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Actions, representations and various algebraic structures

Max Neunhöffer

University of St Andrews

12 November 2008

Actions and representations

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$$A(x, g) := R(g)(x)$$

Group algebras — definition

Let $\mathbb{F}$ be a field and G a finite group.

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$\mathbb{F}G :=$ vector space with basis G, multiplication inherited from G and distributive law:

$$\left(\sum_{g\in G}\lambda_g\cdot g\right)\cdot\left(\sum_{\tilde{g}\in G}\mu_{\tilde{g}}\cdot\tilde{g}\right)=\sum_{g,\tilde{g}\in G}\lambda_g\cdot\mu_{\tilde{g}}\cdot(g\tilde{g})$$

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$\mathbb{F}G := \{f : G \to \mathbb{F}\}$ with pointwise addition and convolution product:

$$(f \cdot h)(g) := \sum_{\tilde{g} \in G} f(g \cdot \tilde{g}^{-1}) \cdot h(\tilde{g})$$

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$\mathbb{F}G :=$ associative $\mathbb{F}$-algebra with generators G and relations $g \cdot \tilde{g} - (g\tilde{g}) = 0$ for $g, \tilde{g} \in G$.

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$\mathbb{F}$: field, G: group, $\mathbb{F}G$: group algebra, V: $\mathbb{F}$-vector space.

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(use finite presentation).
Given $\psi : \mathbb{F}G \to \mathrm{End}_{\mathbb{F}}(V)$, simply restrict $\varphi := \psi|_G$, since

$$\mathbf{1}_V = \psi(\mathbf{1}_G) = \psi(g \cdot g^{-1}) = \psi(g) \cdot \psi(g^{-1}) \qquad \text{for all } g \in G.$$

Modules

Definition (G-module or $\mathbb{F}G$-module)

An $\mathbb{F}$-vector space V together with

- a group homomorphism $\varphi : G \to \mathrm{GL}(V)$,
- or an algebra homomorphism $\psi : \mathbb{F}G \to \mathrm{End}_{\mathbb{F}}(V)$

is called a *G-module over $\mathbb{F}$* or an *$\mathbb{F}G$-module*.

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Definition (Faithful representation/action)

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Note: If a G-module V over $\mathbb{F}$ is faithful, it does not necessarily follow that the corresponding $\mathbb{F}G$-module V is faithful!

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$$\varphi(A(x, g)) = \tilde{A}(\varphi(x), g) \quad \text{for all } x \in X \text{ and all } g \in G.$$

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Equivalently, this means that this diagram commutes:

$$\begin{tikzcd} X \times G \arrow[r, "A"] \arrow[d, "\varphi \times \mathrm{id}_G"'] & X \arrow[d, "\varphi"] \\ \tilde{X} \times G \arrow[r, "\tilde{A}"'] & \tilde{X} \end{tikzcd}$$

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If φ has a G-equiv. inverse, it is called a G-isomorphism.

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Subacts

Let G act on X, i.e. $A : X \times G \to X$.

Definition (G-invariant subset, Subact)

A subset $Y \subseteq X$ is called G-invariant, if

$$y \cdot g \in Y \qquad \text{for all } y \in Y \text{ and all } g \in G.$$

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Definition (Irreducible/simple module)

An $\mathbb{F}G$-module M is called irreducible or simple, if it has no submodules except 0 and M itself.

Factor acts

Let G act on X, i.e. $A : X \times G \to X$.

Definition (G-invariant partition, factor act)

Let $X = \dot{\bigcup}_{i \in I} Y_i$ be partitioned such that

$$\forall\ i \in I \text{ and } g \in G, \text{ we have } Y_i \cdot g \subseteq Y_j \text{ for some } j \in I.$$

We say that the partition is G-invariant and get an action on the set of parts $Y := \{Y_i \mid i \in I\}$:

$$Y_i * g := Y_j \quad \text{if} \quad Y_i \cdot g \subseteq Y_j.$$

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Note: We usually want extra conditions to ensure that Y has the same algebraic structure as X and the new action is a homomorphism of such structures for all g.

