Solution to Exercise 4.11.1

The ordinary character table of $G := M_{11}$ is given as follows:

```
gap> m11:=CharacterTable("M11");
CharacterTable( "M11" )
gap> Display(m11);
M11

      2  4  4  1  3  .  1  3  3   .   .
      3  2  1  2  .  .  1  .  .   .   .
      5  1  .  .  .  1  .  .  .   .   .
     11  1  .  .  .  .  .  .  .   1   1

        1a 2a 3a 4a 5a 6a 8a 8b 11a 11b
     2P 1a 1a 3a 2a 5a 3a 4a 4a 11b 11a
     3P 1a 2a 1a 4a 5a 2a 8a 8b 11a 11b
     5P 1a 2a 3a 4a 1a 6a 8b 8a 11a 11b
    11P 1a 2a 3a 4a 5a 6a 8a 8b  1a  1a

X.1      1  1  1  1  1  1  1  1   1   1
X.2     10  2  1  2  . -1  .  .  -1  -1
X.3     10 -2  1  .  .  1  A -A  -1  -1
X.4     10 -2  1  .  .  1 -A  A  -1  -1
X.5     11  3  2 -1  1  . -1 -1   .   .
X.6     16  . -2  .  1  .  .  .   B  /B
X.7     16  . -2  .  1  .  .  .  /B   B
X.8     44  4 -1  . -1  1  .  .   .   .
X.9     45 -3  .  1  .  . -1 -1   1   1
X.10    55 -1  1 -1  . -1  1  1   .   .

A = E(8)+E(8)^3
  = ER(-2) = i2
B = E(11)+E(11)^3+E(11)^4+E(11)^5+E(11)^9
  = (-1+ER(-11))/2 = b11
```

The element u has to lie in the conjugacy class `3a` and we can read off from the character table that $\mathbf{C}_G(u) = \mathbf{C}_G(P_3)$ and $|\,\mathbf{C}_G(P_3)| = 2^1 \cdot 3^2$. So there is an element v order 2 centralizing u. Moreover, v has to lie in the conjugacy class `2a` and uv, which is an element of order 6, is in `6a`. All irreducible characters of G but χ_9, which is a defect zero character, are in the principal 3-block of G as the following GAP code shows:

```
gap> PrimeBlocks(m11,3);
rec( block := [ 1, 1, 1, 1, 1, 1, 1, 1, 2, 1 ],
  defect := [ 2, 0 ],
```

```
  height := [ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0 ],
  relevant := [ 2, 4, 5, 7, 9 ],
  centralcharacter := [ [ , 165,, 990, 1584,, 990,, 720 ],
      [ , -11,, 22, 0,, -22,, 16 ] ] )
```

It is now easy to check that $-1 = \chi_2(\texttt{6a}) \neq \chi_2(\texttt{3a}) = 1$. and therefore $\chi_2(uv) \neq \chi_2(u)$.