UNIVERSITY OF ST ANDREWS
MT5826 Finite Fields
Tutorial Sheet: Chapter 3

1. Let F be a field of eight elements. Does F possess a subfield isomorphic to a field of four elements? If yes, exhibit such a subfield; if no, explain why not.

2. Determine the subfields of the finite field $\mathbb{F}_{3^{42}}$ and draw the corresponding subfield diagram.

3. (a) Express $\mathbb{F}_9$ as $\mathbb{F}_3(\theta)$ for an appropriate θ. List the elements of $\mathbb{F}_9$.

(b) How many primitive elements does $\mathbb{F}_9$ have?

(c) Is the θ you chose a primitive element?

4. Let F be a field. Prove that if

$$a_0 + a_1x + \cdots + a_nx^n \in F[x]$$

is irreducible, then so is

$$a_n + a_{n-1}x + \cdots + a_0x^n.$$

5. Let μ be the Moebius function. Prove that $\mu(mn) = \mu(m)\mu(n)$ if $\gcd(m, n) = 1$.

6. Prove the reverse implication of the Moebius Inversion Formula (i.e. the direction omitted in the proof given in the notes).

7. How many monic irreducible polynomials in $\mathbb{F}_q[x]$ are there of degree (a)18? (b)20?

8. What is the product of all monic irreducible

(a) quartics in $\mathbb{F}_3[x]$;

(b) polynomials of degree 6 in $\mathbb{F}_2[x]$?