Is there a Sudoku puzzle with 16 clues?

Max Neunhöffer

University of St Andrews

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Sudoku Grids

7	9	3	6	8	4	5	1	2
4	8	6	5	1	2	9	3	7
1	2	5	9	7	3	8	4	6
9	3	2	7	5	1	6	8	4
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Rule

Each row, column and 3 $\times$ 3-block contains the numbers 1 to 9 each exactly once.

Sudoku Puzzles

							1	
4								
	2							
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		8				3		
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How difficult is a Sudoku puzzle?

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$\longrightarrow$ Depends on how much one has to try.

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A computer solves this in $\approx 28\mu s \approx 45000$ clock cycles!

Is there one with 16 clues?

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This puzzle has 17 clues.

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	2							
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This puzzle has 17 clues. None of them can be left out.

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This puzzle has 17 clues. None of them can be left out.

Question:

Are there 16 clues which uniquely define a Sudoku grid?

What is known?

- There are altogether

$$6\,670\,903\,752\,021\,072\,936\,960 \approx 6.671 \cdot 10^{21}$$

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- He reckons "that new 17-clue Sudoku puzzles are becoming rarer to find".
- There is a set of 16 clues which allows exactly two solutions.

Equivalence of Sudokus

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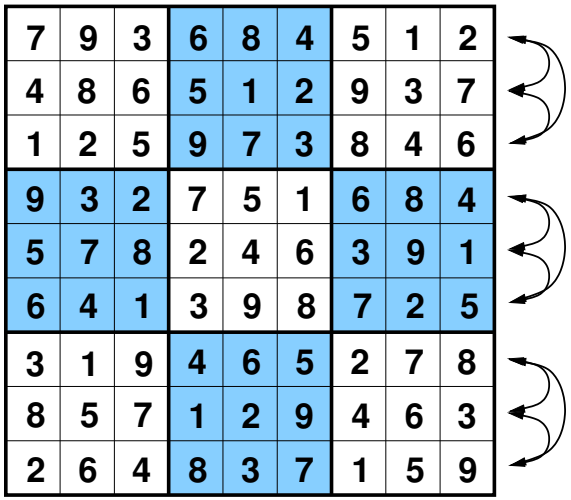
Equivalence transformations:

Equivalence of Sudokus



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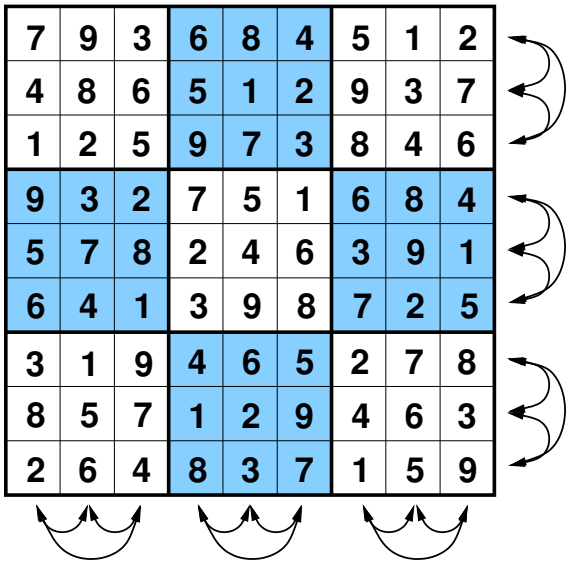
- Permute: rows in a block,

Equivalence of Sudokus



Equivalence transformations:

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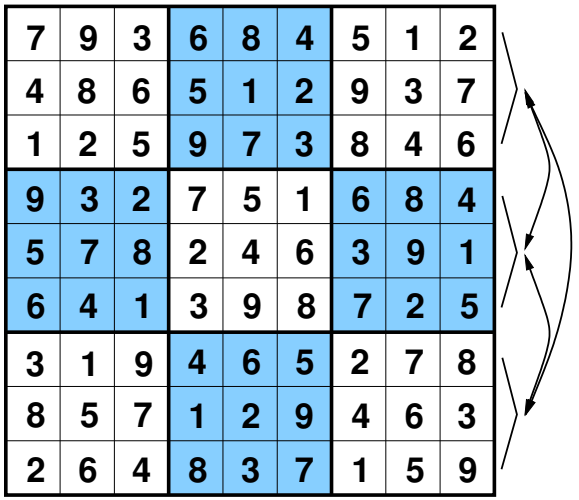
Equivalence of Sudokus



