Solution to Exercise 1.7.1

Clearly

$$A_{a,c,b,d} := \begin{bmatrix} a & b & 0 & 0 \\ 0 & c & 0 & 0 \\ 0 & 0 & c & d \\ 0 & 0 & 0 & a \end{bmatrix} \in A$$

is an idempotent if and only if $\{a, c\} = \{0, 1\}$ or $A_{a,c,b,d} = 1_A = \mathbf{I}_4$. Hence $e_1 := A_{1,0,0,0}$ and $e_2 := A_{0,1,0,0}$ are primitive idempotents and $A = A\,e_1 \oplus A\,e_2$. Obviously $V_1 := \{A_{a,0,0,0} \mid a \in K\}$ is a simple submodule of $A\,e_1$ (yielding the representation $A_{a,c,b,d} \mapsto a$) and $V_2 := \{A_{0,c,0,0} \mid c \in K\}$ is a simple submodule of $A\,e_2$ (yielding the representation $A_{a,c,b,d} \mapsto c$) and

$$A\,e_1/V_1 \cong_A V_2 \qquad \text{and} \qquad A\,e_2/V_2 \cong_A V_1.$$

From Lemma 1.3.2 (a) we conclude, that V_1 and V_2 are up to isomorphism all the simple A-modules. Also $A\,e_1$ is a projective cover of V_2 and $A\,e_2$ is a projective cover of V_1 and the claims follow.

Observe that $e_i A e_i = K \cdot e_i \cong K$ as a ring for $i = 1, 2$, so that A is semi-perfect. Theorem 1.6.27 (a) shows that A is not a symmetric algebract. Theorem 1.6.27 (a) shows that A is not a symmetric algebra.