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Extensions and direct sums

This is only about modules!

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Let

$$0 \longrightarrow W \xrightarrow{i} V \xrightarrow{\pi} U \cong V/W \longrightarrow 0$$

be a module V with a non-trivial submodule.

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This sequence may or may not be split:

$$\begin{tikzcd} 0 \arrow[r] & W \arrow[r, "i"] & V \arrow[r, "\pi"] & U \arrow[l, bend left, "r"] \arrow[r] & 0 \end{tikzcd},$$

i.e. there is $r : U \to W$ with $\pi \circ r = \mathrm{id}_U$.

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If and only if it is split, the module V is isomorphic to the direct sum

$$V \cong W \oplus U.$$

Indecomposability and semisimplicity

Definition (Indecomposable module)

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Lemma (Decomposable implies reducible)

A decomposable module is reducible.

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Ordinary representation theory of groups

For a finite group, the group algebra $\mathbb{C}G$ is semisimple.

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Problem (Classification of simple modules)

Classify the isomorphism types of simple $\mathbb{C}G$-modules,

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Lemma (Characters)

Two representations

$$R_1 : G \to \mathrm{GL}(V) \qquad \text{and} \qquad R_2 : G \to \mathrm{GL}(W)$$

afforded by two $\mathbb{C}G$-modules V and W are isomorphic, if and only if their characters $\chi_1 = \mathrm{Tr} \circ R_1$ and $\chi_2 = \mathrm{Tr} \circ R_2$ are equal.

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The two characters $\chi_i : G \to \mathbb{C}$ are class functions.

Research problems in ordinary rep. theory

Already done:

- Character tables of symmetric groups.

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Research problems in ordinary rep. theory

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- Character tables of alternating groups.

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- Character tables of alternating groups.
- The ATLAS (character tables of simple groups).

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- Some generic character tables.

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- Some generic character tables.

Still to do:

- Determine character tables for more groups.
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- Devise better algorithms to compute tables.

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Modular representation theory of groups

$\mathbb{F}$: field with $\mathrm{char}(\mathbb{F}) \mid |G|$, then $\mathbb{F}G$ is not semisimple.

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Modular representation theory of groups

$\mathbb{F}$: field with $\mathrm{char}(\mathbb{F}) \mid |G|$, then $\mathbb{F}G$ is not semisimple.

The modular rep. theory of groups strives to solve:

Problem (Classification of simple modules)

Classify the isomorphism types of simple $\mathbb{F}G$-modules,

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Lemma (Brauer characters)

Two irreducible representations $R_1 : G \to \mathrm{GL}(V)$ and $R_2 : G \to \mathrm{GL}(W)$ afforded by two $\mathbb{F}G$-modules V and W are isomorphic, if and only if their Brauer characters ψ_1 and ψ_2 are equal.

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The two Brauer characters ψ_i take values in $\mathbb{C}$!

Research problems in modular rep. theory

Already done:

- Brauer tables of some small symmetric groups ($n \leq 18$).

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- Classify indecomposable $\mathbb{F}G$-modules???

Permutation groups

Problem (Permutation group algorithms)

Given $G := \langle g_1, \ldots, g_k \in S_n \rangle \leq S_n$ *on a computer.*
Find efficient algorithms to compute with and in G*:*

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- $\ldots$

All of this is done and works well in nearly linear time:

runtime is bounded by $C \cdot n \cdot k \cdot \log^D(|G|)$.

Open questions for permutation groups

Still to do (in nearly linear time):

- Compute the centraliser $C_G(H)$ for some $H < S_n$.

Open questions for permutation groups

Still to do (in nearly linear time):

- Compute the centraliser $C_G(H)$ for some $H < S_n$.
- Compute the derived subgroup G'.

