

## Midterm Exam

Net duration 120 minutes. We allow 1 formula sheet, but no calculator.  
The total sum of points is 20.

**Question 1 (4 points).**

If possible, calculate the inverse matrix  $A^{-1}$ . If this is not possible, specify  $\text{rk } A$ .  
(Here  $\text{rk } A = \text{Rg } A$  is the rank of  $A$ .)

(1)  $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix} \in \mathbf{R}^{3 \times 3}$ .

(2)  $A = \begin{pmatrix} i & 1 & i \\ i & 0 & 1 \\ i-1 & i & 0 \end{pmatrix} \in \mathbf{C}^{3 \times 3}$ .

**Question 2 (4 points).** Evaluate the integral.

(1)  $\int_2^3 \frac{x^2 + 1}{x^2 - 1} dx$ .

(2)  $\int \frac{dx}{(x^2 + 2)^3}$ .

**Question 3 (4 points).**

Let  $A = \begin{pmatrix} 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 2 \end{pmatrix} \in \mathbf{R}^{4 \times 5}$ , and let  $b = \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} \in \mathbf{R}^4$ .

- (1) Find all solutions of  $Ax = b$ ; i.e. determine  $\{x \in \mathbf{R}^5 : Ax = b\}$ .
- (2) Let  $r = \text{rk } A$ . Find  $r$  column vectors of  $A$  that form a linearly independent tuple and justify your choice. (Here  $\text{rk } A = \text{Rg } A$  is the rank of  $A$ .)

**Question 4 (4 points).** Find the solution  $y$  of the differential equation that satisfies  $y(1) = 2$ .

(1)  $y' = y/x + x$ .

(2)  $y' = x^2y^2$ .

**Question 5 (4 points).** Consider the rotation solid with contour line given by the graph of the function  $f(x) = 1/x$  on  $[1, \infty)$ .

- (1) Is its volume  $V(f, 1, \infty)$  finite? If so, evaluate it.
- (2) Is its surface area  $S(f, 1, \infty)$  finite? If so, evaluate it.