

$O_8^+(3).4 \pmod{7}$ 

|       | blocks          | defect | matrix                                    |
|-------|-----------------|--------|---|
| $G :$ | 1               | 1      | $7 \times 6$                              |
|       | 2               | 1      | $7 \times 6$                              |
|       | 3               | 1      | $7 \times 6$                              |
|       | $4 = \bar{2}$   | 1      | $7 \times 6$                              |
|       | 5               | 1      | $5 \times 3$                              |
|       | 6               | 1      | $5 \times 3$                              |
|       | 7               | 1      | $5 \times 3$                              |
|       | 8               | 0      | $819_1 = \chi_{9,0}, \varphi_{9,0}$       |
|       | 9               | 0      | $819_2 = \chi_{9,1}, \varphi_{9,1}$       |
|       | 10              | 0      | $819_3 = \chi_{9,2}, \varphi_{9,2}$       |
|       | $11 = \bar{9}$  | 0      | $819_4 = \chi_{9,3}, \varphi_{9,3}$       |
|       | 12              | 0      | $1638_1 = \chi_{10,0+}, \varphi_{10,0+}$  |
|       | 13              | 0      | $1638_2 = \chi_{10,1+}, \varphi_{10,1+}$  |
|       | 14              | 0      | $9100_1 = \chi_{12+}, \varphi_{12+}$      |
|       | 15              | 0      | $10920_1 = \chi_{17,0+}, \varphi_{17,0+}$ |
|       | 16              | 0      | $10920_2 = \chi_{17,1+}, \varphi_{17,1+}$ |
|       | 17              | 0      | $21840_1 = \chi_{18+}, \varphi_{18+}$     |
|       | 18              | 0      | $9450_1 = \chi_{23,0}, \varphi_{23,0}$    |
|       | 19              | 0      | $9450_2 = \chi_{23,1}, \varphi_{23,1}$    |
|       | 20              | 0      | $9450_3 = \chi_{23,2}, \varphi_{23,2}$    |
|       | $21 = \bar{19}$ | 0      | $9450_4 = \chi_{23,3}, \varphi_{23,3}$    |
|       | 22              | 0      | $18200_1 = \chi_{25,0}, \varphi_{25,0}$   |
|       | 23              | 0      | $18200_2 = \chi_{25,1}, \varphi_{25,1}$   |
|       | 24              | 0      | $18200_3 = \chi_{25,2}, \varphi_{25,2}$   |
|       | $25 = \bar{23}$ | 0      | $18200_4 = \chi_{25,3}, \varphi_{25,3}$   |
|       | 26              | 0      | $72800_1 = \chi_{26+}, \varphi_{26+}$     |
|       | 27              | 0      | $24192_1 = \chi_{36,0}, \varphi_{36,0}$   |
|       | 28              | 0      | $24192_2 = \chi_{36,1}, \varphi_{36,1}$   |
|       | 29              | 0      | $24192_3 = \chi_{36,2}, \varphi_{36,2}$   |
|       | $30 = \bar{28}$ | 0      | $24192_4 = \chi_{36,3}, \varphi_{36,3}$   |
|       | 31              | 0      | $27300_1 = \chi_{37,0}, \varphi_{37,0}$   |
|       | 32              | 0      | $27300_2 = \chi_{37,1}, \varphi_{37,1}$   |
|       | 33              | 0      | $27300_3 = \chi_{37,2}, \varphi_{37,2}$   |
|       | $34 = \bar{32}$ | 0      | $27300_4 = \chi_{37,3}, \varphi_{37,3}$   |
|       | 35              | 0      | $58240_1 = \chi_{38,0+}, \varphi_{38,0+}$ |
|       | 36              | 0      | $58240_2 = \chi_{38,1+}, \varphi_{38,1+}$ |
|       | 37              | 0      | $116480_1 = \chi_{39+}, \varphi_{39+}$    |
|       | 38              | 0      | $81900_1 = \chi_{44,0+}, \varphi_{44,0+}$ |
|       | 39              | 0      | $81900_2 = \chi_{44,1+}, \varphi_{44,1+}$ |
|       | 40              | 0      | $163800_1 = \chi_{45+}, \varphi_{45+}$    |

