Is there a Sudoku puzzle with 16 hints?

Max Neunhöffer

University of St Andrews

CIRCA meeting 28.1.2010

Sudokus

							1	
4								
	2							
				5		6		4
		8				3		
		1		9				
3			4			2		
	5		1					
			8		7			

Sudokus

7	9	3	6	8	4	5	1	2
4	8	6	5	1	2	9	3	7
1	2	5	9	7	3	8	4	6
9	3	2	7	5	1	6	8	4
5	7	8	2	4	6	3	9	1
6	4	1	3	9	8	7	2	5
3	1	9	4	6	5	2	7	8
8	5	7	1	2	9	4	6	3
2	6	4	8	3	7	1	5	9

Sudokus

7	9	3	6	8	4	5	1	2
4	8	6	5	1	2	9	3	7
1	2	5	9	7	3	8	4	6
9	3	2	7	5	1	6	8	4
5	7	8	2	4	6	3	9	1
6	4	1	3	9	8	7	2	5
3	1	9	4	6	5	2	7	8
8	5	7	1	2	9	4	6	3
2	6	4	8	3	7	1	5	9

Can we choose 16 to make a puzzle?

Sudokus

7				8			1	
1							4	
				4			9	
6						7		5
	1				5			8
8		7						
						1		

Can we choose 16 to make a puzzle?

A possible attack

$$\binom{81}{16} = 33,594,090,947,249,085 \qquad \text{(quadrillions)}$$

A possible attack

$$\binom{81}{16} = 33,594,090,947,249,085 \qquad \text{(quadrillions)}$$

$\Longrightarrow$ do not even think about trying all!

A possible attack

$$\binom{81}{16} = 33,594,090,947,249,085 \qquad \text{(quadrillions)}$$

$\Longrightarrow$ do not even think about trying all!

Idea: We do not have to try all choices.

A possible attack

$$\binom{81}{16} = 33,594,090,947,249,085 \qquad \text{(quadrillions)}$$

$\Longrightarrow$ do not even think about trying all!

Idea: We do not have to try all choices.

We need constraints that the selection of 16 has to fulfill.

Unavoidable sets

7	9	3	6	8	4	5	1	2
4	8	6	5	1	2	9	3	7
1	2	5	9	7	3	8	4	6
9	3	2	7	5	1	6	8	4
5	7	8	2	4	6	3	9	1
6	4	1	3	9	8	7	2	5
3	1	9	4	6	5	2	7	8
8	5	7	1	2	9	4	6	3
2	6	4	8	3	7	1	5	9

Unavoidable sets

7	9	3	6	8	4	5	1	2
4	8	6	5	1	2	9	3	7
1	2	5	9	7	3	8	4	6
9	3	2	7	5	1	6	8	4
5	7	8	2	4	6	3	9	1
6	4	1	3	9	8	7	2	5
3	1	9	4	6	5	2	7	8
8	5	7	1	2	9	4	6	3
2	6	4	8	3	7	1	5	9

Any set of 16 hints cannot avoid all of the yellow positions.

Unavoidable sets

7	9	3	6	8	4	5	1	2
4	8	6	5	1	2	9	3	7
1	2	5	9	7	3	8	4	6
9	3	2	7	5	1	6	8	4
5	7	8	2	4	6	3	9	1
6	4	1	3	9	8	7	2	5
3	1	9	4	6	5	2	7	8
8	5	7	1	2	9	4	6	3
2	6	4	8	3	7	1	5	9

Any set of 16 hints cannot avoid all of the yellow positions.

Unavoidable sets

7	9	3	6	8	4	5	1	2
4	8	6	5	1	2	9	3	7
1	2	5	9	7	3	8	4	6
9	3	2	7	5	1	6	8	4
5	7	8	2	4	6	3	9	1
6	4	1	3	9	8	7	2	5
3	1	9	4	6	5	2	7	8
8	5	7	1	2	9	4	6	3
2	6	4	8	3	7	1	5	9

Any set of 16 hints cannot avoid all of the yellow positions.

