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LOW-DIMENSIONAL REPRESENTATIONS OF QUASI-SIMPLE GROUPS

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Abstract

We determine all absolutely irreducible representations of degree up to 250 of quasi-simple finite groups, excluding groups of Lie type in their defining characteristic. Additional information on the Frobenius-Schur indicators and the Brauer character fields of the representations is also given.

1. *Introduction*

In this paper we classify low-dimensional absolutely irreducible representations of quasi-simple groups. Results of this type seem to have many applications. We have at least one immediate application in mind, namely to the matrix group recognition program in computational group theory.

Our main result is a list of all absolutely irreducible representations of quasi-simple groups up to degree 250, excluding representations of groups of Lie type in their defining characteristic, together with some information on these representations, like character field and Frobenius-Schur indicator.

Theorem 1. *Let G be a quasi-simple finite group and V an absolutely irreducible faithful FG-module of dimension $d \leqslant 250$, where the characteristic of F is not the defining characteristic of G if G is of Lie type. Then $(G, \dim(V))$ (and some additional information) are contained in Tables 1 and 2.*

Previous work of Kondratiev (see [**37**] and the references therein) solved this question for representations up to degree 27. His results are recovered as part of our main theorem. In the case not considered here, of irreducible representations of groups of Lie type in their defining characteristic, Aschbacher (unpublished) obtained a classification up to degree 27. The recent work of Lübeck [**38**] determines all low-dimensional representations of groups of Lie type in their defining characteristic, thus complementing our list. More precisely, Lübeck determines all absolutely irreducible representations of G of degree at least up to $l^3/8$ where l is the Lie rank of G, including all those of degree at most 250.

The construction of the list naturally falls into three parts, according to the classification of finite simple groups. If the group G is of Lie type, we make use of results by various authors on low-dimensional representations in cross-characteristic (see [**16, 17, 24**] and the Landazuri-Seitz-Zalesskii bounds [**49**]. For the alternating groups and their spin-covers, results by James [**28, 29**] and Wales [**55**] can be used. Finally, for the sporadic groups there exists a wealth of published and unpublished tables of Brauer characters computed by different authors. In all three classes, the tables for the smaller groups can be found in the ordinary and modular Atlas [**5, 33**] and also in the computer algebra system GAP [**14**]. Still

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it turns out that in all three families there remain some groups where neither the theoretical results apply to determine all candidate representations, nor are the corresponding tables contained in GAP. In these cases the open questions could be solved with the help of the computer system MOC [**32**].

Throughout our paper we adopt the notation of the Atlas [**5**] for the quasi-simple groups.

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2. *Groups of Lie type*

In this section we describe the proof of correctness of the table for groups of Lie-type in non-defining characteristic.

2.1. *Known results*

The starting point is the result of Landazuri-Seitz-Zalesskii [**49**] giving lower bounds for degrees of non-trivial representations of quasi-simple groups of Lie type in non-defining characteristic. Using this we get down to the following finite list of simple candidate groups whose covering groups possibly have non-trivial irreducible representations of degree at most 250: $\mathrm{L}_2(q)$ $(q \leqslant 499)$, $\mathrm{L}_3(q)$ $(q \leqslant 13)$, $\mathrm{L}_4(q)$ $(q \leqslant 5)$, $\mathrm{L}_5(2)$, $\mathrm{L}_5(3)$, $\mathrm{L}_6(2)$, $\mathrm{L}_7(2)$, $\mathrm{U}_3(q)$ $(q \leqslant 16)$, $\mathrm{U}_4(q)$ $(q \leqslant 5)$, $\mathrm{U}_5(q)$ $(q \leqslant 4)$, $\mathrm{U}_6(2)$, $\mathrm{U}_6(3)$, $\mathrm{U}_7(2)$, $\mathrm{U}_8(2)$, $\mathrm{U}_9(2)$, $\mathrm{S}_4(q)$ $(q \leqslant 19)$, $\mathrm{S}_6(q)$ $(q \leqslant 7)$, $\mathrm{S}_8(q)$ $(q \leqslant 3)$, $\mathrm{S}_{10}(q)$ $(q \leqslant 3)$, $\mathrm{O}_7(3)$, $\mathrm{O}_8^+(2)$, $\mathrm{O}_8^-(2)$, $\mathrm{O}_8^-(3)$, ${}^2B_2(8)$, ${}^2B_2(32)$, $G_2(q)$ $(q \leqslant 5)$, ${}^3D_4(2)$, ${}^3D_4(3)$, ${}^2F_4(2)'$, $F_4(2)$.

From this list we can eliminate the groups whose modular character tables are contained in the Atlas of Brauer Characters [**33**] or in GAP [**14**]. Furthermore, the irreducible representations in non-defining characteristic of $\mathrm{L}_2(q)$ are well-known (see [**3**]), and the relevant information is displayed in Table 1. For linear groups, Guralnick and Tiep [**17**] have described the representations of the first two non-trivial degrees and have given a lower bound for the third smallest degree. Similarly this has been done for unitary groups by the present authors in [**24**], and for the symplectic groups over fields of odd order in [**16**]. Using these results we are left with the groups $\mathrm{L}_4(4)$, $\mathrm{L}_4(5)$, $\mathrm{U}_4(4)$, $\mathrm{S}_4(7)$, $\mathrm{S}_4(8)$, $\mathrm{S}_{10}(2)$, $\mathrm{O}_8^-(3)$, ${}^3D_4(3)$ and $F_4(2)$.

Restriction to $2.\mathrm{O}_7(3)$ (which is in GAP) shows that $2.\mathrm{O}_8^-(3)$ has no faithful characters of degree at most 250, and the characters of the simple group $\mathrm{O}_8^-(3)$ are in GAP.

2.2. *Some groups of Lie type and MOC*

Here we comment on the computation of the results for some of the smaller groups of Lie type not contained in the Modular Atlas. For the linear groups $\mathrm{L}_4(4)$ and $\mathrm{L}_4(5)$ we used the modular representation theory of the general linear groups in the non-defining characteristic case as developed by Dipper and James in a series of papers. In particular we consulted reference [**30**], in which James computed the decomposition matrices for $\mathrm{GL}_n(q)$ up to $n = 10$.

For the unitary group $U_4(4)$ we used the information contained in the diploma thesis of Wings [**61**], as well as the generic character table of $U_4(q)$ computed by Nozawa [**44**]. Partial decomposition matrices for the 4-dimensional symplectic groups have been obtained by White, and completed by Okuyama and Waki in [**45**]. For the case of cyclic defect groups, the decomposition matrices have been computed by Fong and Srinivasan [**7**]. To deal with the groups $S_4(7)$ and $S_4(8)$ we used White's papers [**56**], [**57**] and [**58**]. For the character values we consulted the generic character tables (see [**50**], [**46**] and [**6**]). The desired result for ${}^3D_4(3)$ can be extracted from Geck's work [**8**], and the one for $F_4(2)$ is contained in [**21**].

The decomposition matrices for $F_4(2)$ as well as those for $S_{10}(2)$ and many sporadic groups have been computed with the help of MOC [**32**], a system for dealing with modular characters. This was developed by Jansen, Lux, Parker and the first author beginning in 1984. A description of MOC and its algorithms is given in [**32**], which unfortunately never found a publisher. Some of the algorithms and methods used are also described in [**39**, Section 3] and in [**40**, Section 3].

Let us briefly sketch the principal idea without going into too many details. Let G be a finite group and let p be a prime dividing the order of G. Suppose that the ordinary character tables of G and the p-modular character tables of some (maximal) subgroups of G are known. In a first step we compute a large set B of p-modular Brauer characters and a large set P of projective characters of G. This is done with the usual methods, by tensoring characters or by inducing them from subgroups.

Let $\mathrm{IBr}(G)$ and $\mathrm{IPr}(G)$ denote the set of irreducible Brauer characters and of projective indecomposable characters of G, respectively. In a second step we compute a basic set of Brauer characters BS from B and a basic set of projective characters PS from P. The term basic set here means that the transition matrices X and Y with $\mathrm{BS} = X \cdot \mathrm{IBr}(G)$ and $\mathrm{PS} = Y \cdot \mathrm{IPr}(G)$ are unimodular and have non-negative integral entries. (In the above matrix equations we have identified the sets of characters with their matrices of values, so that, for example, $\mathrm{IBr}(G)$ also stands for the Brauer character table of G.)

Let U denote the square matrix of scalar products between the characters in BS and those in PS. Then $U = XY^t$, by the orthogonality relations. Thus in order to find candidates for $\mathrm{IBr}(G)$ one has to find the factorizations of U into a product of unimodular matrices with non-negative integral entries. If $U = X_1Y_1^t$ is such a factorization, then $X_1^{-1}\cdot\mathrm{BS}$ is a candidate set for the irreducible Brauer characters. The projective characters in P reduce the number of candidates for $\mathrm{IBr}(G)$, and thus for the first factor X_1, since every irreducible Brauer character must have a non-negative scalar product with every character in P. Dually, the Brauer characters in B restrict the number of possibilities for the second factor Y_1^t.

Of course, in each individual problem we used some ad hoc techniques, adapted to the particular problem. The application of these techniques was supported by substantial calculations with GAP and MAPLE [**4**].

3. *The alternating groups*

In this section we collect some results on low-dimensional representations of alternating groups and their covering groups. For this purpose we recall some standard notation. For a field F and a partition $\lambda \vdash n$ let S^λ denote the corresponding Specht module for $\mathfrak{S}_n$ over F. Assume that F has characteristic ℓ. For ℓ-regular partitions λ we write D^λ for the irreducible $F\mathfrak{S}_n$-module indexed by λ, the unique simple composition factor in the socle of S^λ. It is known that the D^λ constitute a complete set of representatives for the irreducible

$F\mathfrak{S}_n$-modules.

3.1. *2-modular representations of* $\mathfrak{A}_n$

We first consider the case of representation over fields of characteristic $\ell = 2$. The 2-modular character table of $\mathfrak{S}_n$ is known for $n \leqslant 14$ [**33, 1**].

Proposition 2. *Let* $n \geqslant 15$, $\ell = 2$ *and* $(n) \neq \lambda \vdash n$ *be 2-regular. Assume that* $\dim(D^\lambda) \leqslant 500$. *Then one of the following cases occurs.*
(1) $\lambda = (n-1,1)$ *and* $n \leqslant 502$, *or*
(2) $\lambda = (n-2,2)$ *and* $n \leqslant 34$, *or*
(3) $\lambda = (12,3)$, $\dim(D^\lambda) = 336$, *or*
(4) $\lambda = (8,7),(9,7)$, $\dim(D^\lambda) = 128$, *or*
(5) $\lambda = (9,8),(10,8)$, $\dim(D^\lambda) = 256$.

Proof. From the known 2-modular character table of $\mathfrak{S}_{14}$ it follows that for $n = 14$ either

$$\lambda \in M_{14} := \{(14),(13,1),(12,2),(11,3),(10,4),(8,6)\}$$

or $\dim(D^\lambda) > 500$. Now let $n = 15$ and assume that $\dim(D^\lambda) \leqslant 500$. Then the restriction $D^\lambda|_{\mathfrak{S}_{14}}$ has to contain some D^μ with $\mu \in M_{14}$ as a submodule. This implies that S^λ is a constituent of the induced module $(S^\mu)^{\mathfrak{S}_{15}}$, hence

$$\lambda \in \{(15),(14,1),\ldots,(9,6),(8,7),(8,6,1)\}$$

by the branching rule for ordinary characters of $\mathfrak{S}_n$. The dimension of D^λ for λ in the above list is known by [**28**, Th. 5.2 and Th. 7.1], and it turns out that $\dim(D^\lambda) \leqslant 500$ only for

$$\lambda \in M_{15} := \{(15),(14,1),(13,2),(12,3),(8,7)\}.$$

The same argument may be applied to obtain $M_{16},\ldots,M_{19}$ yielding in particular $M_{19} = \{(19),(18,1),(17,2)\}$. Now induction and the branching rule allow to complete the proof. The degrees of the relevant D^λ follow from [**28**, Th. 5.2]. □

Benson has determined those 2-regular partitions $\lambda = (\lambda_1 \geqslant \lambda_2 \geqslant \ldots)$ for which the irreducible $\mathfrak{S}_n$-module D^λ splits upon restriction to the alternating group $\mathfrak{A}_n$: the restriction is irreducible if $\lambda_{2j-1} - \lambda_{2j} > 2$ or $\lambda_{2j-1} + \lambda_{2j} \equiv 2 \pmod 4$ for some $j > 0$ [**2**, Th. 1.1]. We thus obtain:

Corollary 3. *Let* $n \geqslant 15$, $\ell = 2$ *and* D *a non-trivial absolutely irreducible* $\mathfrak{A}_n$*-module of dimension* $\dim(D) \leqslant 250$. *Then either* D *is the restriction to* $\mathfrak{A}_n$ *of* $D^{(n-1,1)}$ *with* $n \leqslant 252$ *or of* $D^{(n-2,2)}$ *with* $n \leqslant 24$, *or*

$$(n,\dim(D)) \in \{(15,64),(16,64),(17,128)\}.$$

3.2. *ℓ-modular representations of* $\mathfrak{A}_n$, $\ell > 2$

Now assume that F is a sufficiently large field of characteristic $\ell \neq 2$. The decomposition matrices of $\mathfrak{S}_n$ over F are known for $n \leqslant 14$ (see [**33**]) or can be computed using the Specht-package in GAP written by Andrew Mathas. (In fact, for $\ell \neq 3$ the decomposition matrices can be computed at least up to $n = 20$, but we will not need this here.)

We obtain the following analogue of Proposition 2:

Proposition 4. *Let* $n \geqslant 14$ *and let* M *be an non-trivial absolutely irreducible* $F\mathfrak{A}_n$*-module of dimension* $\dim(M) \leqslant 250$. *Then* $M = D^\lambda$ *for* λ *one of*

(1) $(n-1,1)$ *and* $n \leqslant 252$*, or*
(2) $(n-2,2)$ *and* $n \leqslant 24$*, or*
(3) $(n-2,1^2)$ *and* $n \leqslant 24$*, or*
(4) $(11,3)$ *and* $\ell = 5,7$*, or*
(5) $(7,7)$ *and* $\ell = 5$.