|  | blocks               | defect | matrix                                     |
|--|----------------------|--------|--|
|  | 41                   | 0      | $46592_1 = \chi_{50,0}, \varphi_{50,0}$    |
|  | 42                   | 0      | $46592_2 = \chi_{50,1}, \varphi_{50,1}$    |
|  | 43                   | 0      | $46592_3 = \chi_{50,2}, \varphi_{50,2}$    |
|  | $44 = \overline{42}$ | 0      | $46592_4 = \chi_{50,3}, \varphi_{50,3}$    |
|  | 45                   | 0      | $93184_1 = \chi_{51,0+}, \varphi_{51,0+}$  |
|  | 46                   | 0      | $93184_2 = \chi_{51,1+}, \varphi_{51,1+}$  |
|  | 47                   | 0      | $98280_1 = \chi_{53,0+}, \varphi_{53,0+}$  |
|  | 48                   | 0      | $98280_2 = \chi_{53,1+}, \varphi_{53,1+}$  |
|  | 49                   | 0      | $196560_1 = \chi_{54+}, \varphi_{54+}$     |
|  | 50                   | 0      | $218400_1 = \chi_{59+}, \varphi_{59+}$     |
|  | 51                   | 0      | $54600_1 = \chi_{63,0}, \varphi_{63,0}$    |
|  | 52                   | 0      | $54600_2 = \chi_{63,1}, \varphi_{63,1}$    |
|  | 53                   | 0      | $54600_3 = \chi_{63,2}, \varphi_{63,2}$    |
|  | $54 = \overline{52}$ | 0      | $54600_4 = \chi_{63,3}, \varphi_{63,3}$    |
|  | 55                   | 0      | $109200_1 = \chi_{64,0+}, \varphi_{64,0+}$ |
|  | 56                   | 0      | $109200_2 = \chi_{64,1+}, \varphi_{64,1+}$ |
|  | 57                   | 0      | $66339_1 = \chi_{66,0}, \varphi_{66,0}$    |
|  | 58                   | 0      | $66339_2 = \chi_{66,1}, \varphi_{66,1}$    |
|  | 59                   | 0      | $66339_3 = \chi_{66,2}, \varphi_{66,2}$    |
|  | $60 = \overline{58}$ | 0      | $66339_4 = \chi_{66,3}, \varphi_{66,3}$    |
|  | 61                   | 0      | $132678_1 = \chi_{67,0+}, \varphi_{67,0+}$ |
|  | 62                   | 0      | $132678_2 = \chi_{67,1+}, \varphi_{67,1+}$ |
|  | 63                   | 0      | $163800_2 = \chi_{69,0}, \varphi_{69,0}$   |
|  | 64                   | 0      | $163800_3 = \chi_{69,1}, \varphi_{69,1}$   |
|  | 65                   | 0      | $163800_4 = \chi_{69,2}, \varphi_{69,2}$   |
|  | $66 = \overline{64}$ | 0      | $163800_5 = \chi_{69,3}, \varphi_{69,3}$   |
|  | 67                   | 0      | $327600_1 = \chi_{70,0+}, \varphi_{70,0+}$ |
|  | 68                   | 0      | $327600_2 = \chi_{70,1+}, \varphi_{70,1+}$ |
|  | 69                   | 0      | $737100_1 = \chi_{72+}, \varphi_{78+}$     |
|  | 70                   | 0      | $465920_1 = \chi_{83,0+}, \varphi_{83,0+}$ |
|  | 71                   | 0      | $465920_2 = \chi_{83,1+}, \varphi_{83,1+}$ |
|  | 72                   | 0      | $931840_1 = \chi_{84+}, \varphi_{84+}$     |
|  | 73                   | 0      | $245700_1 = \chi_{89,0}, \varphi_{89,0}$   |
|  | 74                   | 0      | $245700_2 = \chi_{89,1}, \varphi_{89,1}$   |
|  | 75                   | 0      | $245700_3 = \chi_{89,2}, \varphi_{89,2}$   |
|  | $76 = \overline{74}$ | 0      | $245700_4 = \chi_{89,3}, \varphi_{89,3}$   |
|  | 77                   | 0      | $524160_1 = \chi_{90,0+}, \varphi_{90,0+}$ |
|  | 78                   | 0      | $524160_2 = \chi_{90,1+}, \varphi_{90,1+}$ |
|  | 79                   | 0      | $1048320_1 = \chi_{91+}, \varphi_{91+}$    |
|  | 80                   | 0      | $291200_1 = \chi_{96,0}, \varphi_{96,0}$   |