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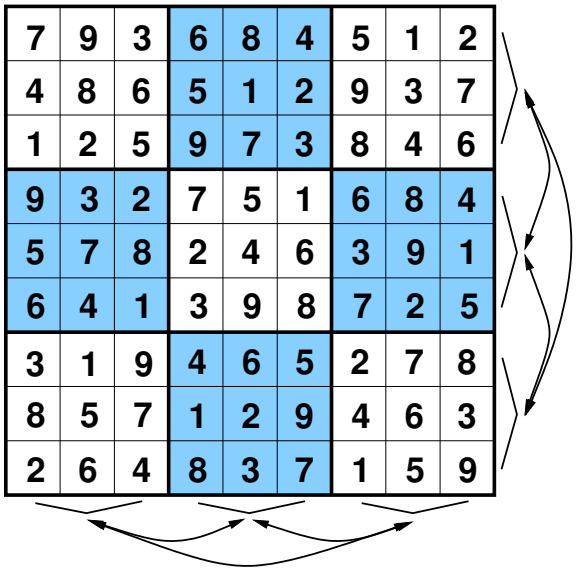

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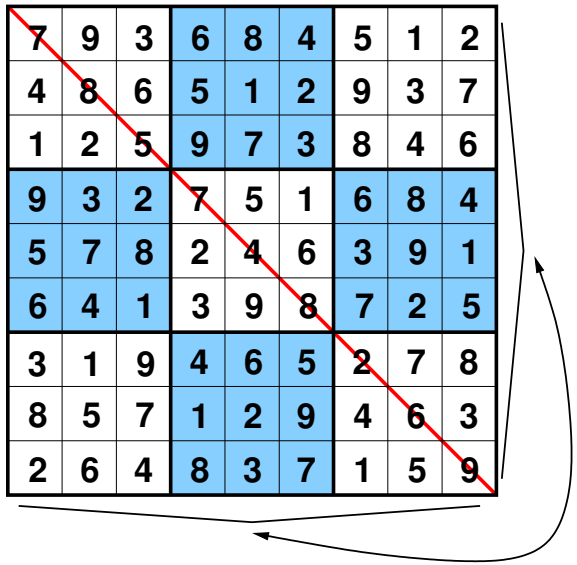

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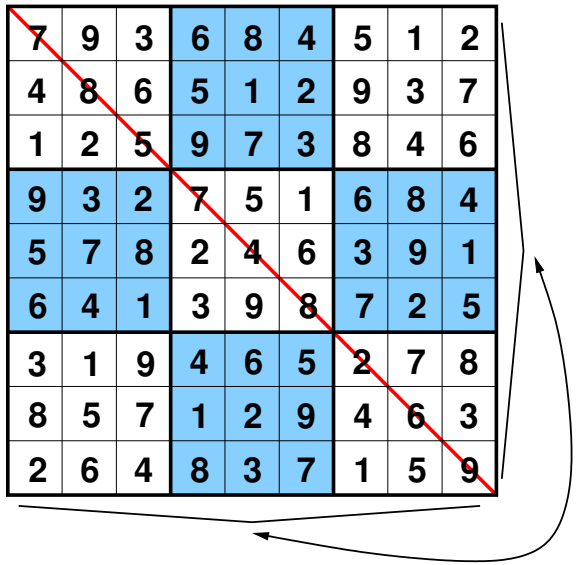

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$\longrightarrow$ All concatenations of these form a group.

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Two Sudoku grids/puzzles are called equivalent if one arises from the other by applying a sequence of equivalence transformations.

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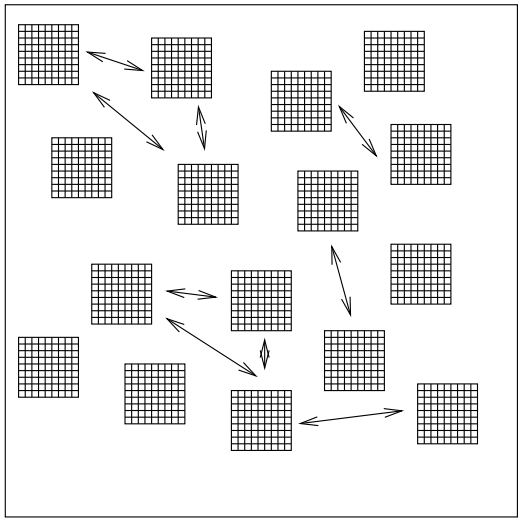

