THE 7-MODULAR DECOMPOSITION MATRICES OF THE SPORADIC O'NAN GROUP

ANNE HENKE, GERHARD HISS, AND JÜRGEN MÜLLER

Abstract. The determination of the modular character tables of the sporadic O'Nan group, its automorphism group and its covering group is completed by the calculation of the 7-modular decomposition numbers. The results are obtained with the assistance of the systems GAP, MOC, and MeatAxe, and by applying new condensation methods.

1. Introduction and results

In this paper we describe the computation of the 7-modular decomposition numbers for the sporadic simple O'Nan group ON, its automorphism group $ON.2$ and its triple covering group $3.ON$. Our results complete the determination of the Brauer character tables for these groups (see [4, 7]; the 3-modular table for $ON.2$ is not published yet). The proof involves far too many details to be presented in this paper. We have tried, however, to give enough information to enable the reconstruction of the proofs for the principal blocks in a suitable computational environment. We do not comment on the proof for the non-principal blocks of $3.ON$. At any rate, the results for these blocks are considerably easier to obtain than those for the principal block. The results for ON and $3.ON$ have been obtained by the first author in her Diploma thesis [2], to which we refer the interested reader for more details.

To find the decomposition numbers, we had to apply both character theoretic and module theoretic methods. In particular we made use of GAP [17], MOC [3, 11], the MeatAxe [13, 14], and Condensation [10, 16]. Of particular power is a new condensation method which allows to condense tensor products of modules [12, 18]. The ordinary character tables we have used were taken from the GAP library. The numbering of the characters used there coincides with the one in the Atlas [1]. We denote, as usual, characters by their degrees, distinguishing characters of equal degree by subscripts. It is a great pleasure for us to acknowledge the help of T. Breuer, who on our request computed the ordinary character tables of some maximal subgroups of $3.ON$.

From now on, blocks and decomposition matrices are understood with respect to the prime 7. The group $3.ON$ has eight blocks of defect zero and three blocks B_1, B_2, B_3 of defect 3. The blocks of defect zero consist of the characters

$$58\,653_1,\quad 85\,064,\quad 116\,963,\quad 143\,374,\quad 175\,616_1,\quad 175\,616_2,\quad 58\,653_2,\quad 58\,653_3,$$

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Table 1. Decomposition numbers of the principal block B_1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
$\underline{1}$	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$\underline{10\,944}$	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$\underline{13\,376_1}$	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$\underline{13\,376_2}$	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.
$\underline{25\,916_1}$	.	.	.	1	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.
$\underline{25\,916_2}$	.	.	.	.	.	1	.	1	1	.	.	.	.	.	.	.	.	.	.
$\underline{26\,752}$	1	1	1	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.
$\underline{32\,395_1}$	1	1	1	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.
$\underline{32\,395_2}$	1	1	1	.	.	.	.	.	.	.	.	.	1	1	.	.	.	.	.
$\underline{37\,696}$	.	.	.	.	1	.	1	.	.	.	.	.	.	.	1	.	.	.	.
$\underline{52\,668}$	.	2	2	.	1	.	1	.	.	.	1	.	1	.	.	.	.	.	.
$\underline{58\,311_1}$	.	1	.	.	.	.	.	2	1	.	.	.	.	.	.	1	.	.	.
$\underline{58\,311_2}$	1	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	1	.	.
$\underline{58\,311_3}$	1	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	1	.
$\underline{64\,790_1}$	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	1	.
$64\,790_2$	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	1	.	.
$\underline{169\,290_1}$	1	1	.	.	1	.	1	.	1	.	.	1	.	1	1	.	1	1	.
$\underline{169\,290_2}$	.	1	1	.	.	.	.	.	.	1	1	.	1	.	.	.	.	.	1
$175\,770_1$	.	2	.	.	.	.	.	.	.	1	.	.	.	.	.	1	.	.	1
$\underline{207\,360_1}$	.	1	.	.	.	.	.	1	1	.	.	.	.	.	1	1	.	.	1
$\underline{207\,360_2}$	1	2	1	.	1	.	1	.	.	.	1	1	1	1	1	.	.	.	1
$207\,360_3$	.	1	.	1	1	1	1	1	2	.	.	1	.	1	1	.	1	1	.
$234\,080_1$	.	1	.	1	1	1	1	.	1	.	1	1	1	1	1	.	.	.	1
$234\,080_2$	.	1	.	1	1	1	1	.	1	.	1	1	1	1	1	.	.	.	1

of which the first six are characters of ON.