Unavoidable sets

7	9	3	6	8	4	5	1	2
4	8	6	5	1	2	9	3	7
1	2	5	9	7	3	8	4	6
9	3	2	7	5	1	6	8	4
5	7	8	2	4	6		9	
6	4	1	3	9	8	7	2	5
3				6		2	7	8
8		7		2			6	
2	6		8	3	7		5	

Any set of 16 hints cannot avoid all of the yellow positions.

Unavoidable sets

7	9	3	6	8	4	5	1	2
4	8	6	5	1	2	9	3	7
1	2	5	9	7	3	8	4	6
9	3	2	7	5	1	6	8	4
5	7	8	2	4	6		9	
6	4	1	3	9	8	7	2	5
3				6		2	7	8
8		7		2			6	
2	6		8	3	7		5	

Any set of 16 hints cannot avoid all of the yellow positions.

Because this Sudoku problem has more than one solution.

The Plan

- Find lots of unavoidable sets.

The Plan

- Find lots of unavoidable sets.
- Solve the constraint satisfaction problem to find all subsets of 16 positions intersecting all unavoidable sets.

The Plan

- Find lots of unavoidable sets.
- Solve the constraint satisfaction problem to find all subsets of 16 positions intersecting all unavoidable sets.
- For each solution run a Sudoku solver and find another as the known solution.

The Plan

- Find lots of unavoidable sets.
- Solve the constraint satisfaction problem to find all subsets of 16 positions intersecting all unavoidable sets.
- For each solution run a Sudoku solver and find another as the known solution.
- Repeat the same for all other $5,472,730,538$ (billions) essentially different filled Sudoku grids.

The Plan

- Find lots of unavoidable sets.
- Solve the constraint satisfaction problem to find all subsets of 16 positions intersecting all unavoidable sets.
- For each solution run a Sudoku solver and find another as the known solution.
- Repeat the same for all other $5,472,730,538$ (billions) essentially different filled Sudoku grids.

Problems:

- Have a program to find 513 unavoidable sets in < 0.1s.

The Plan

- Find lots of unavoidable sets.
- Solve the constraint satisfaction problem to find all subsets of 16 positions intersecting all unavoidable sets.
- For each solution run a Sudoku solver and find another as the known solution.
- Repeat the same for all other $5,472,730,538$ (billions) essentially different filled Sudoku grids.

Problems:

- Have a program to find 513 unavoidable sets in < 0.1s.
- Find all 767 solutions for 16-subsets in 21min.

The Plan

- Find lots of unavoidable sets.
- Solve the constraint satisfaction problem to find all subsets of 16 positions intersecting all unavoidable sets.
- For each solution run a Sudoku solver and find another as the known solution.
- Repeat the same for all other $5,472,730,538$ (billions) essentially different filled Sudoku grids.

Problems:

- Have a program to find 513 unavoidable sets in < 0.1s.
- Find all 767 solutions for 16-subsets in 21min.
- Have a Sudoku solver which solves a Sudoku in $\approx 28\mu$s or ≈ 45000 clock cycles.

The Plan

- Find lots of unavoidable sets.
- Solve the constraint satisfaction problem to find all subsets of 16 positions intersecting all unavoidable sets.
- For each solution run a Sudoku solver and find another as the known solution.
- Repeat the same for all other $5,472,730,538$ (billions) essentially different filled Sudoku grids.

Problems:

- Have a program to find 513 unavoidable sets in < 0.1s.
- Find all 767 solutions for 16-subsets in 21min.
- Have a Sudoku solver which solves a Sudoku in $\approx 28\mu$s or ≈ 45000 clock cycles.
- This needs an estimated amount of

  $6.9 \cdot 10^{12}$ CPU seconds (218659 million years)!