Proof. We can argue essentially as in the proof of the preceding proposition. First let $\ell = 3$. From the known decomposition matrix it follows that the irreducible $\mathfrak{A}_{13}$-modules D with $\dim(D) \leqslant 250$ are the D^λ with

$$\lambda \in M_{3,13} := \{(13),(12,1),(11,2),(11,1^2),(10,3),(10,2,1)\},$$

in particular, all of them are restrictions of irreducible $\mathfrak{S}_{13}$-modules. Thus if D is an irreducible $\mathfrak{A}_{14}$-module of dimension $1 \neq \dim(D) \leqslant 250$ then D is a constituent of D^λ or of $D^{\bar{\lambda}}$ for a partition $\lambda \vdash 14$ which can be obtained by adding a hook of length 1 to some partition $\mu \in M_{3,13}$. The dimensions of the ten relevant D^λ can be determined using [**29**]. It turns out that in fact

$$\lambda \in M_{3,14} := \{(14),(13,1),(12,2),(12,1^2)\}.$$

All of these restrict irreducibly to $\mathfrak{A}_{14}$. Similarly for $n = 15$ we obtain $D = D^\lambda$ for

$$\lambda \in M_{3,15} := \{(15),(14,1),(13,2),(13,1^2)\}$$

and so on. The assertion for $\ell = 3$ follows by induction.

The argument for $\ell \geqslant 5$ is similar. For $\ell = 0$ the degrees of the Specht modules are well known. □

3.3. *ℓ-modular representations of* $2.\mathfrak{A}_n$

Here we study low-dimensional faithful irreducible representations of the covering groups $2.\mathfrak{A}_n$. Since an irreducible 2-modular representation of $2.\mathfrak{A}_n$ is not faithful, we assume that $\ell \neq 2$ throughout this section. Also, we may and will assume that $n \geqslant 14$ since for smaller n the modular tables are contained in [**33**] or in GAP.

We begin by looking at the covering groups of the symmetric groups. Their irreducible representations in characteristic 0 are indexed by partitions of n into distinct parts, with the rule that one partition corresponds to two different representations if these have the same restriction to $2.\mathfrak{A}_n$. Following the notation in the paper of Wales [**55**], the representations of $2.\mathfrak{S}_n$ corresponding to a partition λ are denoted by $<\lambda>$, respectively by $<\lambda>, <\lambda>^*$ if there are two.

Proposition 5. *Let* $n \geqslant 14$ *and* $\ell \neq 13$ *for* $n = 14$*. Then a faithful irreducible* $F2.\mathfrak{S}_n$*-module* M *with* $\dim M \leqslant 500$ *is a constituent of the reduction modulo* ℓ *of* $<n>$ *or of* $<n>^*$.

Proof. The assertion is correct for $n = 14$ and $\ell \neq 13$ and for $n = 15$ and $\ell = 13$. This follows from explicit computations with the character tables using MOC.

Suppose then that $n \geqslant 15$ and $n \geqslant 16$ if $\ell = 13$. Let M be a faithful simple $F2.\mathfrak{S}_{n+1}$-module with $\dim M \leqslant 500$. Then the restriction of M to $2.\mathfrak{S}_n$ contains a faithful irreducible submodule of dimension at most 500. By induction, this is a composition factor of $<n>$ or of $<n>^*$.

It follows that M is a composition factor of the reduction modulo ℓ of $<n>^{2.\mathfrak{S}_{n+1}}$ or of $<n>^{*2.\mathfrak{S}_{n+1}}$. By the branching theorem we have $<n>^{2.\mathfrak{S}_{n+1}} = <n+1> + <n,1>$ if n is odd, and $<n>^{2.\mathfrak{S}_{n+1}} = <n+1> + <n+1>^* + <n,1>$ if n is even. By Table IV of

[**55**], a modular composition factor of $< n, 1 >$ either is a composition factor of $< n+1 >$ or $< n+1 >^*$, or its dimension is larger than 500. This completes the proof. $\square$

This proposition shows that for $n \geqslant 15$ and $n = 14$, $\ell \neq 13$, a faithful irreducible $F2.\mathfrak{A}_n$-module of dimension at most 250 is a composition factor of the restriction to $2.\mathfrak{A}_n$ of the reduction modulo ℓ of $< n >$ or of $< n >^*$. A direct computation with MOC shows that this is true even for $n = 14$ and $\ell = 13$. (Note that this is a cyclic defect case.) The modular composition factors of $< n >$ and $< n >^*$ are determined in [**55**]. In addition, it is also determined there which of them restrict irreducibly to $2.\mathfrak{A}_n$.

We thus obtain from Table III of [**55**]:

Corollary 6. *Let $n \geqslant 14$, $\ell \neq 2$ and let D be a faithful absolutely irreducible $2.\mathfrak{A}_n$-module of dimension $\dim(D) \leqslant 250$. Then $n \leqslant 18$ and D is a composition factor of a basic spin representation of $2.\mathfrak{A}_n$. We have the following cases for $(n, \dim(D))$:*
(1) $(14,32)$ *for $\ell = 7$ (two D's),*
(2) $(14,64)$ *for $\ell \neq 7$ (one D),*
(3) $(15,64)$ *for $\ell = 3,5$ (one D),*
(4) $(15,64)$ *for $\ell \neq 3,5$ (two D's),*
(5) $(16,128)$ *(one D),*
(6) $(17,128)$ *for $\ell = 17$ (one D),*
(7) $(17,128)$ *for $\ell \neq 17$ (two D's),*
(8) $(18,128)$ *for $\ell = 3$ (two D's).*

4. *The sporadic groups*

Our first source was the Modular Atlas [**33**], which contains the modular character tables for ten of the 26 sporadic groups (and their covering groups). Apart from these ten groups, the largest of which is the McLaughlin group., the modular character tables are completely known for the six sporadic groups He, $2.Ru$, $6.Suz$, $3.O'N$, Co_3 and Co_2. Modular character tables for some of the larger groups are also available in some characteristics, for example for the Fischer groups $6.Fi_{22}$ and Fi_{23} in characteristic 5 or for the Conway group $2.Co_1$ in characteristic 7. Moreover, for primes dividing the group order only once, the character degrees can be computed from the Brauer trees given in [**22**].

There are several sources for these character tables, not all of them have been published yet. The only published source on the Held group is Ryba's paper [**47**]. The 2-modular character tables for the covering group of the Rudvalis group were determined by Wilson (unpublished), its 3-modular tables are contained in [**20**] and its 5-modular tables in [**25**]. On the Suzuki group there is only one paper, by Jansen and Müller [**34**]. The relevant papers for the O'Nan group and its covering group are [**35**], by Jansen and Wilson, and [**18**], by Henke, Hiss and Müller. The 2-modular characters of the smallest two Conway groups have been computed by Suleiman and Wilson [**51**], [**52**], its 3-modular characters by Jansen in his PhD-thesis [**31**], and the 5-modular characters of Co_3 by Müller [**43**]. The 5-modular character table for Co_2 is contained in [**32**], as well as the 7-modular table for $2.Co_1$. The 5-modular Brauer characters for the Fischer groups $6.Fi_{22}$ and Fi_{23} have been computed by Hiss, Lux, and White in [**23**] and [**26**]. This completes the list of known modular tables for the sporadic groups and their covering groups. The Web site of MOC [**15**] contains the corresponding decomposition matrices and Brauer character degrees.

The latter reference also contains a list of minimal degrees of non-trivial representations of sporadic groups and their covering groups (except for the double cover of the Baby

Monster), compiled by Christoph Jansen. It follows from this list that we do not have to consider the two Fischer groups Fi_{23}, Fi'_{24}, nor the Baby Monster B or the Monster M. For example, the minimal degree of a non-trivial representation of Fi_{23} is 253. Since Fi_{23} is a subgroup of the double cover $2.B$ of the Baby Monster, the smallest degree of the latter group is well above 250.

The same list also shows that for the Lyons group Ly we only have to worry about representations in characteristic 5, and for the largest Janko group J_4 only about characteristic 2. The case of the Lyons group was handled by Klaus Lux and Alex Ryba (unpublished). They showed, using the condensation techniques (see [**48**] or [**41**]) of the Meat-Axe, that the only non-trivial representation in characteristic 5 of the Lyons group of degree smaller than 250 is the known one of degree 111, constructed by Meyer, Neutsch and Parker [**42**]. Lux and Ryba also settled the case of the Harada-Norton group in characteristic 5 with the same techniques, but this has been checked independently with the methods described in Section 2. The minimal degree of a faithful representation of the Conway group in odd characteristic equals 276. Hence for $2.Co_1$ and odd characteristics we only have to consider blocks containing faithful characters. The diploma thesis of Hensing [**19**] gives enough information on the 5-modular decomposition numbers of $2.Co_1$ to show that our list is correct in this case. Unpublished work of Rosenboom settles the case of Co_1 in characteristic 2, but we have checked this case independently. The case of the smallest Fischer group Fi_{22} and its covering group has been handled by Christoph Jansen (unpublished).

We are thus left with the groups HN, Th, $2.Co_1$ and J_4, the latter only in characteristic 2, and the Conway group only in characteristic 3. To deal with these cases we used the methods and the MOC system described in Section 2.

5. *The Frobenius-Schur indicators*

We briefly comment on the methods used to determine the Frobenius-Schur indicators of the characters occurring in the tables below. Let us begin with the definition. Suppose that G is a finite group, K a field, and M a finitely generated, absolutely irreducible KG-module. Suppose first that M is self-dual. Then M carries a non-degenerate G-invariant bilinear form, which is symmetric or alternating. Moreover, this form is unique up to scalar multiplication. If the characteristic of K is 2, and M is not the trivial module, then the invariant form is alternating. If the characteristic of K is odd and M carries a non-degenerate symmetric bilinear form, this form is the polarization of a quadratic form on M, which is non-degenerate and G-invariant. If M is not self-dual, it does not carry any non-trivial G-invariant bilinear form.

The Frobenius-Schur indicator of M is an integer of the set $\{-1,0,+1\}$. It is defined to be 0, if and only if M is not self-dual. The Frobenius-Schur indicator of M is set to be $+1$, if and only if M carries a non-degenerate G-invariant quadratic form. Thus the indicator of M is -1 if and only if M is self-dual and carries a non-degenerate G-invariant bilinear alternating form, but no G-invariant quadratic form.

In our tables the indicators are represented by one of the three symbols $\{-,\circ,+\}$ with the obvious bijection.

To compute the Frobenius-Schur indicators, we used several well known methods. We begin our discussion with the case that K has odd characteristic p and M is self-dual. Then the Frobenius-Schur indicator of M can be computed from the p-modular decomposition matrix of G and the ordinary character table of G as follows. There is a self-dual ordinary irreducible module V of G whose reduction modulo p contains M with odd multiplicity, and the Frobenius-Schur indicator of M is equal to that of V. This result is due, independently, to Thompson [**53**] and Willems [**59**] (see the survey [**60**], in particular Theorem 2.8 therein).

In our case, the self-dual ordinary irreducible modules containing M with odd multiplicity can easily be found; in fact most of the time M is liftable to a self-dual ordinary irreducible module. Thus it remains to compute the Frobenius-Schur indicators of certain ordinary irreducible modules.

Let V be an irreducible $\mathbb{C}G$-module with character χ. Then the Frobenius-Schur indicator of V is equal to

$$\nu_2(\chi) := \frac{1}{|G|} \sum_{g \in G} \chi(g^2).$$

Thus the indicators in characteristic 0 can be computed from the ordinary character table and the 2-powermap of G. Also, $\nu_2(\chi) = +1$ if and only if $V \cong \mathbb{C} \otimes_{\mathbb{R}} V_0$ for some $\mathbb{R}G$-module V_0, in other words, if and only if χ is afforded by a real representation of G (see Chapter 4 of [**27**]). If the character table of G is not available, we use this latter criterion.

Let χ be real valued. To decide whether or not χ is afforded by a real representation, we use the real Schur index $m_{\mathbb{R}}(\chi)$ (see Definition 10.1 of [**27**]). By the definition of the Schur index and the discussion above we have $\nu_2(\chi) = -1$ if and only if $m_{\mathbb{R}}(\chi) \neq 1$. Since $|\mathbb{C}\colon \mathbb{R}| = 2$, the latter condition is equivalent to $m_{\mathbb{R}}(\chi) = 2$, and implies that $\chi(1)$ is even (see Corollary 10.2(g) and (h) of [**27**]). Suppose that ψ is the character of a real representation of G. Then, by Corollary 10.2(c) of [**27**], the Schur index $m_{\mathbb{R}}(\chi)$ divides the scalar product (ψ, χ). This can be applied in particular in case ψ is induced or restricted from a character of a real representation of a subgroup, respectively overgroup, of G.

Results on Schur indices of characters of groups of Lie type can be found in the papers [**9, 10**] by Rod Gow. We also made use of the results of Przygocki [**46**] on the Schur indices of the symplectic groups and of Tiep [**54**], Cor. 4.5, on the Schur indices of the Weil representations of the unitary groups.

Let us suppose now that the characteristic of K equals 2. Here, no simple method is known for computing the Frobenius-Schur indicators. A collection of various methods is contained in the articles [**12, 13**] of Gow and Willems. We made in particular use of Lemmas 1.1 and 1.2 of [**12**], and of Theorem 1.1 of [**13**] and its proof.

If the KG-module M lifts to an absolutely irreducible $\mathbb{Q}G$-module, then the Frobenius-Schur indicator of M equals $+1$. This is a well known elementary result which is included in the much more general Theorem 2.2 of [**12**].

To determine the indicators of the basic spin representations of the alternating groups, we followed a suggestion of Rod Gow. First, by Corollary 4.3 of [**11**], the spin module of the symplectic group $\mathrm{S}_{2m}(q)$, q even, of degree 2^m has indicator $+1$, if $m \geqslant 3$ (see also Proposition 5.4.9 of [**36**]). The restriction of the spin module of $\mathrm{S}_{2m}(q)$ to the subgroup $\mathfrak{S}_{2m+1}$ or $\mathfrak{S}_{2m+2}$ remains irreducible. Thus the basic spin representation of the symmetric group $\mathfrak{S}_n$ has indicator $+1$ if $n \geqslant 7$. This implies, for example with Lemma 1.2 of [**12**], that the basic spin representations for $\mathfrak{A}_n$ also have indicator $+1$, provided they are self-dual. The splitting of the basic spin representation of $\mathfrak{S}_n$ on restriction to $\mathfrak{A}_n$ has been determined by Benson in Theorems 1.1 and 6.1 of [**2**].