1.1. **The principal block.** The decomposition matrix for the principal block B_1 is given in Table 1. Its columns correspond to the following irreducible Brauer characters:

$$1,\quad 1\,618,\quad 9\,326,\quad 12\,155_1,\quad 1\,221_1,\quad 12\,155_2,$$
$$1\,221_2,\quad 406,\quad 13\,355,\quad 15\,807,\quad 14\,169_1,\quad 7\,281_1,$$
$$14\,169_2,\quad 7\,281_2,\quad 35\,254,\quad 42\,526,\quad 51\,029_1,\quad 51\,029_2,\quad 114\,201.$$

The underlined characters in the first column of Table 1 constitute a basic set of Brauer characters for B_1, i. e. the restrictions to the 7-regular conjugacy classes of these characters are a $\mathbb{Z}$-basis for the group of generalized Brauer characters of B_1. Equivalently, the matrix consisting of the rows of Table 1 corresponding to these characters is invertible over $\mathbb{Z}$.

1.2. **The non-principal blocks.** The two non-principal blocks of positive defect, B_2 and B_3, are complex conjugate to each other. The decomposition matrix of B_2 is given in Table 2. The columns there correspond to the following irreducible

TABLE 2. Decomposition matrix for block B_2

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
$\underline{342_1}$	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$\underline{342_2}$	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$\underline{495_1}$	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$\underline{495_2}$	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$\underline{5\,643_1}$	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.
$\underline{5\,643_2}$	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.	.
$\underline{5\,643_3}$	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.	.
$\underline{52\,668_2}$	.	.	.	.	.	.	.	.	1	1	1	.	.	.	.	.	.	.	.
$52\,668_3$	.	.	.	.	.	.	.	.	1	1	1	.	.	.	.	.	.	.	.
$\underline{58\,311_4}$	.	.	.	.	1	.	1	1	1	1	.	1	1	.	.	.	.	.	.
$\underline{63\,612}$	1	1	1	1	1	.	.	.	.	.	1	.	.	1	.	.	.	.	.
$\underline{111\,321}$	.	.	1	1	2	1	.	.	.	.	.	.	.	.	1	.	.	.	.
$\underline{116\,622}$	1	1	1	1	1	1	1	1	.	.	.	1	1	.	.	1	.	.	.
$\underline{122\,760}$	.	.	.	.	.	.	.	.	1	1	.	1	.	.	.	.	1	.	.
$\underline{169\,290_3}$	.	.	1	1	1	.	.	.	.	.	.	.	1	.	1	.	.	1	.
$\underline{169\,290_4}$	.	.	1	1	2	.	.	.	.	.	.	.	.	1	.	.	1	.	1
$\underline{169\,632_1}$	.	.	.	.	.	.	.	.	1	1	.	.	1	.	.	.	1	.	1
$169\,632_2$	1	1	1	1	2	1	1	1	.	.	.	1	1	.	.	1	.	1	.
$175\,770_2$	1	1	2	2	4	1	1	1	.	.	.	.	2	1	.	.	.	1	1
$\underline{207\,360_4}$	1	1	2	2	3	1	1	1	.	.	.	1	1	1	.	1	.	1	.
$\underline{207\,360_5}$	1	1	1	1	1	.	1	1	1	.	1	1	1	1	.	.	1	.	.
$207\,360_6$	1	1	1	1	3	1	1	1	.	1	.	1	1	1	.	.	1	.	1
$\underline{253\,440_1}$	1	1	1	1	2	.	1	1	1	1	1	1	1	1	.	.	1	.	1
$253\,440_2$	.	.	1	1	2	1	.	.	.	.	.	.	1	.	1	1	.	1	.