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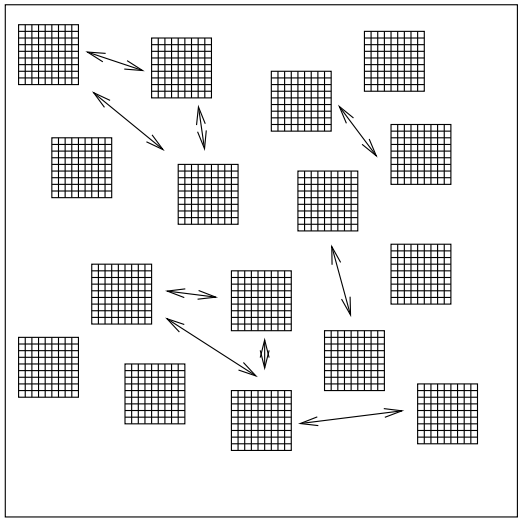

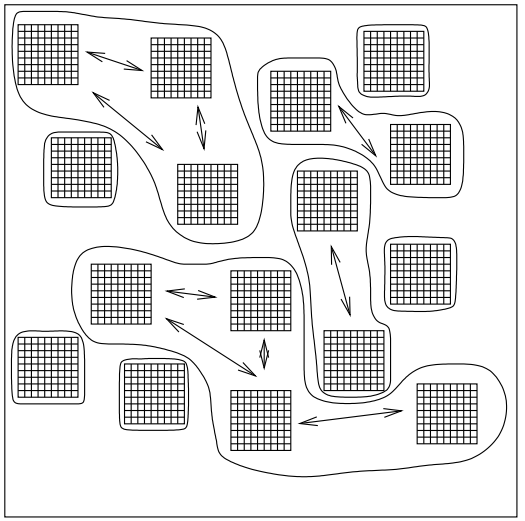

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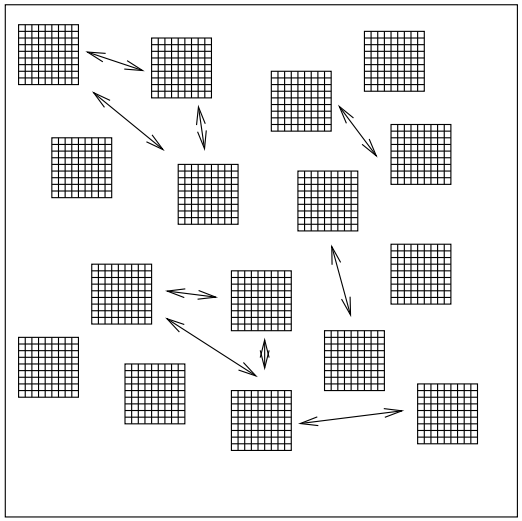

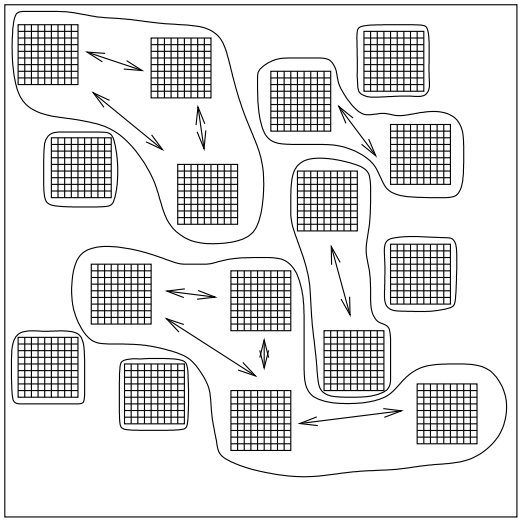

We form equivalence classes or orbits.

→ There are 5 472 730 538 classes (Russell/Jarvis 2006)

`http://www.afjarvis.staff.shef.ac.uk/sudoku/`

Symmetry Breaking

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1. We can renumber to get this left hand 3 × 3-block:

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(Type I)

1	2	3	{4,5,6}	{7,8,9}
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(Type II)

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(Type II)

1	2	3	{4,5,7}	{6,8,9}
4	5	6	{8,9,a}	{7,b,c}
7	8	9	{6,b,c}	{4,5,a}

where $\{a, b, c\} = \{1, 2, 3\}$.