The Frobenius-Schur indicators for the sporadic groups have mainly been determined by Richard Parker and Rob Wilson using the computational methods described in [**52**]. In addition, Rob Wilson computed the indicators of the following representations

d	G	ℓ	ind
32	$2.\mathfrak{A}_{14}$	7	$-$
104	$\mathrm{U}_4(5)$	2	$-$
174	$\mathrm{S}_4(7)$	2	$+$
208	$\mathfrak{A}_{14}$	2	$+$
218	${}^3D_4(3)$	2	$+$

which remained open in a previous version of this paper.

6. *A table of low-dimensional absolutely irreducible representations of quasi-simple groups*

In order to shorten the table of low-dimensional representations we have grouped together some generic cases in the following table. The corresponding entries have been omitted from the large table for $\mathfrak{A}_n$ with $n \geqslant 14$ and for $\mathrm{L}_2(q)$ for $q \neq 9$. The indicators for $\mathrm{L}_2(q)$ when $\ell = 2$ were taken from [**12**, Th. 2.3] for the characters of degree $q \pm 1$, and from [**13**, Th. 1.2] for the Weil character of degree $(q-1)/2$.

It is maybe helpful to point out that in accordance with the formulation of Theorem 1 the following pairs $(G/Z(G), \ell)$ have been omitted from the tables:

$\mathfrak{A}_6 \cong \mathrm{S}_4(2)' \cong \mathrm{L}_2(9)$ in characteristic $\ell = 3$,

$\mathfrak{A}_8 \cong \mathrm{L}_4(2)$ in characteristic $\ell = 2$,

$\mathrm{L}_2(7) \cong \mathrm{L}_3(2)$ in characteristic $\ell = 2, 7$,

$\mathrm{U}_4(2) \cong \mathrm{S}_4(3)$ in characteristic $\ell = 2, 3$,

$G_2(2)' \cong \mathrm{U}_3(3)$ in characteristic $\ell = 3$,

${}^2G_2(3)' \cong \mathrm{L}_2(8)$ in characteristic $\ell = 2$.

The Tits group, that is, the derived group ${}^2F_4(2)'$ of the Ree group of type F_4 over $\mathbb{F}_2$, cannot be obtained as a factor group of the group of fixed points under a Frobenius morphism of an algebraic group, so it is not considered as a group of Lie type in characteristic 2 here.

Let us briefly explain the notation used in the tables. The column headed "d" contains the degrees. The column headed "ℓ" specifies the characteristics of the fields over which the respective representations are defined. An entry $\neq \ell$ for a prime ℓ indicates that a representation of this degree exists for all other characteristics than ℓ, including 0. The "field" column gives the irrationalities of the Brauer characters (using Atlas notation), except for the irrationalities occurring in Table 1. Finally, the last column gives the Frobenius-Schur indicators, which are defined in the section above.

Table 7. *1. Generic examples.*

Alternating groups

d	G	ℓ	field	ind
$n-2$	$\mathfrak{A}_n$	$\ell \mid n$		$+$
$n-1$	$\mathfrak{A}_n$	$\ell \nmid n$		$+$

$\mathrm{L}_2(q)$, $q \equiv 1 \pmod 4$, $\ell \nmid q$

d	G	condition on ℓ	field	ind
$(q-1)/2$	$\mathrm{L}_2(q)$	2	$\sqrt{q}$	$-$
$(q-1)/2$	$2.\mathrm{L}_2(q)$	$\ell \neq 2$	$\sqrt{q}$	$-$
$(q+1)/2$	$\mathrm{L}_2(q)$	$\ell \neq 2$	$\sqrt{q}$	$+$
$q-1$	$\mathrm{L}_2(q)$		$\zeta_{q+1}^{j}+\zeta_{q+1}^{-j}$	$+$
$q-1$	$2.\mathrm{L}_2(q)$	$((q+1)/2)_{\ell'} \neq 1$	$\zeta_{q+1}^{j}+\zeta_{q+1}^{-j}$	$-$
q	$\mathrm{L}_2(q)$	$\ell \nmid (q+1)$		$+$
$q+1$	$\mathrm{L}_2(q)$	$((q-1)/4)_{\ell'} \neq 1$	$\zeta_{q-1}^{2j}+\zeta_{q-1}^{-2j}$	$+$
$q+1$	$2.\mathrm{L}_2(q)$	$\ell \neq 2$	$\zeta_{q-1}^{j}+\zeta_{q-1}^{-j}$	$-$

$\mathrm{L}_2(q)$, $q \equiv 3 \pmod 4$, $\ell \nmid q$

d	G	condition on ℓ	field	ind
$(q-1)/2$	$\mathrm{L}_2(q)$		$\sqrt{-q}$	$\circ$
$(q+1)/2$	$2.\mathrm{L}_2(q)$	$\ell \neq 2$	$\sqrt{-q}$	$\circ$
$q-1$	$\mathrm{L}_2(q)$		$\zeta_{q+1}^{2j}+\zeta_{q+1}^{-2j}$	$+$
$q-1$	$2.\mathrm{L}_2(q)$	$\ell \neq 2$	$\zeta_{q+1}^{j}+\zeta_{q+1}^{-j}$	$-$
q	$\mathrm{L}_2(q)$	$\ell \nmid (q+1)$		$+$
$q+1$	$\mathrm{L}_2(q)$	$((q-1)/2)_{\ell'} \neq 1$	$\zeta_{q-1}^{j}+\zeta_{q-1}^{-j}$	$+$
$q+1$	$2.\mathrm{L}_2(q)$	$((q-1)/2)_{\ell'} \neq 1, \ell \neq 2$	$\zeta_{q-1}^{j}+\zeta_{q-1}^{-j}$	$-$

$\mathrm{L}_2(q)$, $q \equiv 0 \pmod 2$, $\ell \nmid q$

d	G	condition on ℓ	field	ind
$q-1$	$\mathrm{L}_2(q)$		$\zeta_{q+1}^{j}+\zeta_{q+1}^{-j}$	$+$
q	$\mathrm{L}_2(q)$	$\ell \nmid (q+1)$		$+$
$q+1$	$\mathrm{L}_2(q)$	$(q-1)_{\ell'} \neq 1$	$\zeta_{q-1}^{j}+\zeta_{q-1}^{-j}$	$+$

Table 8. *2. Absolutely irreducible representations of quasi-simple groups.*

d	G	ℓ	field	ind
3	$3.\mathfrak{A}_6$	$0,2$	$z3,b5$	$\circ$
3	$3.\mathfrak{A}_6$	5	$z3$	$\circ$
3	$3.\mathfrak{A}_7$	5	$z3,b7$	$\circ$
4	$\mathfrak{A}_6$	2		$-$
4	$2.\mathfrak{A}_6$	$0,5$		$-$
4	$\mathfrak{A}_7$	2	$b7$	$\circ$
4	$2.\mathfrak{A}_7$	7		$-$
4	$2.\mathfrak{A}_7$	$\neq 2,7$	$b7$	$\circ$
4	$4_2.\mathrm{L}_3(4)$	3	$i1,r7$	$\circ$
4	$2.\mathrm{U}_4(2)$	$0,5$	$z3$	$\circ$
5	$\mathfrak{A}_6$	$0,5$		$+$
5	$\mathfrak{A}_7$	7		$+$
5	$\mathrm{U}_4(2)$	$0,5$	$z3$	$\circ$
5	M_{11}	3	$i2,b11$	$\circ$
6	$3.\mathfrak{A}_6$	$0,5$	$z3$	$\circ$
6	$6.\mathfrak{A}_6$	$0,5$	$z3,r2$	$\circ$
6	$\mathfrak{A}_7$	$\neq 7$		$+$
6	$2.\mathfrak{A}_7$	3	$r2$	$-$
6	$3.\mathfrak{A}_7$	$\neq 3$	$z3$	$\circ$
6	$6.\mathfrak{A}_7$	$\neq 2,3$	$z3,r2$	$\circ$
6	$2.\mathrm{L}_3(4)$	3		$+$
6	$6.\mathrm{L}_3(4)$	$\neq 2,3$	$z3$	$\circ$
6	$\mathrm{U}_3(3)$	$\neq 3$		$-$
6	$\mathrm{U}_4(2)$	$0,5$		$+$
6	$3_1.\mathrm{U}_4(3)$	2	$z3$	$\circ$
6	$6_1.\mathrm{U}_4(3)$	$\neq 2,3$	$z3$	$\circ$
6	$2.M_{12}$	3	$i2,i5,b11$	$\circ$
6	$3.M_{22}$	2	$z3,b11$	$\circ$
6	J_2	2	$b5$	$-$
6	$2.J_2$	5		$-$
6	$2.J_2$	$\neq 2,5$	$b5$	$-$
7	$\mathfrak{A}_8$	$\neq 2$		$+$
7	$\mathfrak{A}_9$	3		$+$
7	$\mathrm{U}_3(3)$	$0,7$		$+$
7	$\mathrm{U}_3(3)$	$0,7$	$i1$	$\circ$
7	$\mathrm{S}_6(2)$	$\neq 2$		$+$
7	J_1	11	$b5,c19$	$+$
8	$\mathfrak{A}_6$	$0,2$	$b5$	$+$
8	$\mathfrak{A}_6$	5		$+$
8	$2.\mathfrak{A}_6$	0	$b5$	$-$
8	$\mathfrak{A}_7$	5		$+$
8	$2.\mathfrak{A}_8$	$\neq 2$		$+$
8	$\mathfrak{A}_9$	$\neq 3$		$+$
8	$2.\mathfrak{A}_9$	$\neq 2$		$+$
8	$\mathfrak{A}_{10}$	2		$-$
8	$\mathfrak{A}_{10}$	5		$+$

d	G	ℓ	field	ind
8	$2.\mathfrak{A}_{10}$	5	$r6,r21$	$+$
8	$4_1.\mathrm{L}_3(4)$	5	$i1$	$\circ$
8	$4_1.\mathrm{L}_3(4)$	$\neq 2,5$	$i1,b5$	$\circ$
8	$2.\mathrm{S}_6(2)$	$\neq 2$		$+$
8	$2.\mathrm{O}_8^+(2)$	$\neq 2$		$+$
8	$2.Sz(8)$	5	c_{13}	$+$
9	$\mathfrak{A}_6$	0		$+$
9	$3.\mathfrak{A}_6$	$0,2$	$z3$	$\circ$
9	$3.\mathfrak{A}_7$	7	$z3$	$\circ$
9	$\mathfrak{A}_{10}$	$\neq 2,5$		$+$
9	$\mathfrak{A}_{11}$	11		$+$
9	M_{11}	11		$+$
9	$3.J_3$	2	$z3,b17,b19$	$\circ$
10	$\mathfrak{A}_6$	$0,5$		$+$
10	$2.\mathfrak{A}_6$	$0,5$	$r2$	$-$
10	$\mathfrak{A}_7$	7		$+$
10	$\mathfrak{A}_7$	$\neq 2,7$	$b7$	$\circ$
10	$\mathfrak{A}_{11}$	$\neq 11$		$+$
10	$\mathfrak{A}_{12}$	$2,3$		$+$
10	$2.\mathrm{L}_3(4)$	7		$+$
10	$2.\mathrm{L}_3(4)$	$\neq 2,7$	$b7$	$\circ$
10	$\mathrm{U}_4(2)$	$0,5$	$z3$	$\circ$
10	$\mathrm{U}_5(2)$	$\neq 2$		$-$
10	M_{11}	$\neq 11$		$+$
10	M_{11}	$\neq 2$	$i2$	$\circ$
10	M_{12}	$2,3$		$+$
10	$2.M_{12}$	$\neq 2$	$i2$	$\circ$
10	M_{22}	2	$b7$	$\circ$
10	$2.M_{22}$	7		$+$
10	$2.M_{22}$	$\neq 2,7$	$b7$	$\circ$
11	$\mathfrak{A}_{12}$	$\neq 2,3$		$+$
11	$\mathfrak{A}_{13}$	13		$+$
11	$\mathrm{U}_5(2)$	$\neq 2,3$	$z3$	$\circ$
11	M_{11}	$\neq 2,3$		$+$
11	M_{12}	$\neq 2,3$		$+$
11	M_{23}	2	$b7,i15,b23$	$\circ$
11	M_{24}	2	$b7,i15,b23$	$\circ$
12	$6.\mathfrak{A}_6$	0	$z3,b5$	$\circ$
12	$6.\mathfrak{A}_7$	5	$z3,b7$	$\circ$
12	$\mathfrak{A}_{13}$	$\neq 13$		$+$
12	$12_2.\mathrm{L}_3(4)$	7	$z12,b5$	$\circ$
12	$\mathrm{U}_3(4)$	$\neq 2$		$-$
12	$\mathrm{S}_4(5)$	2	$b5$	$-$
12	$2.\mathrm{S}_4(5)$	$\neq 2,5$	$b5$	$-$
12	$2.G_2(4)$	$\neq 2$		$-$
12	$2.M_{12}$	$\neq 2,3$		$+$