Brauer characters:

$$342_1,\quad 342_2,\quad 495_1,\quad 495_2,\quad 45_1,\quad 5\,598_1,$$
$$5\,643_2,\quad 5\,643_3,\quad 26\,523_1,\quad 945_1,\quad 25\,200_1,\quad 8\,865_1,$$
$$10\,647_1,\quad 36\,693_1,\quad 104\,643_1,\quad 78\,507_1,\quad 86\,427_1,\quad 52\,965_1,\quad 45\,090_1.$$

Again the underlined characters form a basic set. We get the decomposition matrix of B_3 by replacing the ordinary characters in the decomposition matrix of B_2 by their complex conjugates.

1.3. **The principal block of $ON.2$.** The group $ON.2$ has twelve blocks of defect zero. These consist of the extensions of the defect zero characters of ON. Only the principal block is of positive defect. Its decomposition matrix is given in Table 3. The columns there correspond to the following irreducible Brauer characters:

$$1_1,\quad 1_2,\quad 9\,326_1,\quad 1\,618_1,\quad 1\,618_2,\quad 9\,326_2,\quad 24\,310,$$
$$2\,442,\quad 406_1,\quad 406_2,\quad 13\,355_1,\quad 13\,355_2,\quad 15\,807_1,\quad 15\,807_2,\quad 28\,338,$$
$$14\,562,\quad 35\,254_1,\quad 35\,254_2,\quad 42\,526_1,\quad 42\,526_2,\quad 102\,058,\quad 114\,201_1,\quad 114\,201_2.$$

As in the tables above, we have underlined the ordinary characters of a basic set.

TABLE 3. Decomposition numbers of the principal block of $ON.2$

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23
$\underline{1}_1$	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$\underline{1}_2$	.	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$\underline{10\,944}_1$	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$\underline{10\,944}_2$	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$\underline{26\,752}_1$	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
$\underline{51\,832}$	.	.	.	.	.	.	1	.	1	1	1	1	.	.	.	.	.	.	.	.	.	.	.
$\underline{26\,752}_2$	.	1	1	.	1	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	.
$\underline{26\,752}_3$	1	.	.	1	.	1	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.
$\underline{64\,790}$	1	1	1	1	1	1	.	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.
$\underline{37\,696}_1$	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	1	.	.	.	.	.	.
$\underline{37\,696}_2$	.	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.	.	1	.	.	.	.	.
$\underline{52\,668}_1$	.	.	1	1	1	1	.	1	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.
$52\,668_2$	.	.	1	1	1	1	.	1	.	.	.	.	.	.	1	.	.	.	.	.	.	.	.
$\underline{58\,311}_1$	.	.	.	1	.	.	.	.	1	1	1	.	.	.	.	.	.	.	1	.	.	.	.
$\underline{58\,311}_2$	.	.	.	.	1	.	.	.	1	1	.	1	.	.	.	.	.	.	.	1	.	.	.
$\underline{116\,622}$	1	1	.	.	.	.	.	.	.	.	.	.	.	.	.	1	.	.	.	.	1	.	.
$129\,580$	.	.	.	.	.	.	.	.	1	1	1	1	.	.	.	.	.	.	.	.	1	.	.
$\underline{169\,290}_1$	1	.	.	1	.	.	.	1	.	.	1	.	.	.	.	1	.	1	.	.	1	.	.
$\underline{169\,290}_2$	.	1	.	.	1	.	.	1	.	.	.	1	.	.	.	1	1	.	.	.	1	.	.
$\underline{169\,290}_3$	.	.	1	.	1	.	.	.	.	.	.	.	1	.	1	.	.	.	.	.	.	1	.
$\underline{169\,290}_4$	.	.	.	1	.	1	.	.	.	.	.	.	.	1	1	.	.	.	.	.	.	.	1
$175\,770_1$	.	.	.	2	.	.	.	.	.	.	.	.	.	1	.	.	.	.	1	.	.	.	1
$175\,770_2$	.	.	.	.	2	.	.	.	.	.	.	.	1	.	.	.	.	.	.	1	.	1	.
$\underline{207\,360}_1$	.	.	.	1	.	.	.	.	1	.	1	.	.	.	.	.	.	1	1	.	.	.	1
$\underline{207\,360}_2$	.	.	.	.	1	.	.	.	.	1	.	1	.	.	.	.	1	.	.	1	.	1	.
$\underline{207\,360}_3$	.	1	1	1	1	.	.	1	.	.	.	.	.	.	1	1	1	.	.	.	.	1	.
$\underline{207\,360}_4$	1	.	.	1	1	1	.	1	.	.	.	.	.	.	1	1	.	1	.	.	.	.	1
$207\,360_5$	.	.	.	1	.	.	1	1	.	1	1	1	.	.	.	1	1	.	.	.	1	.	.
$207\,360_6$	.	.	.	.	1	.	1	1	1	.	1	1	.	.	.	1	.	1	.	.	1	.	.
$234\,080_1$	.	.	.	.	1	.	1	1	.	.	.	1	.	.	1	1	1	.	.	.	.	1	.
$234\,080_2$	.	.	.	1	.	.	1	1	.	.	1	.	.	.	1	1	.	1	.	.	.	.	1
$234\,080_3$	.	.	.	.	1	.	1	1	.	.	.	1	.	.	1	1	1	.	.	.	.	1	.
$234\,080_4$	.	.	.	1	.	.	1	1	.	.	1	.	.	.	1	1	.	1	.	.	.	.	1

