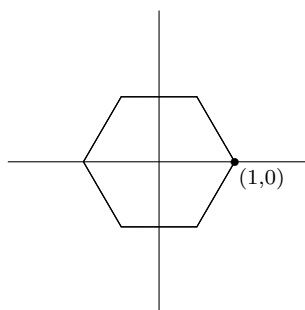


Sheet 1**Problem 1 (4+4 points)**Let $n \geq 3$.

- (1) Determine the symmetry group $\mathcal{D}_{2n} \leq O_2(\mathbf{R})$ of a regular n -gon in \mathbf{R}^2 (with center $(0, 0)$, one vertex $(1, 0)$). Determine $\mathcal{D}_{2n} \cap SO_2(\mathbf{R})$.

Ex. $n = 6$ 

- (2) Show that \mathcal{D}_{2n} is generated by two elements. Determine the conjugacy classes of \mathcal{D}_{2n} and the centre $Z(\mathcal{D}_{2n})$.

Problem 2 (2+2 points)

- (1) Let G be a group. Let M be a set with a transitive G -action. Suppose given $H \leq G$ and $m \in M$. Show that $G = G_m \cdot H$ if and only if H acts transitively on M .
- (2) Let G be a finite group and let $U \leq G$. Let p be the smallest prime divisor of $|U|$. Assume that $p \geq [G : U]$. Show that $U \trianglelefteq G$.
(Hint: Fixed points of action of U on $U \backslash G$.)

Problem 3 (4 points)

Let G be a finite group of order $|G| = 2n$, where n is odd. Show that there exists a normal subgroup of index 2 in G .

(Hint: The action of G on itself by right multiplication yields $G \hookrightarrow \mathcal{S}_G$. Cycle type of an element of order 2? Is the composite $G \hookrightarrow \mathcal{S}_G \rightarrow \{\pm 1\}$ surjective? Kernel?)

Problem 4 (6 points)

Show that there exists an epimorphism $GL_2(\mathbf{F}_3) \rightarrow \mathcal{S}_4$ whose kernel equals the centre of $GL_2(\mathbf{F}_3)$. Determine the image of $SL_2(\mathbf{F}_3)$.

(Remark: Therefore the quotient $PGL_2(\mathbf{F}_3) := GL_2(\mathbf{F}_3)/Z(GL_2(\mathbf{F}_3))$ is isomorphic to \mathcal{S}_4 .)

(Hint: $GL_2(\mathbf{F}_3)$ acts on one-dimensional subspaces of \mathbf{F}_3^2 .)