|  | blocks                 | defect | matrix                                       |
|--|------------------------|--------|--|
|  | 81                     | 0      | $291200_2 = \chi_{96,1}, \varphi_{96,1}$     |
|  | 82                     | 0      | $291200_3 = \chi_{96,2}, \varphi_{96,2}$     |
|  | $83 = \overline{81}$   | 0      | $291200_4 = \chi_{96,3}, \varphi_{96,3}$     |
|  | 84                     | 0      | $1164800_1 = \chi_{97+}, \varphi_{97+}$      |
|  | 85                     | 0      | $419328_1 = \chi_{105,0}, \varphi_{102,0}$   |
|  | 86                     | 0      | $419328_2 = \chi_{105,1}, \varphi_{102,1}$   |
|  | 87                     | 0      | $419328_3 = \chi_{105,2}, \varphi_{102,2}$   |
|  | $88 = \overline{86}$   | 0      | $419328_4 = \chi_{105,3}, \varphi_{102,3}$   |
|  | 89                     | 0      | $838656_1 = \chi_{106,0+}, \varphi_{103,0+}$ |
|  | 90                     | 0      | $838656_2 = \chi_{106,1+}, \varphi_{103,1+}$ |
|  | 91                     | 0      | $465920_3 = \chi_{108,0}, \varphi_{105,0}$   |
|  | 92                     | 0      | $465920_4 = \chi_{108,1}, \varphi_{105,1}$   |
|  | 93                     | 0      | $465920_5 = \chi_{108,2}, \varphi_{105,2}$   |
|  | $94 = \overline{92}$   | 0      | $465920_6 = \chi_{108,3}, \varphi_{105,3}$   |
|  | 95                     | 0      | $931840_2 = \chi_{109,0+}, \varphi_{106,0+}$ |
|  | 96                     | 0      | $931840_3 = \chi_{109,1+}, \varphi_{106,1+}$ |
|  | 97                     | 0      | $491400_1 = \chi_{111,0}, \varphi_{108,0}$   |
|  | 98                     | 0      | $491400_2 = \chi_{111,1}, \varphi_{108,1}$   |
|  | 99                     | 0      | $491400_3 = \chi_{111,2}, \varphi_{108,2}$   |
|  | $100 = \overline{98}$  | 0      | $491400_4 = \chi_{111,3}, \varphi_{108,3}$   |
|  | 101                    | 0      | $716800_1 = \chi_{113,0}, \varphi_{109,0}$   |
|  | 102                    | 0      | $716800_2 = \chi_{113,1}, \varphi_{109,1}$   |
|  | 103                    | 0      | $716800_3 = \chi_{113,2}, \varphi_{109,2}$   |
|  | $104 = \overline{102}$ | 0      | $716800_4 = \chi_{113,3}, \varphi_{109,3}$   |
|  | 105                    | 0      | $716800_5 = \chi_{114,0}, \varphi_{110,0}$   |
|  | 106                    | 0      | $716800_6 = \chi_{114,1}, \varphi_{110,1}$   |
|  | 107                    | 0      | $716800_7 = \chi_{114,2}, \varphi_{110,2}$   |
|  | $108 = \overline{106}$ | 0      | $716800_8 = \chi_{114,3}, \varphi_{110,3}$   |