2. Providing a few representations

In this section we describe the construction of a few representations, which will be needed later. From now on let G denote the simple group ON, $3.G$ its triple covering group, and $G.2$ and $3.G.2$ the extensions of G and $3.G$, respectively, by the non-trivial outer automorphism.

2.1. **Generators.** We start our constructions by accessing the $\mathbb{F}_7$-representation of $3.G.2$ of degree 90 from the data base [19], see also [15]. It is given in terms of two standard generators, X and Y say, where X is a $2B$-element, Y is a $4A$-element, and their product XY is a $22A$-element. The corresponding structure constant confirms that $(2B, 4A, 22A)$ is a rigid triple, see [20]. We let $W := (XY)^2Y$ and observe that $WXYW$ is an element of order 56, hence $A := W^9(WXYW)^{28}W^{-9}$ is a $2A$-element. We let $B := Y$ and find that AB is of order 33. Structure constants show that $(2A, 4A, 33A)$ and $(2A, 4A, 33B)$ both are rigid triples, hence we can choose A and B as our pair of standard generators. Thereby we define the class $33A$ to be the class AB belongs to. We let $x := (XY)^{11}$, which also is a $2B$-element. Hence we can assume that all explicit representations of $3.G$ which occur in the sequel are given in terms of A, B, and their conjugates A^x, B^x. Finally we let $Z := (AB)^{22}$, which is a non-trivial central element.

Restricted to $3.G$, the $\mathbb{F}_7$-representation of degree 90 splits into a pair 45_1, $45_2 = 45_1^*$ of mutually contragradient representations. We define 45_1 to be the module where the scalar action of Z is given by the chosen standard primitive third root of unity, hence 45_1 belongs to block B_2. It is a standard application of the MeatAxe to obtain a few new representations from these by tensoring and symmetrising. We find $45_1 \otimes 45_2 = 1 + 406 + 1618$, $45_2^{[1,1]} = 45_1 + 945_1$, $945_2 = 945_1^*$, $45_2^{[2]} = 495_1 + 495_2 + 45_1$, $495_2 = 495_1^{*x}$, $495_3 = 495_1^*$, $495_4 = 495_1^x$, where 406, 1618 belong to the principal block B_1, $495_{1,2}$, 945_1 belong to B_2 and $495_{3,4}$, 945_2 belong to B_3. Here ^{x} denotes the action of the outer automorphism.

Furthermore, we use the following notation: If π is a partition of $n \in \mathbb{N}$, then V^π denotes the symmetrisation of the module V with respect to the ordinary irreducible representation χ_π of the symmetric group S_n corresponding to π. Hence e. g. $V^{[2]}$ and $V^{[1,1]}$ denote the symmetric and skew square of V, respectively. Note that the Brauer character of V^π can be computed from the Brauer character of V and the character χ_π.

2.2. **Some Brauer characters.** We are now going to compute some Brauer character values for the representations constructed above. First, we have to find element representatives for a suitable subset of conjugacy classes. A consideration of table automorphisms shows that we can choose representatives for the classes $11A$, $16B$, $19A$, $20A$, $31A$ as given in Table 4, where we have put $C := (AB)^4B$ and $D := ABC$. The representatives for the other classes shown there are then given by powermaps.

