Exercise 3.6.12

Verify that the following GAP-function

```
HasOuterAutomorphismsLeavingClassesInvariant:= function( G )
    local aut, inn, tr, reps, good;

    if IsAbelian( G ) then
      return false;
    else

    aut:= AutomorphismGroup( G );
    inn:= InnerAutomorphismsAutomorphismGroup( aut );
    tr:= RightTransversal( aut, inn );
    reps:= List( ConjugacyClasses( G ), Representative );

    good:= First( tr, x -> ( not x in inn ) and
                           ForAll( reps, g -> IsConjugate( G, g, g^x ) ) );
    if good = fail then
      return false;
    else
      return true;
    fi;

    fi;
end;
```

tests whether or not a given group has a non-trivial outer automorphism leaving all conjugacy classes invariant.
Show that the smallest groups with this property have order 32.

This exercise is due to Thomas Breuer.