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3. Some more such arguments . . .

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Question

Fix one Sudoku grid. Can it be the solution to a 16-clue Sudoku puzzle?

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Number of ways to choose 16 out of 81:

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Idea: We do not have to try all choices.

We need constraints that the selection of 16 has to fulfil.

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Definition: Unavoidable Set

Let S be a filled Sudoku grid. A subset U of the 81 positions is called an unavoidable set, if every set of clues uniquely defining S has a number in at least one of the positions in U.

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- How does this help?

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Any set of 16 clues cannot avoid the yellow positions.

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5	7	8	2	4	6		9	
6	4	1	3	9	8	7	2	5
3				6		2	7	8
8		7		2			6	
2	6		8	3	7		5	

Any set of 16 clues cannot avoid the yellow positions.

Unavoidable sets

7	9	3	6	8	4	5	1	2
4	8	6	5	1	2	9	3	7
1	2	5	9	7	3	8	4	6
9	3	2	7	5	1	6	8	4
5	7	8	2	4	6		9	
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Any set of 16 clues cannot avoid the yellow positions.

Because this puzzle has more than one solution.

The Hitting Set Problem

Problem: Hitting Set (resp. Set Covering)

Let M be a set and let $A_1, \ldots, A_k$ be subsets of M.
Find a minimal subset H of M which contains at least one element of every A_i for $1 \leq i \leq k$.

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We want to solve it to use lots of unavoidable sets to reduce the number of 16-clue sets we need to consider.

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Algorithm: Hitting Sets

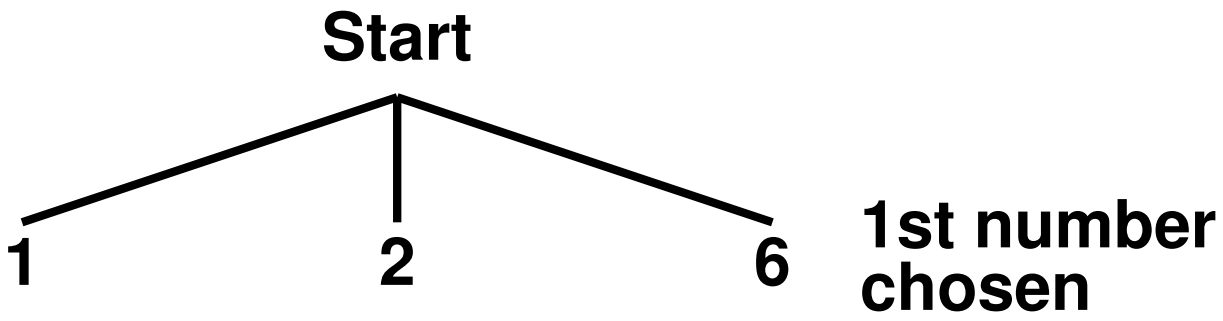
An example Hitting Set Problem

Let $M = \{1, 2, 3, \ldots, 12\}$ and

$$\begin{aligned}\mathcal{A} := \{ & \{1,2,6\},\\ & \{2,3,4,8\},\\ & \{1,7,8,9\},\\ & \{3,4,6,9\},\\ & \{4,6,10,12\},\\ & \{2,10,11,12\},\\ & \{5,7,8,9\},\\ & \{5,7,10,12\},\\ & \{1,3,4,5,11\} \quad \}\end{aligned}$$

Find a 3-subset of M intersecting all members of $\mathcal{A}$ non-trivially.

