G. Hiß, M. Künzer Due Friday, 08.02.08

Group theory, WS 07/08

Sheet 13

Problem 47 (18 points)

Let G be a finite group. Let $U \leq G$. Let $N \trianglelefteq G$. Prove or disprove.

(1) If G is a p-group for some prime p, then $\mathrm{K}_i(G) = \mathrm{Z}_{\ell-i}(G)$ for all $0 \leq i \leq \ell$, where ℓ denotes the nilpotency class of G.

(2) We have $\Phi(N) \trianglelefteq \Phi(G)$.

(3) We have $\Phi(U) \leq \Phi(G)$.

(4) We have $\bigcap_{g \in G} \Phi(U)^g \trianglelefteq \Phi(G)$.

(5) We have $\Phi(G/N) \leq \Phi(G)N/N$.

(6) We have $\Phi(G/N) \geq \Phi(G)N/N$.

Problem 48 (5+3 points)

(1) Let G and H be finite groups. Show that $\Phi(G \times H) = \Phi(G) \times \Phi(H)$.

(2) Let G be a finite group. Show that G is nilpotent if and only if $G' \leq \Phi(G)$.

Problem 49 (4 points)

Let $G \neq 1$ be a finite group. Show that G is cyclic of prime power order if and only if G has a unique maximal subgroup.

(Hint: If $|G|$ is not a prime power, then a unique maximal subgroup of G contains all Sylow subgroups of G.)