d	G	ℓ	field	ind
12	$2.Suz$	3		$-$
12	$3.Suz$	2	$z3$	$\circ$
12	$6.Suz$	$\neq 2,3$	$z3$	$\circ$
13	$\mathfrak{A}_7$	$3,5$		$+$
13	$\mathfrak{A}_8$	$3,5$		$+$
13	$\mathrm{U}_3(4)$	$\neq 2,5$	$z5$	$\circ$
13	$\mathrm{S}_4(5)$	$\neq 2,5$	$b5$	$+$
13	$\mathrm{S}_6(3)$	$\neq 3$	$z3$	$\circ$
13	J_2	3	$b5$	$+$
14	$\mathfrak{A}_7$	$\neq 3,5$		$+$
14	$2.\mathfrak{A}_7$	$\neq 2,3$	$r2$	$-$
14	$\mathfrak{A}_8$	$0,7$		$+$
14	$\mathrm{U}_3(3)$	$\neq 3$		$+$
14	$\mathrm{S}_6(2)$	3		$+$
14	$2.\mathrm{S}_6(3)$	$\neq 2,3$	$z3$	$\circ$
14	$Sz(8)$	$\neq 2$	$i1$	$\circ$
14	$G_2(3)$	$\neq 3$		$+$
14	J_1	11	$b5, c19$	$+$
14	J_2	5		$+$
14	J_2	$\neq 3,5$	$b5$	$+$
14	$2.J_2$	$\neq 2$		$-$
15	$3.\mathfrak{A}_6$	$0,5$	$z3$	$\circ$
15	$\mathfrak{A}_7$	$\neq 2,7$		$+$
15	$3.\mathfrak{A}_7$	$\neq 3$	$z3$	$\circ$
15	$\mathrm{L}_3(4)$	3		$+$
15	$3.\mathrm{L}_3(4)$	$\neq 2,3$	$z3$	$\circ$
15	$\mathrm{U}_4(2)$	$0,5$		$+$
15	$3_1.\mathrm{U}_4(3)$	$\neq 3$	$z3$	$\circ$
15	$\mathrm{S}_6(2)$	$\neq 2,3$		$+$
15	M_{12}	3	$b11$	$\circ$
15	$3.M_{22}$	2	$z3, b11$	$\circ$
16	$2.\mathfrak{A}_7$	7		$-$
16	$2.\mathfrak{A}_8$	7		$-$
16	$\mathfrak{A}_{10}$	2		$+$
16	$2.\mathfrak{A}_{10}$	$\neq 2,5$		$+$
16	$\mathfrak{A}_{11}$	2	$b11$	$\circ$
16	$2.\mathfrak{A}_{11}$	11		$+$
16	$2.\mathfrak{A}_{11}$	$\neq 2,11$	$b11$	$\circ$
16	$\mathfrak{A}_{12}$	2	$z3$	$\circ$
16	$2.\mathfrak{A}_{12}$	3	$i2, i5, r7, b11$	$\circ$
16	$4_2.\mathrm{L}_3(4)$	3	$i1, r7$	$\circ$
16	$2.Sz(8)$	13	$y7$	$+$
16	M_{11}	11		$+$
16	M_{11}	$\neq 3,11$	$b11$	$\circ$
16	M_{12}	11		$+$
16	M_{12}	$\neq 3,11$	$b11$	$\circ$
16	$4.M_{22}$	7	$i1, r11$	$\circ$
18	$3.\mathfrak{A}_7$	5	$z3, b7$	$\circ$
18	$\mathrm{S}_4(4)$	$\neq 2$		$+$

d	G	ℓ	field	ind
18	J_3	3	$b5$	$+$
18	$3.J_3$	5	$z3$	$\circ$
18	$3.J_3$	$\neq 3,5$	$z3, b5$	$\circ$
19	$\mathfrak{A}_8$	7		$+$
19	$\mathfrak{A}_9$	7		$+$
19	$\mathrm{L}_3(4)$	$3,7$		$+$
20	$\mathfrak{A}_7$	2		$-$
20	$2.\mathfrak{A}_7$	$\neq 2,3$		$-$
20	$\mathfrak{A}_8$	$0,5$		$+$
20	$\mathfrak{A}_9$	2	$i15$	$\circ$
20	$\mathrm{L}_3(4)$	$0,5$		$+$
20	$4_2.\mathrm{L}_3(4)$	$\neq 2,3$	$i1$	$\circ$
20	$\mathrm{U}_3(5)$	$\neq 5$		$-$
20	$\mathrm{U}_4(2)$	$0,5$		$+$
20	$2.\mathrm{U}_4(2)$	$0,5$		$-$
20	$2.\mathrm{U}_4(2)$	$0,5$	$z3$	$\circ$
20	$\mathrm{U}_4(3)$	2		$+$
20	$2.\mathrm{U}_4(3)$	$\neq 2,3$		$-$
20	$4.\mathrm{U}_4(3)$	$\neq 2,3$	$i1$	$\circ$
20	M_{22}	11		$+$
20	J_1	2		$+$
20	HS	2		$-$
21	$\mathfrak{A}_7$	$0,7$		$+$
21	$3.\mathfrak{A}_7$	$\neq 2,3$	$z3$	$\circ$
21	$\mathfrak{A}_8$	$0,7$	$i15$	$\circ$
21	$\mathfrak{A}_8$	$\neq 2$		$+$
21	$\mathfrak{A}_9$	$0,7$	$i15$	$\circ$
21	$\mathfrak{A}_9$	$3,5$		$+$
21	$3.\mathrm{L}_3(4)$	$\neq 2,3$	$z3$	$\circ$
21	$\mathrm{U}_3(3)$	$0,7$		$+$
21	$\mathrm{U}_3(3)$	$0,7$	$i1$	$\circ$
21	$\mathrm{U}_3(5)$	$\neq 2,5$		$+$
21	$3.\mathrm{U}_3(5)$	$\neq 3,5$	$z3$	$\circ$
21	$\mathrm{U}_4(3)$	$\neq 2,3$		$+$
21	$3_1.\mathrm{U}_4(3)$	$\neq 2,3$	$z3$	$\circ$
21	$\mathrm{U}_6(2)$	3		$+$
21	$3.\mathrm{U}_6(2)$	$\neq 2,3$	$z3$	$\circ$
21	$\mathrm{S}_6(2)$	$\neq 2$		$+$
21	M_{22}	$\neq 2,11$		$+$
21	$3.M_{22}$	$\neq 2,3$	$z3$	$\circ$
21	M_{23}	23		$+$
21	J_2	5		$+$
21	J_2	$\neq 2,5$	$b5$	$+$
21	HS	5		$+$
21	M^cL	$3,5$		$+$
22	$2.\mathrm{L}_3(4)$	3	$b5$	$+$
22	$\mathrm{U}_6(2)$	$\neq 2,3$		$+$
22	M_{23}	$\neq 2,23$		$+$
22	M_{24}	3		$+$

d	G	ℓ	field	ind
22	J_1	19	$b5$	$+$
22	HS	$\neq 2,5$		$+$
22	M^cL	$\neq 3,5$		$+$
22	Co_3	2		$-$
22	Co_3	3		$+$
22	Co_2	2		$+$
23	$\mathrm{U}_4(2)$	5		$+$
23	M_{24}	$\neq 2,3$		$+$
23	Co_3	$\neq 2,3$		$+$
23	Co_2	$\neq 2$		$+$
24	$3.\mathfrak{A}_7$	$0,2$	$z3, b7$	$\circ$
24	$6.\mathfrak{A}_7$	7	$z3$	$\circ$
24	$6.\mathfrak{A}_7$	$0,5$	$z3, b7$	$\circ$
24	$2.\mathfrak{A}_8$	$\neq 2,7$	$b7$	$\circ$
24	$4_1.\mathrm{L}_3(4)$	3	$i1, r7$	$\circ$
24	$12_1.\mathrm{L}_3(4)$	7	$z12$	$\circ$
24	$12_1.\mathrm{L}_3(4)$	$0,5$	$z12, b7$	$\circ$
24	$\mathrm{U}_4(2)$	0		$+$
24	$\mathrm{S}_4(7)$	2	$b7$	$\circ$
24	$2.\mathrm{S}_4(7)$	$\neq 2,7$	$b7$	$\circ$
24	$2.Sz(8)$	13	$y7$	$+$
24	M_{11}	3		$+$
24	$12.M_{22}$	11	$z24, b7$	$\circ$
24	Co_1	2		$+$
24	$2.Co_1$	$\neq 2$		$+$
25	$\mathrm{S}_4(7)$	$\neq 2,7$	$b7$	$\circ$
25	${}^3D_4(2)$	3		$+$
26	$\mathfrak{A}_9$	2		$+$
26	$\mathfrak{A}_{10}$	2		$+$
26	$2.\mathrm{L}_3(4)$	7		$+$
26	$\mathrm{L}_4(3)$	$\neq 3$		$+$
26	$\mathrm{U}_3(3)$	7		$+$
26	$\mathrm{S}_6(2)$	7		$+$
26	${}^3D_4(2)$	$\neq 2,3$		$+$
26	${}^2F_4(2)'$	2		$+$
26	${}^2F_4(2)'$	$\neq 2$	$i2$	$\circ$
27	$\mathfrak{A}_9$	$\neq 2,7$		$+$
27	$\mathrm{U}_3(3)$	0		$+$
27	$\mathrm{S}_6(2)$	$\neq 2,7$		$+$
27	$3.\mathrm{O}_7(3)$	$\neq 3$	$z3$	$\circ$
27	$3.G_2(3)$	$\neq 3$	$z3$	$\circ$
27	${}^2F_4(2)'$	$\neq 2$	$i1$	$\circ$
27	J_1	11	$b5, c19$	$+$
27	$3.Fi_{22}$	2	$z3, b11$	$\circ$
28	$\mathfrak{A}_8$	$\neq 2,5$		$+$
28	$\mathfrak{A}_9$	$\neq 2,3$		$+$
28	$\mathfrak{A}_{10}$	5		$+$
28	$2.\mathrm{L}_3(4)$	5		$+$
28	$2.\mathrm{L}_3(4)$	$0,7$	$b5$	$+$

d	G	ℓ	field	ind
28	$4_2.\mathrm{L}_3(4)$	5	$i1$	$\circ$
28	$4_2.\mathrm{L}_3(4)$	$\neq 2,5$	$i1, b5$	$\circ$
28	$\mathrm{U}_3(3)$	$0,7$	$i1$	$\circ$
28	$\mathrm{U}_3(5)$	$\neq 5$		$+$
28	$\mathrm{O}_8^+(2)$	$\neq 2$		$+$
28	$2.M_{22}$	5	$i1, r11$	$\circ$
28	$2.HS$	5	$i1, r11$	$\circ$
28	Ru	2		$+$
28	$2.Ru$	$\neq 2$	$i1$	$\circ$
29	$\mathrm{L}_3(5)$	31		$+$
29	$\mathrm{L}_5(2)$	31		$+$
29	M_{12}	11		$+$
30	$\mathrm{L}_3(5)$	$\neq 5,31$		$+$
30	$\mathrm{L}_5(2)$	$\neq 2,31$		$+$
30	$\mathrm{U}_4(2)$	$0,5$		$+$
30	$\mathrm{U}_4(2)$	$0,5$	$z3$	$\circ$
31	$\mathrm{L}_3(5)$	$\neq 2,5$		$+$
31	$\mathrm{L}_3(5)$	$\neq 2,5$	$i1$	$\circ$
31	J_1	7	$c19$	$+$
32	$2.\mathfrak{A}_8$	5	$z3, b7$	$\circ$
32	$2.\mathfrak{A}_{12}$	$\neq 2,3$		$-$
32	$\mathfrak{A}_{13}$	2	$b13$	$+$
32	$2.\mathfrak{A}_{13}$	13		$-$
32	$2.\mathfrak{A}_{13}$	$\neq 2,13$	$b13$	$-$
32	$2.\mathfrak{A}_{14}$	7	$r3, r6, r10, b5, b13, b33$	$-$
32	$\mathrm{U}_3(3)$	$0,2$	$b7$	$\circ$
32	$2.\mathrm{U}_4(2)$	5	$z3$	$\circ$
32	$2.M_{12}$	$\neq 2,3$		$-$
33	$\mathrm{S}_4(4)$	5		$+$
33	$\mathrm{O}_8^-(2)$	7		$+$
34	$\mathfrak{A}_9$	5		$+$
34	$\mathfrak{A}_{10}$	$3,5$		$+$
34	$\mathfrak{A}_{11}$	3		$+$
34	$\mathrm{U}_4(3)$	2		$-$
34	$\mathrm{S}_4(4)$	$\neq 2,5$		$+$
34	$\mathrm{S}_6(2)$	3		$+$
34	$\mathrm{O}_8^-(2)$	$\neq 2,7$		$+$
34	M_{12}	3		$+$
34	M_{22}	2		$-$
34	J_1	19	$b5$	$+$
35	$\mathfrak{A}_7$	$\neq 2,3$		$+$
35	$\mathfrak{A}_8$	$\neq 2$		$+$
35	$\mathfrak{A}_9$	$\neq 2$		$+$
35	$\mathfrak{A}_{10}$	$\neq 2,3$		$+$
35	$\mathfrak{A}_{10}$	5	$r21$	$+$
35	$\mathrm{L}_3(4)$	$\neq 2,3$		$+$
35	$\mathrm{U}_4(3)$	$\neq 2,3$		$+$
35	$\mathrm{S}_6(2)$	$\neq 2$		$+$
35	$\mathrm{S}_8(2)$	$\neq 2$		$+$

d	G	ℓ	field	ind
35	$\mathrm{O}_8^+(2)$	$\neq 2$		$+$
35	$Sz(8)$	13		$+$
35	$Sz(8)$	$\neq 2,13$	c_{13}	$+$
36	$2.\mathfrak{A}_7$	$0,3$		$-$
36	$6.\mathfrak{A}_7$	0	$z3$	$\circ$
36	$\mathfrak{A}_{10}$	$\neq 2,5$		$+$
36	$\mathfrak{A}_{11}$	11		$+$
36	$2.\mathrm{L}_3(4)$	$\neq 2,7$		$+$
36	$4_2.\mathrm{L}_3(4)$	$\neq 2$	$i1$	$\circ$
36	$6.\mathrm{L}_3(4)$	$\neq 2,3$	$z3$	$\circ$
36	$12_2.\mathrm{L}_3(4)$	$\neq 2,3$	$z12$	$\circ$
36	$2.\mathrm{U}_4(2)$	0	$z3$	$\circ$
36	$3_2.\mathrm{U}_4(3)$	$\neq 3$	$z3$	$\circ$
36	$12_2.\mathrm{U}_4(3)$	$\neq 2,3$	$z12$	$\circ$
36	$6.M_{22}$	11	$z12$	$\circ$
36	J_2	$\neq 5$		$+$
36	$2.J_2$	3	$i1$	$\circ$
38	$\mathrm{L}_4(3)$	$2,5$		$+$
39	$\mathrm{L}_4(3)$	$0,13$		$+$
39	$\mathrm{U}_3(4)$	5		$+$
39	$\mathrm{U}_3(4)$	$\neq 2,5$	$b5$	$+$
40	$2.\mathrm{L}_4(3)$	$\neq 2,3$		$+$
40	$4_1.\mathrm{L}_3(4)$	3	$i1$	$\circ$
40	$\mathrm{U}_4(2)$	$0,5$	$z3$	$\circ$
40	$\mathrm{S}_4(5)$	$\neq 5$		$+$
40	$\mathrm{S}_4(9)$	2		$-$
40	$2.\mathrm{S}_4(9)$	$\neq 2,3$		$-$
40	$2.\mathrm{S}_6(2)$	7		$+$
40	$\mathrm{S}_8(3)$	2	$z3$	$\circ$
40	$2.\mathrm{S}_8(3)$	$\neq 2,3$	$z3$	$\circ$
40	$2.Sz(8)$	7		$+$
40	$2.Sz(8)$	$\neq 2,7$	$y7$	$+$
41	$\mathfrak{A}_9$	3		$+$
41	$\mathfrak{A}_{10}$	3		$+$
41	$\mathrm{S}_4(9)$	$\neq 2,3$		$+$
41	$\mathrm{S}_8(3)$	$\neq 2,3$	$z3$	$\circ$
41	J_2	5		$+$
42	$\mathfrak{A}_9$	$0,7$		$+$
42	$\mathfrak{A}_{10}$	$0,7$		$+$
42	$6.\mathrm{L}_3(4)$	$0,7$	$z3,b5$	$\circ$
42	$\mathrm{U}_3(7)$	$\neq 7$		$-$
42	$\mathrm{U}_7(2)$	$\neq 2$		$-$
43	$\mathfrak{A}_8$	5		$+$
43	$\mathfrak{A}_{11}$	5		$+$
43	$\mathfrak{A}_{12}$	5		$+$
43	$\mathrm{U}_3(7)$	$\neq 2,7$		$+$
43	$\mathrm{U}_3(7)$	$\neq 2,7$	$i1$	$\circ$
43	$\mathrm{U}_3(7)$	$\neq 2,7$	$z8$	$\circ$
43	$\mathrm{U}_5(2)$	5		$+$