Backtrack Search in Action



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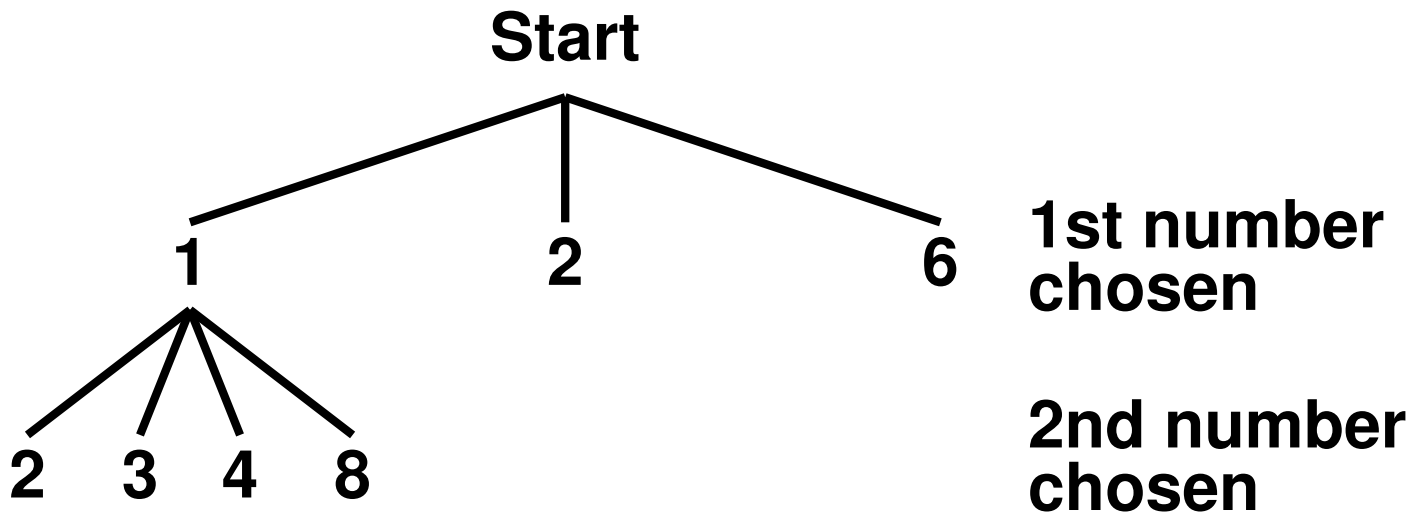

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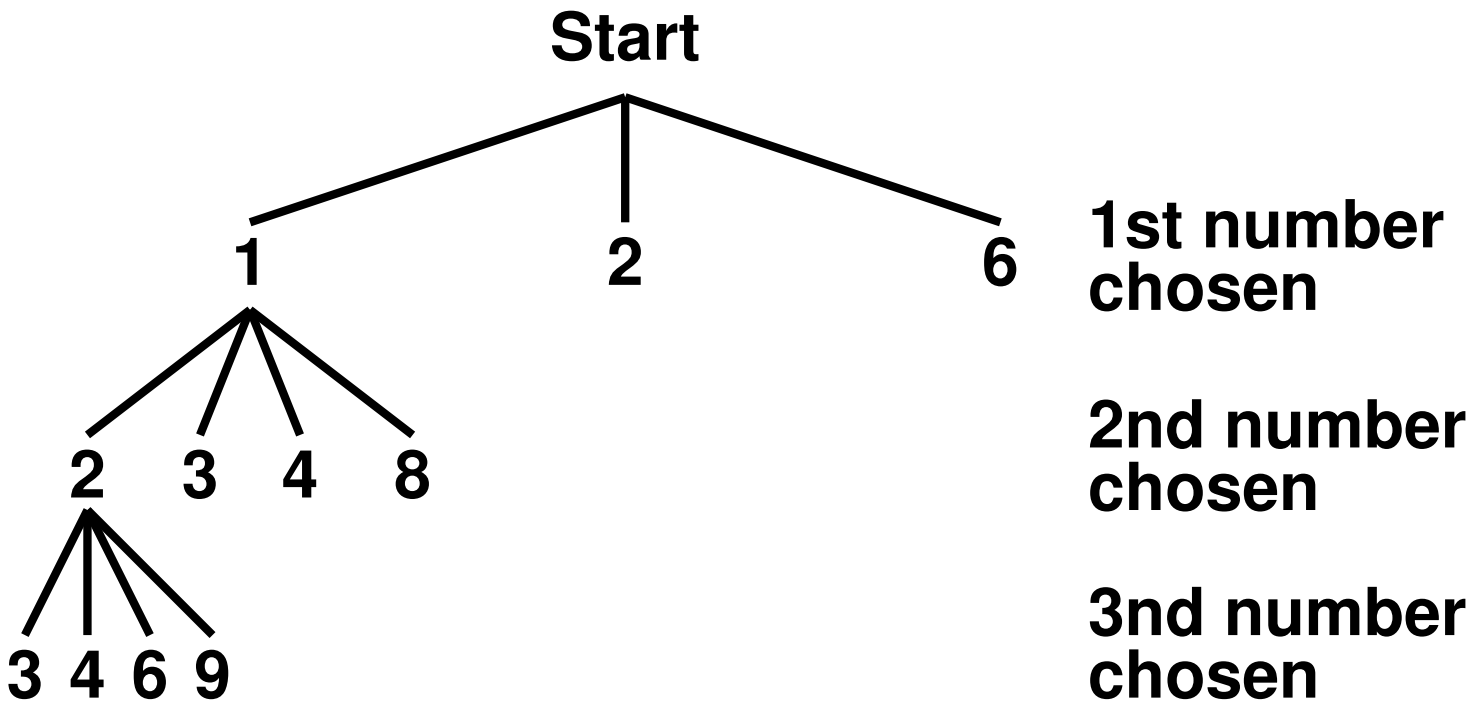

