
*X_3^7+(26340*E(5)+68959*E(5)^2+68959*E(5)^3+26340*E(5)^4)*X_2^2*X_3^8+(1042*E\
(5)+2728*E(5)^2+2728*E(5)^3+1042*E(5)^4)*X_2*X_3^9+(-377*E(5)-987*E(5)^2-987*E\
(5)^3-377*E(5)^4)*X_3^10
```

```
gap> prim:=[f1,f2,f3];;
gap> jac:=List(prim,p->List(x,i->Derivative(p,i)));;

gap> jac[1][1];
6*X_1
+(2*E(5)-2*E(5)^2-2*E(5)^3+2*E(5)^4)*X_2
+(-4*E(5)-2*E(5)^2-2*E(5)^3-4*E(5)^4)*X_3
gap> jac[1][2];
 (2*E(5)-2*E(5)^2-2*E(5)^3+2*E(5)^4)*X_1
+6*X_2
+(4*E(5)+2*E(5)^2+2*E(5)^3+4*E(5)^4)*X_3
gap> jac[1][3];
 (-4*E(5)-2*E(5)^2-2*E(5)^3-4*E(5)^4)*X_1
+(4*E(5)+2*E(5)^2+2*E(5)^3+4*E(5)^4)*X_2
+(-6*E(5)-4*E(5)^2-4*E(5)^3-6*E(5)^4)*X_3

gap> g2:=DeterminantMat(jac);;
> # invariant of degree 15
gap> g2=0*g2;
false

gap> g2;
( 4642832384*E(5)+12155092992*E(5)^2
 +12155092992*E(5)^3+4642832384*E(5)^4)*X_1^12*X_2^2*X_3
+...
```