Exercise for: The GAP package SingerAlg – using the GAP-Julia integration

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The function $m(q, e)$

(See [BHHK20, Sect. 6], [BHHK21, Sect. 2] for more information.)

Fix coprime integers $q > 1$ and $e \geq 1$,
and let $m(q, e)$ denote the smallest positive integer t
such that there exists a sum of t powers of q which is divisible by e.

Equivalently,

$m(q, e)$ is the smallest q-adic digit sum $s_q(ke)$, for $1 \leq k$,
where $s_q(x) = x_1 + x_2 + x_3 + \cdots$
if $x = x_1 + qx_2 + q^2x_3 + \cdots$, with $0 \leq x_i < q$.

(It is sufficient to consider $1 \leq k \leq z$,
where $z = (q^n - 1)/e$, for $n := \mathrm{ord}_e(q)$.)

The function $m(q, e)$

Then we have for example

- $1 \leq m(q, e) \leq e$,
- $m(q, e) = 1$ if and only if $e = 1$,
- $m(q, e) = 2$ if and only if
  either $e = 2$ or $n := \mathrm{ord}_e(q)$ is even with $q^{n/2} \equiv -1 \pmod{e}$,
- $m(q, e) = e$ if and only if $q \equiv 1 \pmod{e}$,
- $\gcd(e, q-1)$ divides $m(q, e)$.

Compute $m(q, e)$

In many situations, $m(q, e)$ is known from theory,
but we do not know a general formula for $m(q, e)$.

Thus it is eventually necessary to compute certain values explicitly.

Write GAP and/or Julia functions that compute $m(q, e)$.

(The SingerAlg package provides implementations in GAP and Julia,
respectively,
via `MinimalDegreeOfSingerAlgebraGAP`
and `Julia.SingerAlg.MinimalDegree`,
respectively.)

References

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