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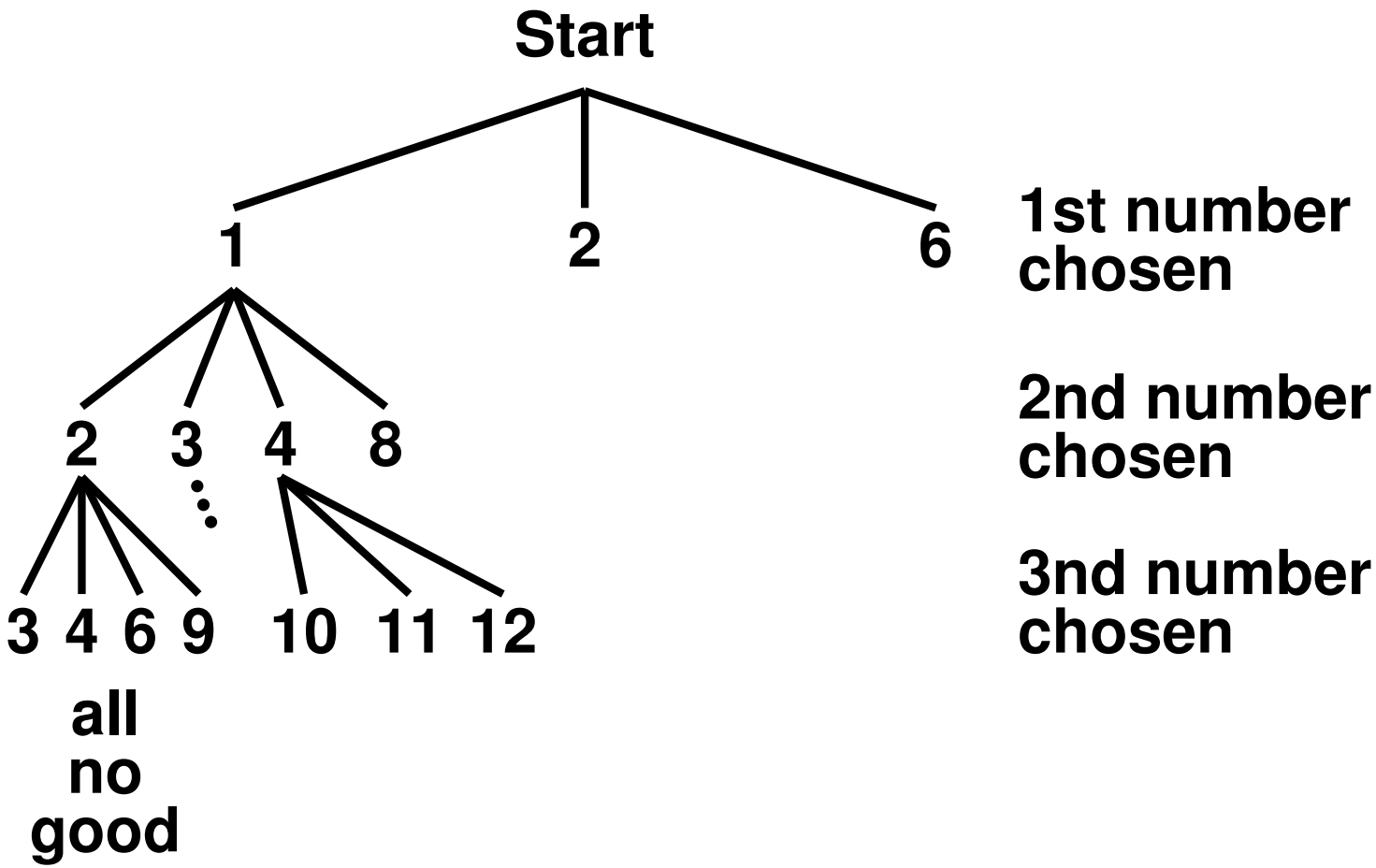