d	G	ℓ	field	ind
43	$\mathrm{U}_7(2)$	$\neq 2,3$	$z3$	$\circ$
43	J_1	19	$b5$	$+$
44	$\mathfrak{A}_{11}$	$\neq 3,5$		$+$
44	$\mathfrak{A}_{12}$	2		$+$
44	$4_2.\mathrm{L}_3(4)$	7	$i1$	$\circ$
44	$\mathrm{U}_5(2)$	$\neq 2,5$		$+$
44	M_{11}	$\neq 3,5$		$+$
44	M_{12}	2		$+$
44	$2.M_{12}$	$\neq 2,5$	$i5$	$\circ$
44	M_{23}	2	$b7$	$\circ$
44	M_{24}	2	$b7$	$\circ$
45	$\mathfrak{A}_8$	7		$+$
45	$\mathfrak{A}_8$	$\neq 2,7$	$b7$	$\circ$
45	$\mathfrak{A}_{11}$	$\neq 2,11$		$+$
45	$\mathfrak{A}_{12}$	3		$+$
45	$\mathrm{L}_3(4)$	7		$+$
45	$\mathrm{L}_3(4)$	$\neq 2,7$	$b7$	$\circ$
45	$3.\mathrm{L}_3(4)$	$0,5$	$z3,b7$	$\circ$
45	$\mathrm{U}_4(2)$	$0,5$	$z3$	$\circ$
45	$3_2.\mathrm{U}_4(3)$	7	$z3$	$\circ$
45	$3_2.\mathrm{U}_4(3)$	$\neq 3,7$	$z3,b7$	$\circ$
45	M_{11}	$\neq 2,11$		$+$
45	M_{12}	$\neq 2,11$		$+$
45	M_{22}	7		$+$
45	M_{22}	$\neq 2,7$	$b7$	$\circ$
45	$3.M_{22}$	7	$z3$	$\circ$
45	$3.M_{22}$	$\neq 3,7$	$z3,b7$	$\circ$
45	M_{23}	7		$+$
45	M_{23}	$\neq 2,7$	$b7$	$\circ$
45	M_{24}	7		$+$
45	M_{24}	$\neq 2,7$	$b7$	$\circ$
45	J_1	7	$c19$	$+$
45	$3.M^cL$	5	$z3,b7$	$\circ$
45	$3.ON$	7		$\circ$
47	$\mathfrak{A}_9$	7		$+$
48	$2.\mathfrak{A}_8$	$\neq 2$		$-$
48	$\mathfrak{A}_9$	$0,2$		$+$
48	$2.\mathfrak{A}_9$	3		$+$
48	$2.\mathfrak{A}_9$	$\neq 2,3$	$i6$	$\circ$
48	$\mathfrak{A}_{10}$	2		$+$
48	$2.\mathfrak{A}_{10}$	3		$+$
48	$2.\mathfrak{A}_{10}$	$\neq 2,3$	$i6$	$\circ$
48	$12_1.\mathrm{L}_3(4)$	$0,7$	$z12,b5$	$\circ$
48	$12_2.\mathrm{L}_3(4)$	$0,7$	$z12,b5$	$\circ$
48	$12_2.\mathrm{L}_3(4)$	5	$z12$	$\circ$
48	$3.\mathrm{U}_3(5)$	$\neq 3,5$	$z3$	$\circ$
48	$2.\mathrm{S}_6(2)$	$\neq 2,7$		$+$
48	$\mathrm{O}_8^+(2)$	3		$+$
48	$2.Sz(8)$	5	c_{13}	$+$

d	G	ℓ	field	ind
48	$12.M_{22}$	5	$z12, b11$	$\circ$
49	$\mathrm{S}_4(4)$	17		$+$
49	$\mathrm{S}_6(2)$	3		$+$
49	M_{22}	3	$b11$	$\circ$
49	J_1	11	$b5, c19$	$+$
49	HS	3	$i5, b11$	$\circ$
50	$\mathrm{S}_4(4)$	$\neq 2, 17$		$+$
50	$\mathrm{S}_8(2)$	3		$+$
50	$\mathrm{O}_8^+(2)$	$\neq 2, 3$		$+$
50	$\mathrm{O}_8^-(2)$	3		$+$
50	$2.J_2$	3	$b5$	$-$
50	$2.J_2$	$\neq 2, 3$	$i1$	$\circ$
50	He	7		$+$
51	$\mathrm{U}_4(4)$	5		$+$
51	$\mathrm{U}_4(4)$	$\neq 2, 5$	$z5$	$\circ$
51	$\mathrm{S}_4(4)$	$\neq 2, 5$	$b5$	$+$
51	$\mathrm{S}_8(2)$	$\neq 2, 3$		$+$
51	$\mathrm{O}_8^-(2)$	$\neq 2, 3$		$+$
51	He	$\neq 7$	$b7$	$\circ$
52	$\mathrm{L}_4(3)$	$\neq 2, 3$		$+$
52	$\mathrm{U}_3(4)$	$\neq 2, 5$	$z5$	$\circ$
52	$\mathrm{U}_4(4)$	$\neq 2, 5$		$+$
52	$2.\mathrm{S}_4(5)$	$\neq 2, 5$	$b5$	$-$
52	${}^3D_4(2)$	$\neq 2$		$+$
52	$2.F_4(2)$	$\neq 2$		$+$
53	$\mathfrak{A}_{12}$	11		$+$
53	$\mathfrak{A}_{13}$	11		$+$
53	M_{12}	11		$+$
54	$\mathfrak{A}_{12}$	$0, 3, 7$		$+$
54	$6.\mathrm{L}_3(4)$	7	$z3$	$\circ$
54	M_{12}	$0, 3$		$+$
54	M_{22}	7		$+$
54	$6.M_{22}$	7	$z3$	$\circ$
55	$\mathfrak{A}_{10}$	5		$+$
55	$\mathfrak{A}_{11}$	5		$+$
55	$\mathfrak{A}_{12}$	$\neq 2, 3$		$+$
55	$\mathfrak{A}_{13}$	13		$+$
55	$\mathrm{L}_3(7)$	$3, 19$		$+$
55	$\mathrm{U}_5(2)$	$\neq 2$		$+$
55	$\mathrm{U}_5(2)$	$\neq 2, 3$	$z3$	$\circ$
55	M_{11}	$\neq 2, 3$		$+$
55	M_{12}	$\neq 2, 3$		$+$
55	M_{22}	$\neq 2, 7$		$+$
55	J_1	19	$b5$	$+$
55	HS	5		$+$
56	$\mathfrak{A}_8$	$0, 7$		$+$
56	$2.\mathfrak{A}_8$	$0, 7$	$z3$	$\circ$
56	$2.\mathfrak{A}_8$	$0, 7$	$i15$	$\circ$
56	$\mathfrak{A}_9$	$\neq 2, 3$		$+$

d	G	ℓ	field	ind
56	$2.\mathfrak{A}_9$	$\neq 2, 3$		$+$
56	$\mathfrak{A}_{10}$	5		$+$
56	$2.\mathfrak{A}_{10}$	5	$r6, r21$	$+$
56	$2.\mathfrak{A}_{11}$	5	$r6, r21$	$+$
56	$4_1.\mathrm{L}_3(4)$	$\neq 2, 3$	$i1$	$\circ$
56	$\mathrm{L}_3(7)$	$0, 2$		$+$
56	$\mathrm{U}_3(8)$	$\neq 2$		$-$
56	$2.\mathrm{U}_4(3)$	$\neq 2, 3$		$+$
56	$2.\mathrm{U}_6(2)$	$\neq 2$		$+$
56	$\mathrm{S}_6(2)$	$\neq 2, 3$		$+$
56	$2.\mathrm{S}_6(2)$	3	$i5$	$\circ$
56	$2.\mathrm{O}_8^+(2)$	$\neq 2$		$+$
56	$2.Sz(8)$	$\neq 2, 13$	c_{13}	$+$
56	$2.M_{22}$	$\neq 2, 5$		$+$
56	$4.M_{22}$	$\neq 2$	$z8$	$\circ$
56	J_1	2	$b5$	$-$
56	J_1	5		$+$
56	J_1	$\neq 5, 19$	$b5$	$+$
56	$2.J_2$	5		$-$
56	$2.J_2$	$0, 7$	$b5$	$-$
56	HS	2		$+$
56	$2.HS$	$\neq 2, 5$		$+$
57	$\mathrm{L}_3(7)$	$\neq 2, 7$		$+$
57	$3.\mathrm{L}_3(7)$	$\neq 3, 7$	$z3$	$\circ$
57	$\mathrm{U}_3(8)$	$\neq 2, 3$	$z3$	$\circ$
57	$3.\mathrm{U}_3(8)$	$\neq 2, 3$	$z9$	$\circ$
57	J_2	3	$b5$	$+$
58	$\mathrm{U}_4(2)$	5		$+$
58	$2.J_2$	7	$b5$	$-$
60	$4_2.\mathrm{L}_3(4)$	3	$i1, r7$	$\circ$
60	$6.\mathrm{L}_3(4)$	$0, 5$	$z3, b7$	$\circ$
60	$12_2.\mathrm{L}_3(4)$	$0, 5$	$z12, b7$	$\circ$
60	$\mathrm{U}_4(2)$	$0, 5$		$+$
60	$2.\mathrm{U}_4(2)$	$0, 5$		$-$
60	$2.\mathrm{U}_4(2)$	$0, 5$	$z3$	$\circ$
60	$\mathrm{U}_5(3)$	$\neq 3$		$-$
60	$\mathrm{S}_4(11)$	2	$b11$	$\circ$
60	$2.\mathrm{S}_4(11)$	$\neq 2, 11$	$b11$	$\circ$
61	$\mathrm{L}_6(2)$	$3, 7$		$+$
61	$\mathrm{U}_5(3)$	$\neq 2, 3$	$i1$	$\circ$
61	$\mathrm{S}_4(11)$	$\neq 2, 11$	$b11$	$\circ$
62	$\mathrm{L}_6(2)$	$0, 5, 31$		$+$
62	$\mathrm{S}_6(5)$	$\neq 5$	$b5$	$-$
63	$\mathrm{L}_3(4)$	5		$+$
63	$\mathrm{L}_3(4)$	$\neq 2, 5$	$b5$	$+$
63	$3.\mathrm{L}_3(4)$	5	$z3$	$\circ$
63	$3.\mathrm{L}_3(4)$	$0, 7$	$z3, b5$	$\circ$
63	$\mathrm{U}_3(4)$	13		$+$
63	$2.\mathrm{S}_6(5)$	$\neq 2, 5$	$b5$	$+$

d	G	ℓ	field	ind
63	$Sz(8)$	5		$+$
63	J_2	$\neq 2,5$		$+$
64	$\mathfrak{A}_8$	0		$+$
64	$2.\mathfrak{A}_8$	0		$-$
64	$\mathfrak{A}_{10}$	2		$+$
64	$2.\mathfrak{A}_{10}$	$0,7$		$+$
64	$\mathfrak{A}_{13}$	2		$-$
64	$\mathfrak{A}_{13}$	3		$+$
64	$\mathfrak{A}_{14}$	$2,3$		$+$
64	$2.\mathfrak{A}_{14}$	$\neq 2,7$		$-$
64	$\mathfrak{A}_{15}$	2	$b15$	$\circ$
64	$2.\mathfrak{A}_{15}$	$3,5$		$-$
64	$2.\mathfrak{A}_{15}$	$\neq 2,3,5$	$b15$	$\circ$
64	$\mathfrak{A}_{16}$	2	$b7,b15,b39,b55$	$\circ$
64	$\mathrm{L}_3(4)$	0		$+$
64	$2.\mathrm{L}_3(4)$	$0,7$		$+$
64	$4_1.\mathrm{L}_3(4)$	$0,7$	$i1$	$\circ$
64	$4_2.\mathrm{L}_3(4)$	0	$i1$	$\circ$
64	$\mathrm{U}_3(4)$	$0,3$		$+$
64	$\mathrm{U}_4(2)$	0		$+$
64	$2.\mathrm{U}_4(2)$	0		$-$
64	$\mathrm{S}_4(5)$	2		$-$
64	$\mathrm{S}_4(5)$	3		$+$
64	$2.\mathrm{S}_6(2)$	5		$+$
64	$2.\mathrm{S}_6(2)$	$0,7$	$i5$	$\circ$
64	$Sz(8)$	$0,7$		$+$
64	$2.Sz(8)$	$0,7$		$+$
64	$G_2(3)$	$\neq 3$	$z3$	$\circ$
64	$G_2(4)$	3		$+$
64	$2.M_{22}$	11		$+$
64	$4.M_{22}$	3	$i1$	$\circ$
64	J_1	11	$b5,c19$	$+$
64	J_2	2	$b5$	$+$
64	$2.J_2$	5		$-$
64	$2.J_2$	0	$b5$	$-$
64	Suz	3		$+$
65	$\mathfrak{A}_{13}$	$\neq 2,3,11$		$+$
65	$\mathrm{L}_4(3)$	$\neq 2,3$		$+$
65	$\mathrm{U}_3(4)$	$0,13$	$z5$	$\circ$
65	$\mathrm{U}_3(4)$	$\neq 2,3$		$+$
65	$\mathrm{S}_4(5)$	$0,13$		$+$
65	$Sz(8)$	$\neq 2,7$	$y7$	$+$
65	$G_2(4)$	$\neq 2,3$		$+$
66	$\mathfrak{A}_{10}$	7		$+$
66	$\mathfrak{A}_{11}$	7		$+$
66	$\mathfrak{A}_{13}$	$\neq 2,13$		$+$
66	$\mathfrak{A}_{14}$	7		$+$
66	$\mathrm{U}_5(2)$	$\neq 2,3$	$z3$	$\circ$
66	M_{12}	$\neq 2,3$		$+$

