Conference abstracts

First annual meeting of the DFG collaborative research center SFB-TRR 195

Reimer Behrends, Thomas Breuer, Sebastian Gutsche and Bill Hart - OSCAR: Open Source Computer Algebra Resource

OSCAR is the visionary, new computer algebra system which is currently developed as part of the TRR. We will give an update on the integration work that is being done to combine the four "cornerstone" systems: GAP, Singular, Antic and polymake.

We will give a demonstration of Singular.jl, which forms a link between Antic and Singular, and demonstrate the mathematical capabilities of interfacing Singular to Antic.

We will also report on the state of integrating GAP with Julia/Antic/Singular, and discuss benefits and challenges. Finally, we will lay out our roadmap for ongoing and future development.

As OSCAR is developed for the TRR, descriptions of problems that can be solved with OSCAR are needed and welcome. Examples of use cases for OSCAR will help us to adjust the system to the needs of people in the TRR.

Janko Böhm - Massively parallel methods with applications to determining smoothness of algebraic varieties

Many constructions in algebraic geometry rely on checking smoothness of the constructed variety. Ihe Jacobian criterion is often not efficient for this task since it aims at computing the structure of the singular locus. Based on an idea in Hironaka's proof of resolution of singularities, we have developed a new algorithm for determining smoothness of algebraic varieties. The algorithm is inherently parallel and does not involve the calculation of codimension-sized minors of the Jacobian matrix. We also describe a hybrid method which combines the new method with the Jacobian criterion, thus making use of the strengths of both approaches.

Using the computer algebra system Singular in conjunction with the GPI-Space framework developed at ITWM Kaiserslautern, we provide the necessary infrastructure for massively parallel computations in algebraic geometry. As a test case, we have modeled the smoothness test algorithm using this infrastructure. We discuss the performance, and the potential of this approach in further applications.

This is joint work with Wolfram Decker, Anne Frühbis-Krüger, Franz-Josef Pfreundt, Mirko Rahn and Lukas Ristau.

Marvin Hahn - Piecewise polynomiality properties of Hurwitz-type counts

Hurwitz numbers count branched genus g, degree d coverings of the Riemann sphere with fixed ramification data. These enumerative objects are deeply connected to Gromov-Witten theory. In recent years several related notions of Hurwitz-type counts and combinatorial interpolations between them appeared in the literature and the uncovering the connections between these counts and Gromov Witten theory - similar to the classical case - has become an active field of research. One of the crucial necessary proporties for such connections to exist is a (piecewise) polynomial behaviour of the Hurwitz-type counts in the initial ramification data. This talk is centered around the polynomial structure and wall-crossing behaviour of interpolations between double, monotone double and Grothendieck dessin d'enfants double Hurwitz numbers. Parts of this talk are based on joint work with Reinier Kramer and Danilo Lewanski.

Lars Kastner - Parallel Enumeration of Triangulations

We report on the implementation of an algorithm for computing the set of all regular triangulations of finitely many points in Euclidean space. This algorithm, which we call down-flip reverse search, can be restricted, e.g., to computing full triangulations only; this case is particularly relevant for tropical geometry. Most importantly, down-flip reverse search allows for massive parallelization, i.e., it scales well even for many cores. Our implementation allows to compute the triangulations of much larger point sets than before. This is joint work with Charles Jordan and Michael Joswig.

Viktor Levandovskyy - Factorization of noncommutative polynomials with applications

I will talk on the research direction, pursued together with A. Heinle, M. Giesbrecht and J. Bell (University of Waterloo, Canada), which resulted in a series of recent papers.

As in the classical commutative case, we are interested in factorizing polynomials over non-commutative rings. Let us start with a field K and a finitely presented associative K-algebra A, which is a domain.

It turns out, that there are at least two distinct notions of a factorization of polynomials over A. One of them originates from the ring theory (N. Jacobson, P. M. Cohn) and uses a weak notion of association relation (called left or right similarity), what is at the same time hard to approach algorithmically. On the contrary, in applications we'd like to use the classical association relation, i.e. when two elements differ by a factor, which is nonzero central unit.

The results from [1] give long-seeked conditions for a given algebra A to be a *finite factorization domain*, i. e. a domain, where every nonunit has at most finite number of factorizations. Over such domains a factorization procedure thus becomes into an algorithm. Examples, bounds and counterexamples will be given. Over the well-known class of ubiquitous G-algebras (aka PBW aka algebras of solvable type), we provide a factorization algorithm [2] , its' smarter graded-driven version for graded algebras [3,4] and a factorizing Gröbner algorithm [2]. All of these are implemented in SINGULAR:PLURAL [5]. We view the factorizing Gröbner algorithm as the only general possibility to obtain a weaker analogon to the primary decomposition from the commutative algebra.

Recent complexity results and applications of the mentioned algorithms will be presented.

1. J. Bell, A. Heinle and V. Levandovskyy. On noncommutative finite factorization domains. *Transactions of the AMS*, 369:2675–2695, 2017.
2. A. Heinle and V. Levandovskyy. A Factorization Algorithm for G-Algebras and its Applications. *Journal of Symbolic Computation*, `https://doi.org/10.1016/j.jsc.2017.06.005`, 2017.
3. M. Giesbrecht, A. Heinle and V. Levandovskyy. Factoring Linear Partial Differential Operators in n Variables. *Journal of Symbolic Computation*, 75:127–148, 2016.
4. A. Heinle and V. Levandovskyy. Factorization of $\mathbb{Z}$-homogeneous Polynomials in the First (q-) Weyl Algebra. *Springer LNM*, to appear. 2017.
5. G.-M. Greuel, V. Levandovskyy, H. Schönemann and O. Motsak. PLURAL. A SINGULAR 4.1 Subsystem for Computations with Non–commutative Polynomial Algebras. Centre for Computer Algebra, University of Kaiserslautern, 2000–2017. `http://www.singular.uni-kl.de`.