For both blocks B_1 and B_2 the matrix of the values of the 19 ordinary characters in its basic set on the $1A$-class and the 18 classes of Table 4 turns out to be invertible. Hence a Brauer character in one of these blocks is uniquely determined by its values on these classes. Using the data on Conway polynomials and irrationalities given in [6, Appendix 1], GAP and the MeatAxe, we compute the Brauer character values of 45_1, 406, 495_1, 945_1 on the classes of Table 4. Using the equations given in 2.1, we find the Brauer character values for all the other irreducible representations known so far.

2.3. **Permutation representations.** We will now construct two permutation representations needed in the sequel. We start with the $\mathbb{F}_4$-representation of $3.G$ of degree 153 accessible in the data base [19]. Guided by the construction of this representation described in [7], it is easy to find a one-dimensional subspace of

TABLE 4. Representatives for some conjugacy classes

Class	Repr.	Class	Repr.
$2A$	$((AB)^2B)^{30}$	$16B$	$(D^2CD^3CDC)^3$
$4A$	$((AB)^2B)^{15}$	$16C$	$((D^2CD^3CDC)^9)^x$
$4B$	$(D^2CD^3CDC)^{36}$	$16D$	$((D^2CD^3CDC)^3)^x$
$5A$	$((AB)^2B)^{12}$	$19A$	$(AB)^3B$
$8A$	$(D^2CD^3CDC)^{18}$	$19B$	$((AB)^3B)^2$
$8B$	$((D^2CD^3CDC)^{18})^x$	$19C$	$((AB)^3B)^4$
$10A$	$((AB)^2B)^6$	$20A$	$((AB)^2B)^3$
$11A$	$(AB)^3$	$20B$	$((AB)^2B)^{-3}$
$16A$	$(D^2CD^3CDC)^9$	$31A$	$(D^3C)^{-1}$

the underlying space whose orbit under the action of G is of length 122 760, giving a permutation representation of G on the cosets of a maximal subgroup isomorphic to $L_3(7):2$. Considering the orbit of a non-trivial vector in this one-dimensional subspace instead, we get a permutation representation of $3.G$ of degree 368 280, i. e. again on the cosets of a subgroup isomorphic to $L_3(7):2$. Analogously, starting with the $\mathbb{F}_9$-representation of $G.2$ of degree 154 also given in [19] we obtain a permutation representation of G on 245 520 points, i. e. on the cosets of a subgroup isomorphic to $L_3(7)$. The corresponding permutation characters are given by $122\,760 = 1 + 10\,944 + 26\,752 + 32\,395_1 + 52\,668$ and $245\,520 = 1 + 10\,944 + 26\,752 + 32\,395_1 + 37\,696 + 52\,668 + 85\,064$.

2.4. **Condensation.** Let F be a field, A a finite-dimensional F-algebra and $e \in A$ an idempotent, i. e. $0 \neq e = e^2$. We then have an additive exact functor, the condensation functor, from the category of finite-dimensional right A-modules to the category of finite-dimensional right eAe-modules. It maps an A-module V to the eAe-module Ve and an A-homomorphism $\alpha \in Hom_A(V, W)$ to the restriction of α to Ve. If S is a simple A-module then either $Se = \{0\}$ or Se is a simple eAe-module. We are interested in the special situation of fixed point condensation, i. e. $A := FH$ is the group algebra of a finite group H, K is a subgroup of H whose order is invertible in F and $e = e_K := |K|^{-1} \sum_{k \in K} k \in FK \subseteq FH$. Since Ve is the set of vectors of V fixed by K, the dimension of the condensed module Ve is given by the scalar product of the trivial character of K with the Brauer character of V restricted to K.

In the computational context we work in, we are given a set of generators $\{h_1, h_2, \ldots\}$ for H, and an FH-module V of a special nature, i. e. in our cases a permutation module or a tensor product. We then compute matrices for the action of the elements $\{eh_1e, eh_2e, \ldots\}$ on Ve, using the MeatAxe version written by M. Ringe [14] and its tensor condensation package written by M. Wiegelmann [18]. We then analyze the submodule structure of Ve, considered as module for the condensation subalgebra $\mathcal{K}$ of $eFHe$ generated by $\{eh_1e, eh_2e, \ldots\}$, which might be properly contained in $eFHe$. As $Ve \subseteq V$, for a $\mathcal{K}$-submodule W of Ve we can form the smallest FH-submodule of V containing W; this process is called uncondensing.

