Solution to Exercise 4.7.8

First observe that $C := \mathbf{C}_G(P) = \mathbf{C}_G(P)P \cong P \times D_{10}$. We denote the irreducible characters of P by $\lambda_1 = 1_P, \lambda_2, \lambda_3$ and the irreducible characters of D_{10} by $\rho_1, \ldots, \rho_4$. It follows that there are four 3-blocks b_1, b_2, b_3, b_4 of the direct product $P \times D_{10}$ all with defect group P, see Theorem 4.6.12. The block b_j consists of the ordinary irreducible characters $\lambda_i \cdot \rho_j$ for $i = 1, \ldots, 4$ and their canonical characters are $\lambda_1\rho_i$ for $i = 1, \ldots, 4$. By Clifford theory applied to C and $N := N_G P)$ the characters $\lambda_1\rho_j$ extend in two different ways giving $(\lambda_1\rho_j)^+$ and $(\lambda_1\rho_j)^-$. Moreover the characters $\lambda_2\rho_j$ fuse with $\lambda_3\rho_j$ that means $(\lambda_2\rho_j)^N = (\lambda_2\rho_j)^N$ is irreducible. This implies that there are four blocks B_1, B_2, B_3, B_4 for N where B_j consists of the characters $(\lambda_1\rho_j)^+, (\lambda_1\rho_j)^-, (\lambda_2\rho_j)^N$ and the canonical character corresponding to B_j is $\lambda_1\rho_j$ lying in b_j.