d	G	ℓ	field	ind
66	$6.M_{22}$	7	$z3$	$\circ$
66	$6.M_{22}$	$0,5,11$	$z3,b7$	$\circ$
66	$3.Suz$	$\neq 3$	$z3$	$\circ$
69	J_1	11	$b5,c19$	$+$
70	$\mathfrak{A}_8$	$\neq 2,3$		$+$
70	$2.\mathrm{L}_3(4)$	$\neq 2,3$		$+$
70	$\mathrm{U}_4(3)$	2	$z3$	$\circ$
70	$2.\mathrm{U}_4(3)$	$\neq 2,3$		$+$
70	$2.\mathrm{U}_4(3)$	$\neq 2,3$	$z3$	$\circ$
70	$\mathrm{S}_6(2)$	$\neq 2,3$		$+$
70	M_{22}	2	$b11$	$\circ$
70	J_2	5		$+$
70	J_2	$0,7$	$b5$	$+$
71	$\mathrm{L}_3(8)$	73		$+$
72	$2.\mathfrak{A}_9$	7	$z3,r2$	$\circ$
72	$\mathrm{L}_3(8)$	$\neq 2,73$		$+$
72	$\mathrm{U}_3(9)$	$\neq 3$		$-$
73	$\mathrm{L}_3(8)$	$\neq 2,7$	$z7$	$\circ$
73	$\mathrm{U}_3(9)$	$\neq 2,3$		$+$
73	$\mathrm{U}_3(9)$	$\neq 3,5$	$z5$	$\circ$
75	$\mathfrak{A}_{10}$	$0,5$		$+$
75	$\mathrm{U}_3(4)$	$\neq 2,13$	$d13$	$\circ$
75	J_1	7		$+$
76	$\mathfrak{A}_{14}$	13		$+$
76	$\mathfrak{A}_{15}$	13		$+$
76	J_1	2		$-$
76	J_1	$\neq 7,11$		$+$
77	$\mathfrak{A}_{14}$	$\neq 2,3,13$		$+$
77	${}^2F_4(2)'$	3		$+$
77	J_1	$\neq 2,3,19$		$+$
77	J_1	$\neq 2,5$	$b5$	$+$
77	HS	$\neq 2,5$		$+$
77	Fi_{22}	3		$+$
78	$\mathfrak{A}_9$	2		$+$
78	$\mathfrak{A}_{14}$	$\neq 2,7$		$+$
78	$\mathfrak{A}_{15}$	$3,5$		$+$
78	$6_1.\mathrm{U}_4(3)$	5	$z3$	$\circ$
78	$\mathrm{S}_4(5)$	$\neq 2,5$	$b5$	$+$
78	$\mathrm{S}_6(3)$	$\neq 3$		$+$
78	$\mathrm{O}_7(3)$	$\neq 3$		$+$
78	$G_2(3)$	$\neq 3$		$+$
78	$G_2(4)$	$\neq 2$		$+$
78	${}^2F_4(2)'$	$\neq 2,3$		$+$
78	M_{12}	5		$+$
78	$3.M_{22}$	5	$z3$	$\circ$
78	J_3	2	$b5,b17$	$+$
78	Suz	3		$+$
78	$3.Suz$	$\neq 2,3$	$z3$	$\circ$
78	Fi_{22}	2		$+$

d	G	ℓ	field	ind
78	Fi_{22}	$\neq 2,3$		$+$
80	$4_1.\mathrm{L}_3(4)$	$0,5$	$i1,r7$	$\circ$
80	$4_2.\mathrm{L}_3(4)$	$0,5$	$i1,r7$	$\circ$
80	$2.\mathrm{U}_4(2)$	$0,5$		$-$
80	J_3	2	$b17$	$+$
81	$\mathrm{U}_4(2)$	0		$+$
83	$\mathfrak{A}_9$	5		$+$
83	$\mathrm{L}_4(4)$	$5,17$		$+$
83	$\mathrm{S}_6(2)$	5		$+$
83	$\mathrm{O}_8^+(2)$	5		$+$
83	$\mathrm{O}_8^-(2)$	17		$+$
84	$\mathfrak{A}_9$	$0,7$		$+$
84	$\mathfrak{A}_{10}$	$\neq 2,5$		$+$
84	$\mathfrak{A}_{11}$	11		$+$
84	$3.\mathrm{L}_3(4)$	$0,7$	$z3$	$\circ$
84	$12_2.\mathrm{L}_3(4)$	$0,7$	$z12$	$\circ$
84	$\mathrm{L}_4(4)$	$0,3,7$		$+$
84	$\mathrm{U}_3(5)$	$\neq 2,5$		$+$
84	$3.\mathrm{U}_3(5)$	$\neq 3,5$	$z3$	$\circ$
84	$3_1.\mathrm{U}_4(3)$	2	$z3$	$\circ$
84	$6_1.\mathrm{U}_4(3)$	$0,7$	$z3$	$\circ$
84	$12_1.\mathrm{U}_4(3)$	$\neq 2,3$	$z12$	$\circ$
84	$\mathrm{S}_4(13)$	2	$b13$	$-$
84	$2.\mathrm{S}_4(13)$	$\neq 2,13$	$b13$	$-$
84	$\mathrm{S}_6(2)$	$0,7$		$+$
84	$\mathrm{O}_8^+(2)$	$0,7$		$+$
84	$\mathrm{O}_8^-(2)$	$0,5,7$		$+$
84	$2.M_{12}$	3	$i5,b11$	$\circ$
84	$3.M_{22}$	11	$z3$	$\circ$
84	$3.M_{22}$	2	$z3,b11$	$\circ$
84	J_2	2		$+$
84	$2.J_2$	$0,7$		$-$
84	J_3	$2,3$	$b19$	$\circ$
85	$\mathrm{L}_4(4)$	$\neq 2,3$	$z3$	$\circ$
85	$\mathrm{U}_8(2)$	3		$+$
85	$\mathrm{U}_8(2)$	$\neq 2,3$	$z3$	$\circ$
85	$\mathrm{S}_4(4)$	$\neq 2,3$		$+$
85	$\mathrm{S}_4(13)$	$\neq 2,13$	$b13$	$+$
85	$\mathrm{S}_8(2)$	$\neq 2,3$		$+$
85	J_2	5		$+$
85	J_3	19		$+$
85	J_3	$0,5,17$	$b19$	$\circ$
86	$\mathrm{U}_8(2)$	$\neq 2,3$		$+$
88	$4.M_{22}$	5	$z8,b7$	$\circ$
89	$\mathfrak{A}_{10}$	7		$+$
89	$\mathfrak{A}_{11}$	5		$+$
89	$\mathfrak{A}_{12}$	5		$+$
89	$\mathfrak{A}_{15}$	7		$+$

d	G	ℓ	field	ind
89	$\mathfrak{A}_{16}$	7		$+$
89	$\mathrm{L}_3(9)$	$7,13$		$+$
89	$\mathrm{L}_4(3)$	13		$+$
89	$\mathrm{U}_4(3)$	7		$+$
89	$\mathrm{S}_4(5)$	13		$+$
89	J_1	7	$c19$	$+$
89	J_2	7		$+$
90	$\mathfrak{A}_{10}$	$0,3$		$+$
90	$\mathfrak{A}_{15}$	$\neq 7,13$		$+$
90	$\mathfrak{A}_{16}$	2		$+$
90	$2.\mathrm{L}_3(4)$	$\neq 2,7$		$+$
90	$6.\mathrm{L}_3(4)$	$0,5$	$z3$	$\circ$
90	$\mathrm{L}_3(9)$	$0,2,5$		$+$
90	$\mathrm{L}_4(3)$	$0,5$		$+$
90	$\mathrm{U}_4(3)$	$0,5$		$+$
90	$3_1.\mathrm{U}_4(3)$	2	$z3$	$\circ$
90	$6_2.\mathrm{U}_4(3)$	$\neq 2,3$	$z3$	$\circ$
90	$\mathrm{S}_4(5)$	$0,3$		$+$
90	$\mathrm{O}_7(3)$	2		$+$
90	$G_2(3)$	2		$+$
90	$6.M_{22}$	11	$z12$	$\circ$
90	J_2	$\neq 2,7$		$+$
91	$\mathfrak{A}_{15}$	$\neq 2,3,5$		$+$
91	$\mathrm{L}_3(9)$	$\neq 2,3$		$+$
91	$\mathrm{L}_3(9)$	$\neq 2,3$	$i1$	$\circ$
91	$\mathrm{L}_3(9)$	$\neq 2,3$	$z8$	$\circ$
91	$\mathrm{S}_6(2)$	3		$+$
91	$\mathrm{S}_6(3)$	$\neq 2,3$	$z3$	$\circ$
91	$\mathrm{O}_7(3)$	$\neq 2,3$		$+$
91	$Sz(8)$	$\neq 2,5$		$+$
91	$G_2(3)$	$\neq 2,3$		$+$
91	M_{12}	11		$+$
92	$2.G_2(4)$	5		$-$
94	$\mathrm{L}_5(2)$	7		$+$
94	$\mathrm{S}_6(2)$	7		$+$
96	$\mathrm{L}_3(5)$	31		$+$
96	$\mathrm{L}_3(5)$	$\neq 5,31$	$x31$	$\circ$
96	$\mathrm{L}_3(7)$	3		$+$
96	$3.\mathrm{L}_3(7)$	$\neq 3,7$	$z3$	$\circ$
96	$12.M_{22}$	11	$z12$	$\circ$
98	$\mathrm{S}_6(2)$	3		$+$
98	M_{12}	5		$+$
98	M_{22}	2		$-$
98	M_{22}	5		$+$
98	HS	5		$+$
99	M_{12}	$\neq 2,5$		$+$
99	M_{22}	$0,3,11$		$+$
99	$3.M_{22}$	$0,7,11$	$z3$	$\circ$

d	G	ℓ	field	ind
100	$\mathfrak{A}_{11}$	2		$+$
100	$\mathfrak{A}_{12}$	2		$+$
100	$\mathrm{U}_5(2)$	3		$+$
101	$\mathfrak{A}_9$	7		$+$
101	$\mathfrak{A}_{10}$	7		$+$
101	J_2	7		$+$
101	He	2	$b7$	$\circ$
103	$\mathfrak{A}_{16}$	$3,5$		$+$
103	$\mathfrak{A}_{17}$	$3,5$		$+$
103	$G_2(3)$	7		$+$
104	$2.\mathfrak{A}_9$	3		$+$
104	$\mathfrak{A}_{16}$	$0,11,13$		$+$
104	$\mathrm{U}_3(5)$	2		$-$
104	$\mathrm{U}_4(5)$	2		$-$
104	$\mathrm{U}_4(5)$	3		$+$
104	$\mathrm{U}_4(5)$	$\neq 3,5$	$z3$	$\circ$
104	$2.\mathrm{U}_4(5)$	$0,7,13$	$z3$	$\circ$
104	$2.\mathrm{U}_4(5)$	$\neq 2,5$		$-$
104	$\mathrm{S}_4(5)$	$\neq 5$		$+$
104	$2.\mathrm{S}_4(5)$	$0,13$		$-$
104	$2.\mathrm{S}_6(2)$	$3,5$		$+$
104	$\mathrm{O}_7(3)$	2		$+$
104	$2.\mathrm{O}_8^+(2)$	$3,5$		$+$
104	$2.Sz(8)$	$0,13$		$+$
104	$G_2(3)$	$0,13$		$+$
104	$2.G_2(4)$	$\neq 2,5$	$b5$	$-$
104	M_{23}	3	$b11,b23$	$\circ$
104	M^cL	3	$b11$	$\circ$
104	He	5	$r21$	$+$
105	$\mathfrak{A}_9$	$\neq 2,3$		$+$
105	$\mathfrak{A}_{16}$	$\neq 2$		$+$
105	$\mathfrak{A}_{17}$	17		$+$
105	$\mathrm{U}_3(5)$	$0,7$		$+$
105	$3.\mathrm{U}_3(5)$	$0,7$	$z3$	$\circ$
105	$3_1.\mathrm{U}_4(3)$	$\neq 2,3$	$z3$	$\circ$
105	$\mathrm{U}_4(5)$	$0,7,13$		$+$
105	$\mathrm{S}_6(2)$	$\neq 2,3$		$+$
105	$\mathrm{S}_6(3)$	$\neq 2,3$		$+$
105	$\mathrm{O}_7(3)$	$\neq 2,3$		$+$
105	$3.M_{22}$	$0,5,7$	$z3,b11$	$\circ$
106	J_1	11	$b5,c19$	$+$
108	$2.M_{12}$	11		$+$
109	$\mathfrak{A}_{11}$	3		$+$
109	${}^2F_4(2)'$	5	$r2,r3,b13$	$+$
110	$\mathfrak{A}_{11}$	$0,5,11$		$+$
110	$\mathrm{U}_3(11)$	$\neq 11$		$-$
110	$\mathrm{U}_5(2)$	$\neq 2$		$-$
110	$\mathrm{U}_5(2)$	$\neq 2,3$	$z3$	$\circ$
110	$2.M_{12}$	$\neq 2,3$	$i2$	$\circ$