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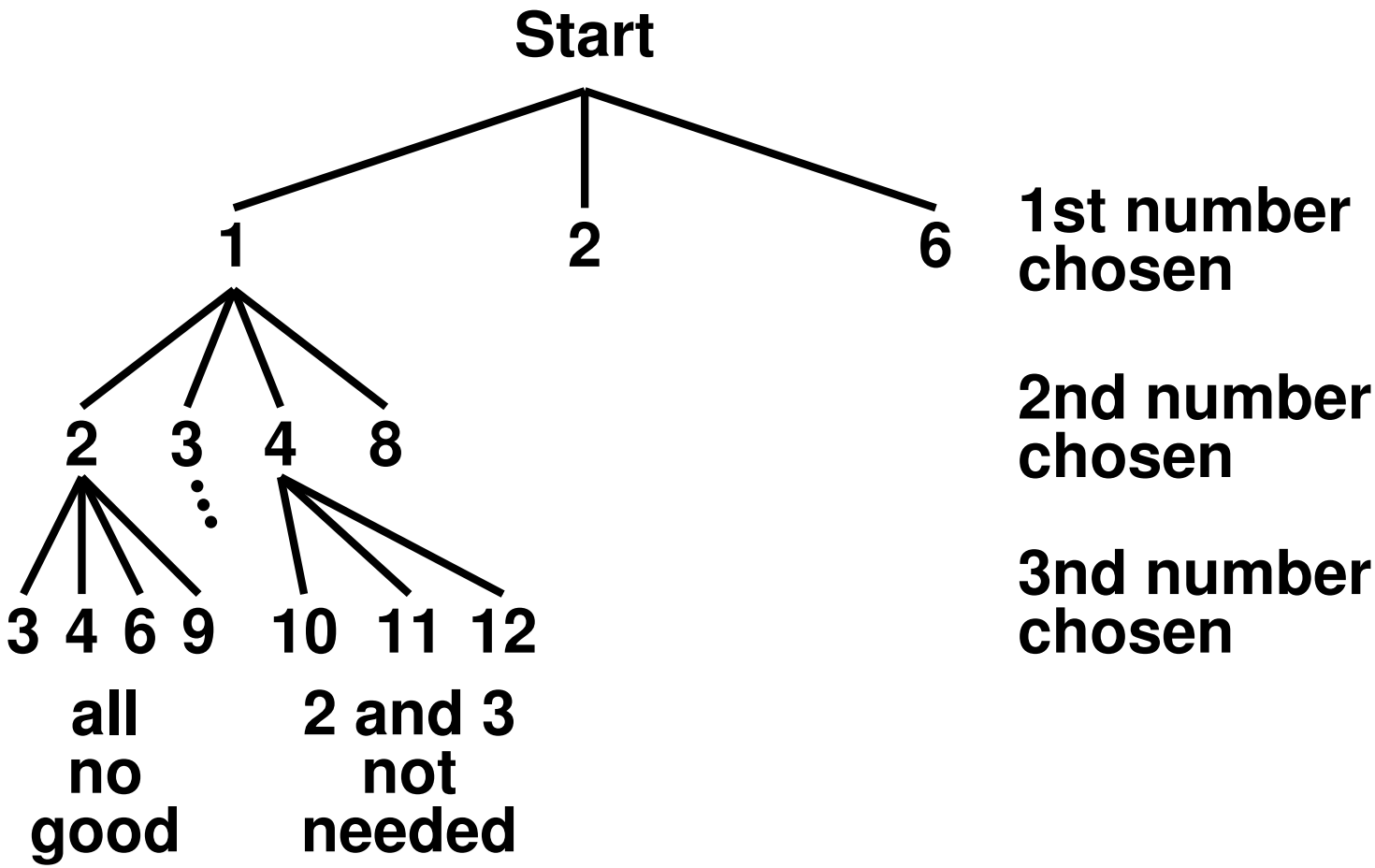