Let B be a block of FH, containing exactly l simple modules $\{S_1, \ldots, S_l\}$ up to isomorphism. We now describe a method to check that $S_k e \neq \{0\}$ holds for all the S_k. Let $\{V_1, \ldots, V_n\}$ be a set of FH-modules and let $\{T_1, \ldots, T_m\}$ be the set of simple $\mathcal{K}$-modules occuring up to isomorphism as constituents of the $V_i e$'s. Let $M = (m_{ij}) \in \mathbb{Z}^{n\times m}$ be defined by $[V_i e] = \sum_{j=1}^{m} m_{ij}[T_j]$, where the terms in brackets denote the corresponding elements of the Grothendieck group of the category of finite-dimensional $\mathcal{K}$-modules.

Proposition. *If M has rank l, then $S_k e \neq \{0\}$ for all simple B-modules S_k, $1 \leq k \leq l$.*

Proof. By definition, M factors as $M = XY$, where the rows of $X \in \mathbb{Z}^{n\times l}$ record the multiplicities of the S_k's in the V_i's, and the rows of $Y \in \mathbb{Z}^{l\times m}$ give the decomposition of the $S_k e$'s into the T_j's. If M has rank l, then Y has rank l, and hence $S_k e \neq \{0\}$ for all $1 \leq k \leq l$. ♯

Note that M possibly has rank l, only if X also has. Hence to apply this criterion it is necessary that $\{[V_1], \ldots, [V_n]\}$ generates a sublattice of finite index in the Grothendieck group of the category of finite-dimensional B-modules.

2.5. **Applying Condensation.** We are now going to find generators for a suitable condensation subgroup $K < 3.G$. We choose a subgroup $K \cong 11 : 10$. Note that the fusion of the conjugacy classes of K into those of $3.G$ is uniquely determined by a consideration of element orders. Using this fusion, the scalar product of the restriction of a character of $3.G$ to K with the trivial character of K can be computed using GAP.

As the envisaged subgroup K is contained in a maximal subgroup of $3.G$ isomorphic to J_1, we first find generators for such a maximal subgroup, which is the centralizer in $3.G$ of a $2B$-element. Let $C_1 := (AB)^2B(AB)^3B$, $C_2 := (AB)^5B$, $A_1 := B^{-1}AB$, $A_2 := C_1AC_1^{-1}$, $A_3 := C_2AC_2^{-1}$, $B_1 := (A_1A_1^x)^6(A_2A_2^x)^{14}$, $B_3 := B_1(A_3A_3^x)^{14}$, $B_2 := (B_1B_3)^2B_3(B_1B_3)^3B_3B_1B_3^2$. It can be checked, e. g. using the permutation representations constructed above, that $\langle B_1, B_2\rangle$ in fact is a subgroup of $3.G$ isomorphic to J_1. Now let $Y_1 := (B_1B_2)^3$, $Y_2 := (B_1B_2)^2B_2$, $Z_1 := Y_2Y_1Y_2^{-1}$.It turns out that Y_1 and Z_1 are of order 2, while Y_1Z_1 is of order 11. Hence we have to find an element of order 5 centralizing Y_1 and normalizing Y_1Z_1. We have $C_{J_1}(Y_1) \cong C_2 \times A_5$, it turns out to be generated by $X_1 := (Y_1B_2^2Y_1B_2^{-2})^3(Y_1B_2^4Y_1B_2^{-4})^5$ and $X_2 := (Y_1B_2^2Y_1B_2^{-2})^3(Y_1B_2^{-2}Y_1B_2^2)^3$. It is easy to find a suitable element of order 5 in this subgroup. We let $Y_3 := (X_1X_2)^4X_2$, $Y_4 := X_1X_2Y_3^2(X_1X_2)^{-1}$, $Z_2 := Y_1Y_4$. Then Y_4 is an element of order 5 having the properties we have looked for, Z_2 is of order 10, and $K := \langle Z_1, Z_2\rangle$ is a subgroup of $3.G$ isomorphic to $11 : 10$.

