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Control Theory – Tutorial 0

To be discussed on Tuesday, April 21

Recap: Smith form. Let $\mathcal{D}$ be a commutative domain and $A \in \mathcal{D}^{n\times m}$. If A is equivalent to a matrix

$$S = \begin{bmatrix} D & 0 \\ 0 & 0 \end{bmatrix} \in \mathcal{D}^{n\times m},$$

where $D = \operatorname{diag}(d_1, \ldots, d_r)$ for some $0 \neq d_i \in \mathcal{D}$ with $d_1 | \ldots | d_r$, then S is said to be a Smith form of A. Under which conditions on $\mathcal{D}$ does every matrix have a Smith form? Express the determinantal ideals of A in terms of $d_1, \ldots, d_r$.

Use the determinantal ideals to determine the Smith forms of (i) $A = \operatorname{diag}(4, 6) \in \mathbb{Z}^{2\times 2}$ and (ii) $A = \operatorname{diag}(6, 10, 15) \in \mathbb{Z}^{3\times 3}$. (iii) Let $i \in \mathbb{Z}$. Compute the Smith form of

$$A = \begin{bmatrix} i & i+1 & i+2 \\ i+3 & i+4 & i+5 \\ i+6 & i+7 & i+8 \end{bmatrix} \in \mathbb{Z}^{3\times 3}$$

via elementary row and column operations.

Recap: Jordan form. Let K be a field and $A \in K^{n\times n}$. If A is similar to a matrix

$$J = \begin{bmatrix} J_1 & & \\ & \ddots & \\ & & J_k \end{bmatrix},$$

where each J_i has the form

$$J_i = \begin{bmatrix} \lambda_i & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & \lambda_i \end{bmatrix} \in K^{n_i \times n_i}$$

for some $\lambda_i \in K$ and $\sum_{i=1}^k n_i = n$, then J is said to be a Jordan form of A. Under which conditions on K does every square matrix have a Jordan form? Interpret the λ_i in terms of A. What can be said about the numbers $\mu(\lambda) := |\{l \mid J_{ll} = \lambda\}|$, $\nu(\lambda) := |\{i \mid (J_i)_{11} = \lambda\}|$, $\kappa(\lambda) := \max\{n_i \mid (J_i)_{11} = \lambda\}$? How can the Jordan form of A be determined from the Smith form of $sI - A \in K[s]^{n\times n}$? Do this for (i) $S = \operatorname{diag}(1, 1, 1, (s-1)^2, (s-1)^3)$ and (ii) $S = \operatorname{diag}(1, 1, 1, 1, (s-a)^3(s-b)^2)$ for $a, b \in K$ (distinguish the cases $a = b$ and $a \neq b$).