| <b>Block 1:</b>           | $\varphi_{1,0}$ | $\varphi_{8,0}$ | $\varphi_{16,2}$ | $\varphi_{24,0}$ | $\varphi_{82,2}$ | $\varphi_{101,2}$ |                              |
|---------------------------|-----------------|-----------------|------------------|------------------|------------------|-------------------|------------------------------|
| $1_1 = \chi_{1,0}$        | 1               | .               | .                | .                | .                | .                 | $\varphi_{1,0} = 1_1$        |
| $300_1 = \chi_{8,0}$      | 1               | 1               | .                | .                | .                | .                 | $\varphi_{8,0} = 299_1$      |
| $2808_3 = \chi_{16,2}$    | .               | .               | 1                | .                | .                | .                 | $\varphi_{16,2} = 2808_3$    |
| $17550_1 = \chi_{24,0}$   | .               | 1               | .                | 1                | .                | .                 | $\varphi_{24,0} = 17251_1$   |
| $218700_3 = \chi_{82,2}$  | .               | .               | .                | 1                | 1                | .                 | $\varphi_{82,2} = 201449_3$  |
| $332800_3 = \chi_{101,2}$ | .               | .               | 1                | .                | .                | 1                 | $\varphi_{101,2} = 329992_3$ |
| $531441_3 = \chi_{112,2}$ | .               | .               | .                | .                | 1                | 1                 |                              |

| <b>Block 2:</b>           | $\varphi_{1,1}$ | $\varphi_{8,1}$ | $\varphi_{16,3}$ | $\varphi_{24,1}$ | $\varphi_{82,3}$ | $\varphi_{101,3}$ |
|---------------------------|-----------------|-----------------|------------------|------------------|------------------|-------------------|
| $1_2 = \chi_{1,1}$        | 1               | .               | .                | .                | .                | .                 |
| $300_2 = \chi_{8,1}$      | 1               | 1               | .                | .                | .                | .                 |
| $2808_4 = \chi_{16,3}$    | .               | .               | 1                | .                | .                | .                 |
| $17550_2 = \chi_{24,1}$   | .               | 1               | .                | 1                | .                | .                 |
| $218700_4 = \chi_{82,3}$  | .               | .               | .                | 1                | 1                | .                 |
| $332800_4 = \chi_{101,3}$ | .               | .               | 1                | .                | .                | 1                 |
| $531441_4 = \chi_{112,3}$ | .               | .               | .                | .                | 1                | 1                 |

$$\begin{aligned} \varphi_{1,1} &= 1_2 \\ \varphi_{8,1} &= 299_2 \\ \varphi_{16,3} &= 2808_4 \\ \varphi_{24,1} &= 17251_2 \\ \varphi_{82,3} &= 201449_4 \\ \varphi_{101,3} &= 329992_4 \end{aligned}$$

| <b>Block 3:</b>           | $\varphi_{1,2}$ | $\varphi_{8,2}$ | $\varphi_{16,0}$ | $\varphi_{24,2}$ | $\varphi_{82,0}$ | $\varphi_{101,0}$ |
|---------------------------|-----------------|-----------------|------------------|------------------|------------------|-------------------|
| $1_3 = \chi_{1,2}$        | 1               | .               | .                | .                | .                | .                 |
| $300_3 = \chi_{8,2}$      | 1               | 1               | .                | .                | .                | .                 |
| $2808_1 = \chi_{16,0}$    | .               | .               | 1                | .                | .                | .                 |
| $17550_3 = \chi_{24,2}$   | .               | 1               | .                | 1                | .                | .                 |
| $218700_1 = \chi_{82,0}$  | .               | .               | .                | 1                | 1                | .                 |
| $332800_1 = \chi_{101,0}$ | .               | .               | 1                | .                | .                | 1                 |
| $531441_1 = \chi_{112,0}$ | .               | .               | .                | .                | 1                | 1                 |

$$\begin{aligned} \varphi_{1,2} &= 1_3 \\ \varphi_{8,2} &= 299_3 \\ \varphi_{16,0} &= 2808_1 \\ \varphi_{24,2} &= 17251_3 \\ \varphi_{82,0} &= 201449_1 \\ \varphi_{101,0} &= 329992_1 \end{aligned}$$

