Homework 12

Last homework sheet. To be handed in before 28.05.

Problem 57.

Let $A \in K^{n\times n}$ be a matrix. Let $t \in K$ be a parameter. Calculate the determinant $\det A$.
For which parameters t is A invertible?

(1) $A = \begin{pmatrix} 1 & 3 \\ -2 & t \end{pmatrix} \in \mathbf{R}^{2\times 2}$.

(2) $A = \begin{pmatrix} 89 & 90 & 91 \\ 70 & 71 & 72 \\ 17 & 18 & 19 \end{pmatrix} \in \mathbf{R}^{3\times 3}$ (no parameter, just decide on invertibility).

(3) $A = \begin{pmatrix} i & 1 & 0 \\ -1 & i & t \\ 1 & t & 1 \end{pmatrix} \in \mathbf{C}^{3\times 3}$.

(4) $A = \begin{pmatrix} -1 & 1 & 1 & t \\ -1 & -1 & 1 & 1 \\ -1 & -1 & -1 & 1 \\ -1 & t & -1 & 1 \end{pmatrix} \in \mathbf{R}^{4\times 4}$.

(5) $A = \begin{pmatrix} 1 & i & i & i & 1 \\ i & 1 & 1 & i & 1 \\ 1 & i & 1 & i & 1 \\ t & i & t & i & t \\ i & t & t & i & 1 \end{pmatrix} \in \mathbf{C}^{5\times 5}$.

Problem 58.

Let f be a function in several variables with values in $\mathbf{R}$. Determine the critical points of f. Calculate the principal minors of the Hesse matrix in these critical points. If possible, use them to decide whether the respective critical point is a local maximum or a local minimum. (If this criterion fails, then do not make an assertion).

(1) $f(x, y) = x^2 + xy + y^2$.

(2) $f(x, y) = y^5 - 4xy^3 + x^4$.

(3) $f(x, y, z) = xz + yz - x^2 - 2y^2 - z^2 - 10x - 10z$.

(4) $f(x, y, z) = (x + y + z)e^{-xyz}$.

Problem 59. Let $v_1 = \begin{pmatrix} 2 \\ 5 \\ 0 \end{pmatrix}$, $v_2 = \begin{pmatrix} 3 \\ -2 \\ 0 \end{pmatrix}$ and $v_3 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$ be vectors in $\mathbf{R}^3$.

(1) Use a determinant to show that (v_1, v_2, v_3) is linearly independent.

(2) Find a nonzero vector $u \in \mathbf{R}^3$ which is orthogonal to v_1 and to $v_1 \times v_2$.

(3) Find a nonzero vector $w \in \operatorname{span}(v_1, v_2)$ which is orthogonal to v_3.

Problem 60. (True or false?)

Prove or disprove.

(1) Let $f(x, y)$ be a function in two variables, and assume $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ to be a critical point of f. If the principal minors in $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ are not both positive, then f does not have a local minimum in $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

(2) Given $A \in \mathbf{R}^{n\times n}$ with n odd, and with $A^{\mathrm{t}} = -A$. Then A is not invertible.