d	G	ℓ	field	ind
110	J_3	19	$b17,y9$	$+$
110	Suz	2	$b5,b13,r21$	$+$
111	$\mathrm{U}_3(11)$	$\neq 2,11$		$+$
111	$\mathrm{U}_3(11)$	$\neq 2,11$	$i1$	$\circ$
111	$3.\mathrm{U}_3(11)$	$\neq 3,11$	$z3$	$\circ$
111	$3.\mathrm{U}_3(11)$	$0,5,37$	$z12$	$\circ$
111	Ly	5		$+$
112	$2.\mathfrak{A}_9$	$0,7$		$+$
112	$2.\mathrm{S}_6(2)$	$0,7$		$+$
112	$2.\mathrm{O}_8^+(2)$	$0,7$		$+$
112	J_4	2		$+$
114	$6_1.\mathrm{U}_4(3)$	7	$z3$	$\circ$
115	$\mathfrak{A}_9$	7		$+$
118	$\mathfrak{A}_{17}$	2		$+$
118	$\mathfrak{A}_{18}$	2		$+$
118	$\mathrm{S}_8(2)$	$3,5$		$+$
119	$\mathfrak{A}_{17}$	$\neq 2,3,5$		$+$
119	$\mathrm{L}_5(3)$	11		$+$
119	$\mathrm{U}_5(2)$	11		$+$
119	$\mathrm{S}_8(2)$	$0,7,17$		$+$
119	J_1	11	$c19$	$+$
120	$\mathfrak{A}_9$	$0,5$		$+$
120	$2.\mathfrak{A}_9$	$0,5$	$z3$	$\circ$
120	$\mathfrak{A}_{11}$	$\neq 2,11$		$+$
120	$\mathfrak{A}_{12}$	3		$+$
120	$\mathfrak{A}_{17}$	$\neq 2,17$		$+$
120	$\mathfrak{A}_{18}$	3		$+$
120	$12_1.\mathrm{L}_3(4)$	$0,5$	$z12$	$\circ$
120	$\mathrm{L}_5(3)$	$\neq 3,11$		$+$
120	$\mathrm{U}_4(3)$	2		$+$
120	$2.\mathrm{U}_4(3)$	$\neq 2,3$		$+$
120	$4.\mathrm{U}_4(3)$	$\neq 2,3$	$i1$	$\circ$
120	$6_1.\mathrm{U}_4(3)$	$0,5$	$z3$	$\circ$
120	$12_1.\mathrm{U}_4(3)$	$\neq 2,3$	$z12$	$\circ$
120	$\mathrm{U}_5(2)$	$0,5$		$+$
120	$2.\mathrm{U}_6(2)$	3		$+$
120	$6.\mathrm{U}_6(2)$	$\neq 2,3$	$z3$	$\circ$
120	$\mathrm{S}_6(2)$	$0,5$		$+$
120	$2.\mathrm{S}_6(2)$	$\neq 2,3$		$+$
120	M_{12}	$0,5$		$+$
120	$2.M_{12}$	$0,5$		$+$
120	$2.M_{22}$	$\neq 2,11$		$+$
120	$6.M_{22}$	$0,5,11$	$z3$	$\circ$
120	$12.M_{22}$	$\neq 2,3$	$z24$	$\circ$
120	M_{23}	2		$+$
120	M_{24}	2		$+$
120	J_1	$\neq 11,19$	$c19$	$+$
120	$2.HS$	5	$i1$	$\circ$
121	$\mathrm{L}_5(3)$	$\neq 2,3$		$+$

d	G	ℓ	field	ind	d	G	ℓ	field	ind
121	$S_{10}(3)$	$\neq 3$	$z3$	$\circ$	131	$\mathfrak{A}_{11}$	$3,7$		$+$
122	$2.S_{10}(3)$	$\neq 2,3$	$z3$	$\circ$	131	$\mathfrak{A}_{12}$	$3,7$		$+$
123	$L_5(2)$	5		$+$	131	$L_3(11)$	$7,19$		$+$
124	$\mathfrak{A}_{10}$	7		$+$	132	$\mathfrak{A}_{11}$	$0,11$		$+$
124	$L_3(5)$	$\neq 5$		$+$	132	$\mathfrak{A}_{12}$	$0,11$		$+$
124	$L_3(5)$	2		$-$	132	$L_3(11)$	$0,2,3,5$		$+$
124	$L_3(5)$	$\neq 2,5$	$i1$	$\circ$	132	$12_1.U_4(3)$	5	$z12$	$\circ$
124	$L_3(5)$	$0,31$	$y24'$	$\circ$	132	HS	2		$-$
124	$L_5(2)$	$0,3,31$		$+$	132	HN	2	$b5$	$+$
124	$U_3(5)$	7		$+$	133	$\mathfrak{A}_9$	5		$+$
124	$Sz(32)$	$\neq 2$	$i1$	$\circ$	133	$\mathfrak{A}_{10}$	5	$r21$	$+$
124	$G_2(5)$	$\neq 5$		$+$	133	$\mathfrak{A}_{11}$	5	$r21$	$+$
124	${}^2F_4(2)'$	3	$b13$	$+$	133	$L_3(11)$	$\neq 2,11$		$+$
124	J_2	7		$+$	133	$L_3(11)$	$\neq 5,11$	$z5$	$\circ$
125	$L_3(5)$	0		$+$	133	$U_3(8)$	$\neq 2$		$+$
125	$L_7(2)$	127		$+$	133	$S_6(2)$	5		$+$
125	$U_3(5)$	0		$+$	133	M_{22}	5		$+$
126	$\mathfrak{A}_{10}$	$\neq 2,5$		$+$	133	J_1	$\neq 2,11$		$+$
126	$\mathfrak{A}_{11}$	11		$+$	133	J_1	$0,7,19$	$b5$	$+$
126	$\mathfrak{A}_{11}$	$\neq 2,11$	$b11$	$\circ$	133	J_2	3		$+$
126	$\mathfrak{A}_{12}$	3	$b11,i35$	$\circ$	133	HS	5		$+$
126	$L_7(2)$	$\neq 2,127$		$+$	133	Ru	5		$+$
126	$U_3(5)$	$\neq 2,5$		$+$	133	HN	5		$+$
126	$U_3(5)$	$\neq 2,5$	$i2$	$\circ$	133	HN	$\neq 2,5$	$b5$	$+$
126	$3.U_3(5)$	$0,7$	$z3$	$\circ$	134	$\mathfrak{A}_9$	5		$+$
126	$3.U_3(5)$	$0,7$	$z3,i2$	$\circ$	134	$\mathfrak{A}_{18}$	17		$+$
126	$3_2.U_4(3)$	$\neq 2,3$	$z3$	$\circ$	134	$\mathfrak{A}_{19}$	17		$+$
126	$6_1.U_4(3)$	$\neq 2,3$	$z3$	$\circ$	134	$S_8(2)$	17		$+$
126	$6_2.U_4(3)$	$\neq 2,3$	$z3$	$\circ$	135	$\mathfrak{A}_{18}$	$\neq 2,17$		$+$
126	$6_2.U_4(3)$	$\neq 2,3$	$z12$	$\circ$	135	$S_8(2)$	$\neq 2,17$		$+$
126	$S_4(7)$	2		$+$	136	$\mathfrak{A}_{18}$	$\neq 2,3$		$+$
126	$S_4(7)$	$\neq 2,7$		$+$	136	$\mathfrak{A}_{19}$	19		$+$
126	$2.M_{22}$	11		$+$	140	$U_4(3)$	$\neq 2,3$		$+$
126	$2.M_{22}$	$\neq 2,11$	$b11$	$\circ$	140	$4.U_4(3)$	$\neq 2,3$	$i1$	$\circ$
126	$6.M_{22}$	$0,5,7$	$z3,b11$	$\circ$	141	$S_6(2)$	5		$+$
126	J_2	$0,7$		$+$	142	Suz	2		$+$
126	$2.J_2$	$\neq 2,5$	$b5$	$-$	143	$\mathfrak{A}_{12}$	3		$+$
126	$3.J_3$	2	$z3,b17,b19$	$\circ$	143	$\mathfrak{A}_{13}$	3		$+$
126	$3.M^cL$	11	$z3$	$\circ$	143	Suz	$\neq 2,3$		$+$
126	$3.M^cL$	$\neq 3,11$	$z3,b11$	$\circ$	144	$\mathfrak{A}_{11}$	2		$+$
126	Co_3	3	$i5,b11,b23$	$\circ$	144	$2.\mathfrak{A}_{11}$	$0,3,7$		$+$
128	$2.\mathfrak{A}_{11}$	11		$+$	144	$\mathfrak{A}_{12}$	5		$+$
128	$2.\mathfrak{A}_{12}$	11		$+$	144	$\mathfrak{A}_{12}$	2	$i35,z3$	$\circ$
128	$2.\mathfrak{A}_{16}$	$\neq 2$		$+$	144	$2.\mathfrak{A}_{12}$	3	$i2,i5,r7$	$\circ$
128	$\mathfrak{A}_{17}$	2	$b17$	$+$	144	$\mathfrak{A}_{13}$	5		$+$
128	$2.\mathfrak{A}_{17}$	17		$+$	144	$\mathfrak{A}_{13}$	2	$i35,z3$	$\circ$
128	$2.\mathfrak{A}_{17}$	$\neq 2,17$	$b17$	$+$	144	$2.\mathfrak{A}_{13}$	3	$i2,i5,r7$	$\circ$
128	$2.\mathfrak{A}_{18}$	3	$r2,r5,r14,b17,b65,b77$	$+$	144	$U_3(5)$	$\neq 5,7$	$b7$	$\circ$
130	$S_4(5)$	$0,13$		$+$	144	$3.U_3(5)$	$0,2$	$z3,b7$	$\circ$

d	G	ℓ	field	ind
144	$S_4(17)$	2	$b17$	$-$
144	$2.S_4(17)$	$\neq 2,17$	$b17$	$-$
144	M_{12}	$0,2$		$+$
144	$4.M_{22}$	7	$i1$	$\circ$
144	$4.M_{22}$	$0,3,11$	$i1,r7$	$\circ$
144	$12.M_{22}$	7	$z12$	$\circ$
144	$12.M_{22}$	$0,5,11$	$z12,b7$	$\circ$
145	$S_4(17)$	$\neq 2,17$	$b17$	$+$
147	$O_8^+(2)$	3		$+$
150	$3_1.U_4(3)$	2	$z3,b7$	$\circ$
150	$S_4(7)$	$\neq 2,7$	$b7$	$\circ$
151	$\mathfrak{A}_{19}$	3		$+$
151	$\mathfrak{A}_{20}$	3		$+$
152	$2.\mathfrak{A}_{10}$	7		$+$
152	$\mathfrak{A}_{19}$	$\neq 3,17$		$+$
152	$\mathfrak{A}_{20}$	2		$+$
152	$L_3(7)$	$\neq 3,7$		$+$
152	$2.O_8^+(2)$	7		$+$
153	$\mathfrak{A}_{12}$	5		$+$
153	$\mathfrak{A}_{19}$	$\neq 2,19$		$+$
153	$\mathfrak{A}_{20}$	5		$+$
153	$3_2.U_4(3)$	5	$z3$	$\circ$
153	$S_4(4)$	$\neq 2$		$+$
153	$O_{10}^-(2)$	$3,5$		$+$
153	$3.M_{22}$	5	$z3$	$\circ$
153	J_3	3	$b5$	$+$
153	$3.J_3$	5	$z3$	$\circ$
153	$3.J_3$	$\neq 3,5$	$z3,b5$	$\circ$
153	$3.M^cL$	5	$z3,b7$	$\circ$
153	He	7		$+$
153	He	$\neq 2,7$	$b7$	$\circ$
153	$3.ON$	2	$z3$	$\circ$
154	$\mathfrak{A}_{12}$	$0,7,11$		$+$
154	$L_4(5)$	$2,3,13$		$+$
154	$O_8^-(2)$	3		$+$
154	$O_{10}^-(2)$	$\neq 2,3,5$		$+$
154	M_{22}	$0,7,11$		$+$
154	$2.M_{22}$	$\neq 2,5$	$i1$	$\circ$
154	HS	$\neq 2,5$		$+$
154	ON	3	$r7$	$+$
155	$\mathfrak{A}_{10}$	5		$+$
155	$\mathfrak{A}_{11}$	7		$+$
155	$L_3(5)$	$0,31$		$+$
155	$L_3(5)$	$0,31$	$i1$	$\circ$
155	$L_4(5)$	$0,31$		$+$
155	$L_5(2)$	$\neq 2$		$+$
155	$S_{10}(2)$	$\neq 2$		$+$
155	$O_{10}^+(2)$	$\neq 2$		$+$
156	$2.L_4(5)$	$\neq 2,5$		$+$
156	$4.L_4(5)$	$\neq 2,5$	$i1$	$\circ$
156	$U_3(13)$	$\neq 13$		$-$
156	$S_4(5)$	$\neq 2,5$		$+$
156	$2.S_4(5)$	$\neq 2,5$		$-$
157	$U_3(13)$	$\neq 2,13$		$+$
157	$U_3(13)$	$\neq 7,13$	$z7$	$\circ$
160	$\mathfrak{A}_9$	2		$+$
160	$2.\mathfrak{A}_9$	$0,5$		$+$
160	$\mathfrak{A}_{10}$	$\neq 3,7$		$+$
160	$2.\mathfrak{A}_{10}$	5	$r6,r21$	$+$
160	$2.\mathfrak{A}_{12}$	$0,5,7$	$b11$	$\circ$
160	$2.O_8^+(2)$	$0,5$		$+$
160	$2.M_{12}$	$0,5$	$b11$	$\circ$
160	$4.M_{22}$	11	$i1$	$\circ$
160	$4.M_{22}$	$\neq 2,11$	$i1,r11$	$\circ$
160	J_2	$0,2$		$+$
162	$\mathfrak{A}_9$	$0,3$		$+$
162	$3.G_2(3)$	2	$z3,b13$	$\circ$
164	$\mathfrak{A}_{11}$	2		$-$
164	$\mathfrak{A}_{12}$	2		$-$
165	$\mathfrak{A}_{11}$	$0,11$		$+$
165	$\mathfrak{A}_{12}$	$\neq 2,3$		$+$
165	$\mathfrak{A}_{13}$	13		$+$
165	$U_5(2)$	$\neq 2,3$		$+$
167	$S_6(3)$	13		$+$
167	$O_7(3)$	13		$+$
167	$G_2(3)$	13		$+$
168	$\mathfrak{A}_9$	$0,7$		$+$
168	$2.\mathfrak{A}_9$	5		$+$
168	$2.\mathfrak{A}_9$	$0,7$	$i15$	$\circ$
168	$2.\mathfrak{A}_{10}$	5	$r21$	$+$
168	$S_6(2)$	$\neq 2,3$		$+$
168	$2.S_6(2)$	$\neq 2,3$		$+$
168	$S_6(3)$	$\neq 3,13$		$+$
168	$O_7(3)$	$0,5,7$		$+$
168	$2.O_8^+(2)$	5		$+$
168	$G_2(3)$	$0,7$		$+$
169	$\mathfrak{A}_{20}$	19		$+$
169	$\mathfrak{A}_{21}$	19		$+$
170	$\mathfrak{A}_{20}$	$\neq 2,3,19$		$+$
170	$U_9(2)$	$\neq 2$		$-$
171	$\mathfrak{A}_{20}$	$\neq 2,5,$		$+$
171	$\mathfrak{A}_{21}$	$3,7$		$+$
171	$3.U_9(2)$	$\neq 2,3$	$z3$	$\circ$
171	$S_6(7)$	$\neq 7$	$b7$	$\circ$
171	$O_8^-(2)$	7		$+$
171	$3.J_3$	$\neq 2,3$	$z3$	$\circ$
171	$3.J_3$	$0,17,19$	$z3,b5$	$\circ$
172	$2.S_6(7)$	$\neq 2,7$	$b7$	$\circ$

