UNIVERSITY OF ST ANDREWS
MT5827 Lie Algebras
Tutorial Sheet: Chapters 1 and 2

1. Show that the following two subspaces of $\mathbb{C}^{1\times 3}$ are equal:

- $\text{Span}([1,0,-1],[0,2,1],[1,2,0])$ and
- $\left\{[x,y,z] \in \mathbb{C}^{1\times 3} \middle| z = y/2 - x\right\}$.

Determine their dimension.

2. Show that $\mathbb{C}^{1\times 3}$ has the following direct sum decomposition:

$$\mathbb{C}^{1\times 3} = \text{Span}([1,2,3]) \oplus \left\{[x,y,z] \in \mathbb{C}^{1\times 3} \mid z = x - y\right\}.$$

3. Let $V := \mathbb{C}^{1\times 3}$ and $W := \text{Span}([1,1,1])$. Find a complement of W in V, that is, a subspace U of V such that $V = W \oplus U$.

4. Let $L := \mathbb{R}^{1\times 3}$ be the 3-dimensional real row space with the following product:

$$[[a,b,c],[x,y,z]] := [a,b,c] \times [x,y,z] := [bz - cy, cx - az, ay - bx].$$

Show that this product fulfills the Jacobi identity.

5. Let L be a Lie algebra over a field $\mathbb{F}$ and H a subspace of L (not necessarily a subalgebra). Use the Jacobi identity to show that both the normaliser $N_L(H)$ and the centraliser $C_L(H)$ are Lie subalgebras of L.

6. Let $L = \mathbb{C}^{n\times n}$ with $n \geq 2$. Show that the subspace K of skew-symmetric matrices, i.e. $\left\{A \in \mathbb{C}^{n\times n} \mid A^t = -A\right\}$ where A^t is the transposed matrix of A, is not an ideal.

7. Let L be any Lie algebra over a field $\mathbb{F}$. Show that the members of the lower central series

$$L = L^0 \supseteq L^1 = [L,L] \supseteq L^2 \supseteq \cdots$$

are in fact ideals in L.

8. Let L be any Lie algebra over a field $\mathbb{F}$. Show that the members of the derived series

$$L = L^{(0)} \supseteq L^{(1)} = [L,L] \supseteq L^{(2)} \supseteq \cdots$$

are in fact ideals in L.