Alessandro Paolini - Bad primes and character degrees in finite groups of Lie type

Let $q = p^f$ with p a prime. Let G be a finite group of Lie type defined over the field with q elements, and let U be a Sylow p-subgroup of G. We first show that if p is a bad prime for G, then there always exist irreducible characters of U whose degrees are not powers of q. We then highlight the importance of characters of U of degree of the form q^n/p and their construction towards the determination of the ℓ-decomposition numbers of G when p is a bad prime for G and $\ell \neq p$.

Sebastian Posur - A constructive approach to finitely presented functors

A common data structure for finitely presented modules over a ring R is given by matrices over R, where an $n \times m$-matrix M is interpreted as the cokernel of its induced map between free modules $R^{1\times n} \xrightarrow{M} R^{1\times m}$. Homomorphisms between two such cokernels can be modeled by certain equivalence classes of commutative squares, a fact following from R being a projective module. In this talk we abstract this data structure in a categorical and constructive way to abelian categories with enough projectives. The upshot is an algorithmic approach to the category of finitely presented functors over the category of finitely presented modules.

Yue Ren - Generic binding polynomials of decoupled biomolecules of type $(n, 2)$

A ligand is a substance that forms a complex with a biomolecule to serve a biological purpose, e.g. oxygen binding to a fixed number of sites of hemoglobin for transportation in the bloodstream. In some models, the average number of occupied binding sites is a rational function of the ligand concentration. As such, the denominator of the rational function, the so-called binding polynomial, is essential for characterizing the binding behavior of the system.

In this talk, we will give an introduction into the biological background as well as the algebraic framework of ligand binding. More specifically, the decoupled molecules of type $(n, 2)$ with a fixed binding polynomial are the solutions to a system of $3n + 2$ unknowns: the $n + 2$ site energies and the $2n$ interaction energies $w_{i,j}$. We show that, generically, the number of solutions to this system equals $4(n!)^3$. These come in $2(n!)^2$ classes under relabelling of the sites.

Mahsa Sayyary - Computing the undulation invariant of plane quartics

Undulation points of a plane curve $P(x, y, z) = 0$ are points at which the tangent line is tangent to the curve with multiplicity at least 4. In 1852 A. Cayley and G. Salmon proved that for a plane curve of degree d there exists a unique (up to rescaling) function F on the coefficients of the plane curve P which vanishes whenever P has an undulation point. An explicit formula for F was only found for the cases $d = 4$ and $d = 5$ in 2013 by A.Popolitov and Sh. Shakirov. They claimed that they used some technical tools to find F, including Maple.

I made a code in M2 which computes this invariant for $d = 4$. The main part of my talk would be explaining how to express the complicated polynomial F for $d = 4$, as the determinant of a relatively simple matrix M of size 21×21. Also I would briefly explain my code and its difficulties.

Isabel Stenger - Constructing torsion-free numerical Godeaux surfaces

Numerical Godeaux surfaces provide the first case in the geography of minimal surfaces of general type. It is known that the torsion group of such a surface is cyclic of order $m \leq 5$ and a full classification has been given for $m = 3, 4, 5$ by Reid, Miyaoka. In my talk

I will discuss a homological approach to construct a numerical Godeaux surface X based on a former project of Frank-Olaf Schreyer. The main idea is to study the syzygies of the canonical ring $R(X)$ considered as a module over some (weighted) graded polynomial ring. We focus on the case $\mathrm{Tors}(X) = 0$ and pay particular emphasis to the genus-4 fibration given by the bicanonical system.

Timo de Wolff - Lopsided Approximation of Amoebas

Given an Laurent polynomial $f \in \mathbb{C}[z_1^{\pm 1}, \ldots, z_n^{\pm 1}]$, the *amoeba* $\mathcal{A}(f)$ (introduced by Gelfand, Kapranov, and Zelevinsky '94) is the image of its variety $\mathcal{V}(f) \subseteq (\mathbb{C}^*)^n = (\mathbb{C} \setminus \{0\})^n$ under the $\mathrm{Log}|\cdot|$-map

$$\mathrm{Log}|\cdot| : (\mathbb{C}^*)^n \to \mathbb{R}^n, \quad (z_1, \ldots, z_n) \mapsto (\log|z_1|, \ldots, \log|z_n|).$$

Amoebas are related to various mathematical subjects like complex analysis, the topology of real algebraic curves, nonnegativity of polynomials, dynamical systems, and particularly tropical geometry.

We demonstrate that a theoretical amoeba approximation method due to Purbhoo can be used efficiently in practice. To do this, we resolve the main bottleneck in Purbhoos method by exploiting relations between cyclic resultants. We provide a SINGULAR/SAGE implementation of this algorithm, which shows a significant speedup when our specialized cyclic resultant computation is used, versus a general purpose resultant algorithm.

This talk is based on joint work with Jens Forsgård, Laura Matusevich, and Nathan Mehlhop; see `https://arxiv.org/abs/1608.08663`.