We are now able to condense the permutation representations constructed in Section 2.3 and a few tensor products of the matrix representations constructed in Section 2.1 with respect to K. Let $F := \mathbb{F}_7$, $e := e_K$ and $\mathcal{K} := \langle eAe, eBe, eA^xe, eB^xe\rangle$. The $\mathcal{K}$-constituents of the condensed representations are given in Table 5, where the columns correspond to the irreducible $\mathcal{K}$-modules occurring in the condensed modules. The condensed permutation representation of degree 368 280 turns out to have a socle constituent of dimension 3. Uncondensing this submodule, we obtain representations 342_1 of $3.G$, belonging to block B_2, and $342_2 = 342_1^{*x}$, $342_3 = 342_1^{*}$,

Table 5. Some condensed modules

<table>
<tr><th></th><th>1</th><th>5</th><th>10_1</th><th>10_2</th><th>15</th><th>67_1</th><th>67_2</th><th>90</th><th>106_1</th><th>106_2</th><th>121</th></tr>
<tr><td>$P_{122\,760}$</td><td>2</td><td>.</td><td>1</td><td>1</td><td>5</td><td colspan="2">1</td><td>5</td><td>.</td><td>.</td><td>.</td></tr>
<tr><td>$P_{245\,520}$</td><td>3</td><td>.</td><td colspan="2">4</td><td>5</td><td colspan="2">1</td><td>5</td><td>.</td><td>.</td><td>.</td></tr>
<tr><td>$342_1 \otimes 342_4$</td><td>.</td><td>2</td><td>.</td><td>.</td><td>1</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>1</td></tr>
<tr><td>$495_1 \otimes 495_4$</td><td>.</td><td>2</td><td>.</td><td>.</td><td>4</td><td>.</td><td>.</td><td>1</td><td>.</td><td>.</td><td>1</td></tr>
<tr><td>$495_1 \otimes 495_3$</td><td>3</td><td>.</td><td>.</td><td>.</td><td>2</td><td colspan="2">2</td><td>2</td><td>.</td><td>.</td><td>.</td></tr>
<tr><td>$495_2 \otimes 495_4$</td><td>3</td><td>.</td><td>.</td><td>.</td><td>2</td><td colspan="2">2</td><td>2</td><td>.</td><td>.</td><td>.</td></tr>
<tr><td>$406 \otimes 406$</td><td>3</td><td>1</td><td>1</td><td>1</td><td>6</td><td>2</td><td>2</td><td>2</td><td>2</td><td>2</td><td>1</td></tr>
</table>

<table>
<tr><th></th><th>129_1</th><th>129_2</th><th>150</th><th>317</th><th>392</th><th>470_1</th><th>470_2</th><th>1 035</th><th>533</th><th>767</th><th>1 304</th></tr>
<tr><td>$P_{122\,760}$</td><td>2</td><td>1</td><td>1</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr>
<tr><td>$P_{245\,520}$</td><td colspan="2">3</td><td>1</td><td>1</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>1</td><td>.</td></tr>
<tr><td>$342_1 \otimes 342_4$</td><td>.</td><td>.</td><td>.</td><td>.</td><td>1</td><td>.</td><td>.</td><td>.</td><td>1</td><td>.</td><td>.</td></tr>
<tr><td>$495_1 \otimes 495_4$</td><td>.</td><td>.</td><td>1</td><td>.</td><td>2</td><td>.</td><td>.</td><td>1</td><td>.</td><td>.</td><td>.</td></tr>
<tr><td>$495_1 \otimes 495_3$</td><td colspan="2">1</td><td>.</td><td>.</td><td>.</td><td>1</td><td>.</td><td>.</td><td>.</td><td>.</td><td>1</td></tr>
<tr><td>$495_2 \otimes 495_4$</td><td colspan="2">1</td><td>.</td><td>.</td><td>.</td><td>.</td><td>1</td><td>.</td><td>.</td><td>.</td><td>1</td></tr>
<tr><td>$406 \otimes 406$</td><td>.</td><td>.</td><td>.</td><td>.</td><td>1</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td><td>.</td></tr>
</table>

$342_4 = 342_1^x$. The $\mathcal{K}$-constituents of the condensed tensor product $342_1 \otimes 342_4$ are also shown in Table 5.