d	G	ℓ	field	ind
174	$\mathrm{S}_4(7)$	2		$+$
174	$6.M_{22}$	11	$z12$	$\circ$
174	HS	11		$+$
175	$\mathrm{S}_4(7)$	$\neq 2,7$		$+$
175	$\mathrm{O}_8^+(2)$	$\neq 2,3$		$+$
175	J_2	$\neq 2,3$		$+$
175	HS	$0,5,7$		$+$
176	$\mathrm{U}_5(2)$	$\neq 2,3$		$+$
176	$2.\mathrm{U}_6(2)$	$\neq 2,3$		$+$
176	M_{12}	$0,11$		$+$
176	$4.M_{22}$	$0,11$	$i1$	$\circ$
176	$2.HS$	$\neq 2,5$	$i1$	$\circ$
176	$2.Fi_{22}$	3	$b13$	$+$
180	$\mathrm{S}_4(19)$	2	$b19$	$\circ$
180	$2.\mathrm{S}_4(19)$	$\neq 2,19$	$b19$	$\circ$
181	$\mathrm{L}_3(13)$	$3,61$		$+$
181	$\mathrm{S}_4(19)$	$\neq 2,19$	$b19$	$\circ$
182	$\mathrm{L}_3(13)$	$0,2,7$		$+$
182	$\mathrm{U}_6(3)$	$\neq 3$		$-$
182	$2.\mathrm{U}_6(3)$	$\neq 2,3$	$i1$	$\circ$
182	$2.\mathrm{S}_6(3)$	$\neq 2,3$		$-$
182	$2.\mathrm{S}_6(3)$	$\neq 2,3$	$z3$	$\circ$
182	$\mathrm{O}_7(3)$	$\neq 2,3$		$+$
182	$G_2(3)$	$\neq 2,3$		$+$
183	$\mathrm{L}_3(13)$	$\neq 2,13$		$+$
183	$\mathrm{L}_3(13)$	$\neq 2,13$	$i1$	$\circ$
183	$3.\mathrm{L}_3(13)$	$0,7,61$	$z12$	$\circ$
183	$3.\mathrm{L}_3(13)$	$\neq 3,13$	$z3$	$\circ$
183	$\mathrm{U}_6(3)$	$\neq 2,3$		$+$
185	$\mathrm{O}_{10}^+(2)$	$3,17$		$+$
186	$\mathfrak{A}_{11}$	2		$+$
186	$\mathrm{L}_3(5)$	$\neq 2,5$		$+$
186	$\mathrm{S}_{10}(2)$	3		$+$
186	$\mathrm{O}_{10}^+(2)$	$\neq 2,3,17$		$+$
186	$\mathrm{O}_{10}^-(2)$	3		$+$
187	$\mathrm{S}_{10}(2)$	$\neq 2,3$		$+$
187	$\mathrm{O}_{10}^-(2)$	$\neq 2,3$		$+$
188	$\mathfrak{A}_{11}$	5		$+$
188	$\mathfrak{A}_{21}$	$2,5$		$+$
188	$\mathfrak{A}_{22}$	$2,5$		$+$
188	$\mathrm{U}_4(3)$	5		$+$
189	$\mathfrak{A}_9$	$\neq 2,5$		$+$
189	$\mathfrak{A}_{21}$	$\neq 2,5,19$		$+$
189	$\mathrm{L}_4(4)$	5		$+$
189	$\mathrm{L}_4(4)$	$\neq 2,5$	$b5$	$+$
189	$3.\mathrm{U}_3(8)$	$\neq 2,3$	$z3$	$\circ$
189	$\mathrm{U}_4(3)$	$0,7$		$+$
189	$3_2.\mathrm{U}_4(3)$	$\neq 3,5$	$z3$	$\circ$
189	$\mathrm{S}_6(2)$	$\neq 2,5$		$+$

d	G	ℓ	field	ind
189	$3.G_2(3)$	13	$z3$	$\circ$
189	$3.G_2(3)$	$0,7$	$z3,b13$	$\circ$
189	J_2	5		$+$
189	J_2	$\neq 2,5$	$b5$	$+$
190	$\mathfrak{A}_{21}$	$\neq 2,3,7$		$+$
190	$\mathfrak{A}_{22}$	11		$+$
190	M_{22}	11		$+$
190	$2.J_2$	5		$-$
194	$\mathrm{S}_6(3)$	7		$+$
194	$\mathrm{O}_7(3)$	7		$+$
195	$\mathrm{S}_6(3)$	$0,5,13$		$+$
195	$\mathrm{O}_7(3)$	$0,5,13$		$+$
196	$\mathfrak{A}_{13}$	5		$+$
196	$\mathfrak{A}_{14}$	5		$+$
196	$\mathrm{S}_4(8)$	$\neq 2$		$+$
196	$\mathrm{S}_6(2)$	3		$+$
196	${}^3D_4(2)$	$\neq 2$		$+$
198	$\mathfrak{A}_{10}$	2		$+$
198	$\mathfrak{A}_{11}$	2		$+$
199	$\mathfrak{A}_{10}$	7		$+$
199	$\mathfrak{A}_{11}$	7		$+$
199	J_2	7		$+$
200	$\mathfrak{A}_{10}$	2		$+$
200	$2.\mathrm{S}_4(7)$	$\neq 2,7$	$b7$	$\circ$
201	$\mathrm{S}_6(2)$	7		$+$
202	$2.J_2$	5		$-$
203	$\mathrm{S}_8(2)$	3		$+$
203	$\mathrm{O}_8^-(2)$	$3,5$		$+$
204	$3_1.\mathrm{U}_4(3)$	2	$z3$	$\circ$
204	$\mathrm{U}_5(4)$	$\neq 2$		$\circ$
204	$\mathrm{S}_4(4)$	$\neq 2,5$	$b5$	$+$
204	$\mathrm{O}_8^-(2)$	$\neq 2,3$		$+$
205	$5.\mathrm{U}_5(4)$	$\neq 5$	$z5$	$\circ$
207	$\mathfrak{A}_{13}$	11		$+$
207	$\mathrm{S}_4(4)$	17		$+$
208	$\mathfrak{A}_{13}$	$\neq 3,5,11$		$+$
208	$\mathfrak{A}_{14}$	2		$+$
208	$\mathfrak{A}_{22}$	$3,7$		$+$
208	$\mathfrak{A}_{23}$	$3,7$		$+$
208	$\mathrm{L}_4(3)$	$2,5$		$+$
208	$2.\mathrm{L}_4(3)$	$\neq 2,3$	$i2$	$\circ$
208	$\mathrm{S}_4(5)$	$\neq 5$	$b5$	$+$
208	$2.\mathrm{S}_4(5)$	$\neq 2,5$	$b5$	$-$
208	M_{23}	7		$+$
208	$2.Suz$	3		$-$
209	$\mathfrak{A}_{22}$	$\neq 2,3,5,7$		$+$
209	J_1	$0,11,19$		$+$
210	$\mathfrak{A}_{10}$	$0,7$		$+$
210	$\mathfrak{A}_{11}$	$\neq 2,11$		$+$

d	G	ℓ	field	ind
210	$\mathfrak{A}_{12}$	3		+
210	$\mathfrak{A}_{22}$	$\neq 2,11$		+
210	$\mathfrak{A}_{23}$	23		+
210	$\mathrm{U}_4(3)$	$\neq 2,3$		+
210	$2.\mathrm{U}_4(3)$	$\neq 2,3$	$i1$	∘
210	$3_1.\mathrm{U}_4(3)$	$\neq 2,3$	$z3$	∘
210	$6_1.\mathrm{U}_4(3)$	$\neq 2,3$	$z3$	∘
210	$\mathrm{U}_6(2)$	3		+
210	$3.\mathrm{U}_6(2)$	$\neq 2,3$	$z3$	∘
210	$\mathrm{S}_6(2)$	$\neq 2,3$		+
210	$\mathrm{O}_8^+(2)$	$\neq 2,3$		+
210	M_{22}	$\neq 2,11$		+
210	$2.M_{22}$	$\neq 2,11$		+
210	$3.M_{22}$	$\neq 2,3$	$z3$	∘
210	$6.M_{22}$	$0,5,7$	$z3$	∘
210	$6.M_{22}$	$\neq 2,3$	$z12$	∘
210	M_{23}	23		+
210	HS	5		+
210	M^cL	$3,5$		+
214	J_3	19	$b17,y9$	+
216	$\mathfrak{A}_9$	0		+
216	$2.\mathfrak{A}_{10}$	$0,3$		+
216	$12_1.\mathrm{U}_4(3)$	$0,7$	$z12$	∘
216	$12_2.\mathrm{U}_4(3)$	$\neq 2,3$	$z12$	∘
216	$\mathrm{S}_6(2)$	0		+
216	$2.J_2$	$0,3$		−
217	$\mathfrak{A}_{10}$	5		+
217	$\mathrm{L}_5(2)$	$\neq 2$		+
217	$\mathrm{L}_6(2)$	$\neq 2$		+
218	${}^3D_4(3)$	2		+
218	${}^3D_4(3)$	73		+
219	${}^3D_4(3)$	$\neq 2,3,73$		+
220	$\mathfrak{A}_{13}$	$\neq 2,13$		+
220	$\mathfrak{A}_{14}$	7		+
220	$\mathrm{U}_4(4)$	5		+
220	$\mathrm{U}_5(2)$	$\neq 2,3$	$z3$	∘
220	M_{23}	2	$b7,b23$	∘
220	M_{24}	2	$b7,b23$	∘
220	$2.Suz$	$\neq 2,3$		−
221	$\mathfrak{A}_{12}$	7		+
221	$\mathfrak{A}_{13}$	7		+
221	$\mathrm{U}_4(4)$	$\neq 2$	$b5$	+
223	$\mathrm{S}_4(7)$	5		+
224	$2.\mathfrak{A}_9$	$0,7$		+
224	$\mathfrak{A}_{10}$	$\neq 2,5$		+
224	$4.\mathrm{U}_4(3)$	$\neq 2,3$	$z12$	∘
224	$\mathrm{S}_4(7)$	$\neq 2,5,7$		+
224	$2.\mathrm{O}_8^+(2)$	$\neq 2,5$		+
224	J_2	$0,7$	$b5$	+

d	G	ℓ	field	ind
225	$\mathfrak{A}_{10}$	$0,5$		+
225	$\mathrm{S}_4(4)$	$\neq 2,17$	$d17$	+
225	J_2	$\neq 2,7$		+
229	$\mathfrak{A}_{23}$	11		+
229	$\mathfrak{A}_{24}$	11		+
229	$\mathrm{U}_6(2)$	3		+
229	M_{23}	11		+
229	M_{24}	11		+
230	$\mathfrak{A}_{23}$	$\neq 3,7,11$		+
230	$\mathfrak{A}_{24}$	2		+
230	M_{23}	$0,5,23$		+
230	M^cL	$2,5$		+
230	Co_3	$2,5$		+
230	Co_2	2		+
231	$\mathfrak{A}_{11}$	$0,7,11$		+
231	$\mathfrak{A}_{23}$	$\neq 2,23$		+
231	$\mathfrak{A}_{24}$	3		+
231	$\mathrm{U}_6(2)$	$\neq 2,3$		+
231	$3.\mathrm{U}_6(2)$	$\neq 2,3$	$z3$	∘
231	M_{22}	$\neq 2,5$		+
231	$3.M_{22}$	$\neq 2,3$	$z3$	∘
231	M_{23}	$\neq 2,23$		+
231	M_{23}	$\neq 2,3,5$	$i15$	∘
231	M_{24}	$3,5$		+
231	M_{24}	$\neq 2,3,5$	$i15$	∘
231	HS	$\neq 2,5$		+
231	M^cL	$0,7,11$		+
231	Co_3	3		+
233	$\mathfrak{A}_{13}$	5		+
233	$\mathfrak{A}_{14}$	5		+
234	$\mathrm{L}_4(3)$	$0,13$		+
236	$2.J_2$	3		−
238	$\mathrm{S}_8(2)$	$\neq 2,3$		+
240	$\mathrm{U}_3(16)$	$\neq 2$		−
241	$\mathrm{U}_3(16)$	$\neq 2,17$	$z17$	∘
244	J_3	2	$b17$	+
245	$\mathrm{O}_8^-(3)$	13		+
246	$\mathrm{O}_8^-(3)$	$\neq 3,13$		+
246	${}^2F_4(2)'$	2		+
246	He	2	$b17$	+
248	$\mathrm{S}_4(5)$	2	$b5$	+
248	Th	all		+

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