UNIVERSITY OF ST ANDREWS
MT5827 Lie Algebras
Tutorial Sheet 2: Chapter 2

1. Assume that L_1 and L_2 are Lie algebras over $\mathbb{C}$. Form $L := L_1 \oplus L_2$, the vector space direct sum. Show that L is a Lie algebra with the Lie product

$$[(x, y), (u, v)] := ([x, u], [y, v]),$$

where we denote elements of L as pairs (x, y) with $x \in L_1$ and $y \in L_2$.

2. Show that if L is a soluble (nilpotent, respectively) Lie algebra and $\varphi : L \to H$ is a Lie algebra homomorphism, then the image $L\varphi$ is soluble (nilpotent, respectively) as well.

3. Let L be a Lie algebra of dimension n and suppose $Z := Z(L)$, the centre of L, has dimension at least $n - 1$. Prove that L is abelian.

4. Let L be a 3-dimensional Lie algebra over $\mathbb{C}$ with basis (x, y, z) such that

$$[x, y] = 0, \quad [x, z] = x \quad \text{and} \quad [y, z] = y.$$

Find bases for L^1 and L^2. What is the centre of L?

5. Compute the radical $\mathrm{rad}(L)$ for $L = \mathrm{Lie}(\mathbb{C}^{2\times 2})$.

6. Let L be a Lie algebra over $\mathbb{C}$ with no no-zero abelian ideals. Prove that L has no non-zero soluble ideals.

7. Assume $L_1, \ldots, L_k$ are simple Lie algebras over $\mathbb{C}$. We form their direct sum $L := L_1 \oplus \cdots \oplus L_k$ as in Exercise 1. Show that L is semisimple and that all L_i are minimal ideals in L.