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Open questions for permutation groups

Still to do (in nearly linear time):

- Compute the centraliser $C_G(H)$ for some $H < S_n$.
- Compute the derived subgroup G'.
- Compute intersections of $G, H < S_n$.

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Still to do (in nearly linear time):

- Compute the centraliser $C_G(H)$ for some $H < S_n$.
- Compute the derived subgroup G'.
- Compute intersections of $G, H < S_n$.
- Compute conjugacy classes of permutation groups.

Open questions for permutation groups

Still to do (in nearly linear time):

- Compute the centraliser $C_G(H)$ for some $H < S_n$.
- Compute the derived subgroup G'.
- Compute intersections of $G, H < S_n$.
- Compute conjugacy classes of permutation groups.
- Test $G, H < S_n$ for conjugacy.

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Matrix and projective groups

Problem (Matrix group algorithms)

Given $G := \langle M_1, \ldots, M_k \in \mathrm{GL}(\mathbb{F}_q^n) \rangle \leq \mathrm{GL}(\mathbb{F}_q^n)$ *on a computer.*

Ultimate goal: Answer similar questions as for permutation groups.

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This is largely unsolved!

Matrix and projective groups

Problem (Matrix group algorithms)

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Problem (Projective group algorithms)

Given $G := \left\langle \bar{M}_1, \ldots, \bar{M}_k \in \mathrm{PGL}(n, q) \right\rangle \leq \mathrm{PGL}(n, q)$ on a computer.
Ultimate goal: Answer similar questions as for permutation groups.

Constructive recognition

Problem (Constructive recognition)

Let $\mathbb{F}_q$ be the field with q elements und

$$M_1, \dots, M_k \in \mathrm{GL}(\mathbb{F}_q^n).$$

Find for $G := \langle M_1, \dots, M_k \rangle$:

- *The group order $|G|$ and*
- *an algorithm that, given $M \in \mathrm{GL}(\mathbb{F}_q^n)$,*
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- *The runtime should be bounded from above by a polynomial in n, k and $\log q$.*

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- *A Monte Carlo Algorithmus is enough.*

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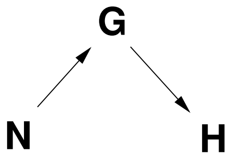
- *The runtime should be bounded from above by a polynomial in n, k and $\log q$.*
- *A Monte Carlo Algorithmus is enough. (Verification!)*

Recursion: composition trees

We get a tree:



Up arrows: inclusions

Down arrows: homomorphisms

Recursion: composition trees

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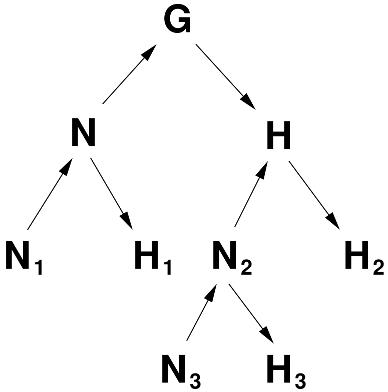

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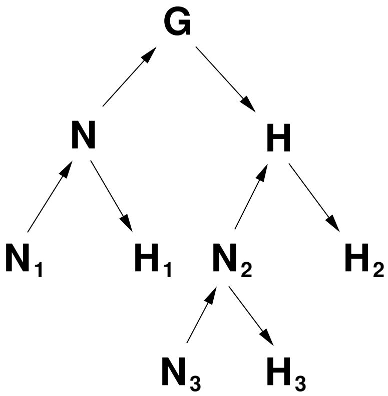

Up arrows: inclusions

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Old idea, improvements are still being made

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Enumerating large orbits

Orbit enumerations play an important role in

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To get a feeling:

- To enumerate an orbit of 1140000 vectors in $\mathbb{F}_2^{760}$ needs around 90 seconds.

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- To enumerate 95% of the same orbit with better tricks takes 1.1 seconds.

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To get a feeling:

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Finding better ways to enumerate orbits is a current research topic.