| <b>Block 4:</b>           | $\varphi_{1,3}$ | $\varphi_{8,3}$ | $\varphi_{16,1}$ | $\varphi_{24,3}$ | $\varphi_{82,1}$ | $\varphi_{101,1}$ |
|---------------------------|-----------------|-----------------|------------------|------------------|------------------|-------------------|
| $1_4 = \chi_{1,3}$        | 1               | .               | .                | .                | .                | .                 |
| $300_4 = \chi_{8,3}$      | 1               | 1               | .                | .                | .                | .                 |
| $2808_2 = \chi_{16,1}$    | .               | .               | 1                | .                | .                | .                 |
| $17550_4 = \chi_{24,3}$   | .               | 1               | .                | 1                | .                | .                 |
| $218700_2 = \chi_{82,1}$  | .               | .               | .                | 1                | 1                | .                 |
| $332800_2 = \chi_{101,1}$ | .               | .               | 1                | .                | .                | 1                 |
| $531441_2 = \chi_{112,1}$ | .               | .               | .                | .                | 1                | 1                 |

$$\begin{aligned} \varphi_{1,3} &= 1_4 \\ \varphi_{8,3} &= 299_4 \\ \varphi_{16,1} &= 2808_2 \\ \varphi_{24,3} &= 17251_4 \\ \varphi_{82,1} &= 201449_2 \\ \varphi_{101,1} &= 329992_2 \end{aligned}$$

| <b>Block 5:</b>           | $\varphi_{2,0+}$ | $\varphi_{30,0+}$ | $\varphi_{72,0+}$ |
|---------------------------|------------------|-------------------|-------------------|
| $520_1 = \chi_{2,0+}$     | 1                | .                 | .                 |
| $46800_1 = \chi_{30,0+}$  | 1                | 1                 | .                 |
| $379080_2 = \chi_{76,1+}$ | .                | 1                 | 1                 |
| $332800_6 = \chi_{102,1}$ | .                | .                 | 1                 |
| $332800_8 = \chi_{102,3}$ | .                | .                 | 1                 |

$$\begin{aligned} \varphi_{2,0+} &= 520_1 \\ \varphi_{30,0+} &= 46280_1 \\ \varphi_{72,0+} &= 332800_1 \end{aligned}$$

| <b>Block 6:</b>           | $\varphi_{2,1+}$ | $\varphi_{30,1+}$ | $\varphi_{72,1+}$ |
|---------------------------|------------------|-------------------|-------------------|
| $520_2 = \chi_{2,1+}$     | 1                | .                 | .                 |
| $46800_2 = \chi_{30,1+}$  | 1                | 1                 | .                 |
| $379080_1 = \chi_{76,0+}$ | .                | 1                 | 1                 |
| $332800_5 = \chi_{102,0}$ | .                | .                 | 1                 |
| $332800_7 = \chi_{102,2}$ | .                | .                 | 1                 |

$$\begin{aligned} \varphi_{2,1+} &= 520_2 \\ \varphi_{30,1+} &= 46280_2 \\ \varphi_{72,1+} &= 332800_2 \end{aligned}$$

| <b>Block 7:</b>            | $\varphi_{3+}$ | $\varphi_{31+}$ | $\varphi_{73+}$ |
|----------------------------|----------------|-----------------|-----------------|
| $1040_1 = \chi_{3+}$       | 1              | .               | .               |
| $93600_1 = \chi_{31+}$     | 1              | 1               | .               |
| $758160_1 = \chi_{77+}$    | .              | 1               | 1               |
| $665600_1 = \chi_{103,0+}$ | .              | .               | 1               |
| $665600_2 = \chi_{103,1+}$ | .              | .               | 1               |

$$\begin{aligned} \varphi_{3+} &= 1040_1 \\ \varphi_{31+} &= 92560_1 \\ \varphi_{73+} &= 665600_1 \end{aligned}$$