Solution to Exercise 4.12.3

Solution to part b). This is easily done using the decomposition matrix given in part a) and observing the two different ways in which a decomposition matrix can be interpreted. Since the decomposition matrix D_B is a 3 by 2 matrix, it means that we have three irreducible ordinary characters in the block say χ, χ', χ''. Moreover, there are two irreducible Brauer characters, say φ_1, φ_2 in the block. The decomposition matrix D_B as given in a) tells us that χ restricted to the 3-regular classes is φ_1 and χ' restricted to the 3-regular classes is φ_2. The third row of D_B gives that χ'' is the sum of φ_1 and φ_2 on the 3-regular classes and hence $\chi''(g) = \chi(g) + \chi'(g)$ on the 3-regular classes of G. We now use the second interpretation of D_B: the columns of D_B give the decomposition of the projective indecomposable characters Φ_{φ_1} and Φ_{φ_2} as sums of the ordinary irreducible characters in the block that means $\Phi_{\varphi_1} = \chi + \chi''$ and $\Phi_{\varphi_2} = \chi' + \chi''$. Since projective characters are zero on the 3-singular classes this shows that $\chi''(g) = -\chi(g) = -\chi'(g)$ for all 3-singular elements g in G.