Exercise 4.7.9

Comparing Theorem 4.7.7 with Theorem 4.4.8 it might perhaps be tempting to conjecture that the somewhat complicated condition ((4.28) for all $g \in G$) in Theorem 4.7.7 can be replaced by the simpler one

$$\omega_{\theta^G}((g^G)^+) - \omega_\chi((g^G)^+) \in pR \qquad \text{for all} \quad g \in G_{p'} \tag{1}$$

or perhaps by the stronger one

$$\omega_{\theta^G}((g^G)^+) - \omega_\chi((g^G)^+) \in pR \qquad \text{for all} \quad g \in G. \tag{2}$$

(a) Show that the condition (4.28) for all $g \in G$ implies (1).
Hint: Look at the proof of Theorem 4.4.8.

(b) Give an example for groups $H \leq G$, characters $\theta \in \mathrm{Irr}(b), b \in \mathrm{Bl}_p(H)$, $\chi \in \mathrm{Irr}(G)$, satisfying (1), such that b^G is not defined.

(c) Give an example for groups $H \leq G$, characters $\theta \in \mathrm{Irr}(b), b \in \mathrm{Bl}_p(H)$, $\chi \in \mathrm{Irr}(B), B \in \mathrm{Bl}_p(G)$, such that $b^G = B$ is defined, but (2) is violated.
Hint: For $p = 2$ the smallest examples have order 40.