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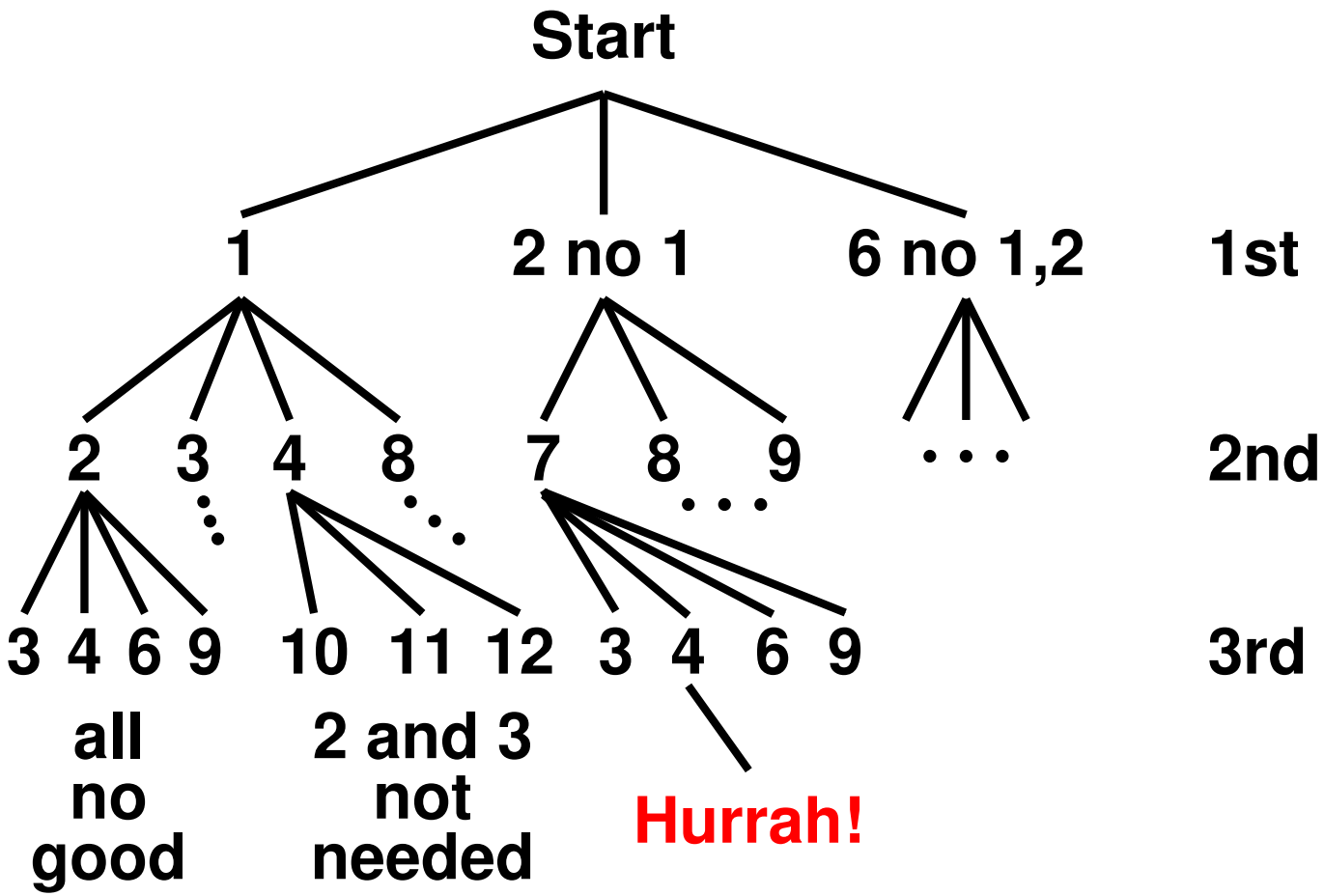

Unique solution: {2,4,7}

Proof that it works

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Proof: It works!

Let H be a solution. Then it intersects all A_i for $1 \leq i \leq k$.
It is found in exactly one leaf of the tree!

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Improvement: Whenever our candidate set C contains an already known unavoidable set U, we remove U from C.

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