3. Remarks on the proof for the principal block

3.1. **Subgroup Fusions.** We are going to induce Brauer characters and projective characters from the first, third, fifth and sixth maximal subgroup of G. We have $M_1 \cong L_3(7):2$, $M_3 \cong J_1$, $M_5 \cong (3^2:4\times A_6)^{\cdot}2$ and $M_6 \cong 3^4:2_-^{1+4}D_{10}$, see [1]. The ordinary character tables of these subgroups are available through GAP. Since we have already chosen element representatives for the conjugacy classes of G given in Table 4, we have to adjust the fusions of the conjugacy classes of these subgroups into those of G accordingly. A consideration of table automorphisms shows that we can choose the subgroup fusions of M_5 and M_6, whose orders are not divisible by 7, as are given in GAP. For M_1 and M_3 we have to take their 7-modular Brauer character tables into account. As M_1 and M_3 are also subgroups of $3.G$, we can restrict 45_1 from $3.G$ to M_1 and M_3. From the known Brauer character values, see 2.2, we find that $45_1|_{M_1}$ decomposes into $8+37$, while $45_1|_{M_3}$ is irreducible. This immediately determines the fusion from M_3 to G. The analogous analysis for M_1 leads to two possible cases, which correspond to the fact that there are two conjugacy classes of subgroups isomorphic to $L_3(7):2$ in G.

3.2. **A basic set of projective characters for the principal block.** Table 6 describes a set of projective characters, understood to be projected to the principal block B_1. We have used the symbol $\uparrow_i$ to denote the induction of characters from a maximal subgroup M_i, $i \in \{1,3,5,6\}$. The character $\chi_{22} = 343_2$ of M_1 is of 7-defect zero. For M_5 and M_6, whose orders are not divisible by 7, we have $\chi_{10}(1) = 1$, $\chi_{17}(1) = 2$, and $\chi_1(1) = \chi_2(1) = 1$, $\chi_5(1) = 5$, $\chi_{11}(1) = \chi_{12}(1) = 4$, respectively.

TABLE 6. The basic set $\mathcal{PS}$ of projective characters

Char.	Origin
Φ_1	$\chi_2 \uparrow_6$
Φ_2	$406 \otimes 58\,653_1$
Φ_3	$406 \otimes 85\,064$
Φ_4	$45_1 \otimes 58\,653_3$
Φ_5	$45_2 \otimes 58\,653_2$
Φ_6	$\frac{1}{2}\chi_{12} \uparrow_6$
Φ_7	$\chi_{22} \uparrow_1$
Φ_8	$\chi_{17} \uparrow_5$
Φ_9	$342_4 \otimes 58\,653_2$
Φ_{10}	$342_3 \otimes 58\,653_2$

Char.	Origin
Φ_{11}	$1\,618 \otimes 58\,653_1$
Φ_{12}	$\frac{1}{2}\chi_{11} \uparrow_6$
Φ_{13}	$1\,618 \otimes 85\,064$
Φ_{14}	$945_1 \otimes 58\,653_3$
Φ_{15}	$\frac{1}{6}(945_2 \otimes 58\,653_2 + \tilde{\Phi})$
Φ_{16}	$\chi_{10} \uparrow_5$
Φ_{17}	$495_3 \otimes 58\,653_2$
Φ_{18}	$\chi_5 \uparrow_6$
Φ_{19}	$\chi_1 \uparrow_6$

The projections to B_1 of $\chi_{11} \uparrow_6$ and $\chi_{12} \uparrow_6$ both are twice ordinary characters, hence Φ_6 and Φ_{12} are projective characters. Let $\tilde{\Phi} := 5\Phi_4 + 5\Phi_5 + 4\Phi_6 + 2\Phi_7 + 5\Phi_8 + 4\Phi_{11} + \Phi_{14}$. Then the projection to B_1 of $945_2 \otimes 58\,653_2 + \tilde{\Phi}$ is six times an ordinary character, and thus Φ_{15} is a projective character as well.

The matrix of scalar products between $\mathcal{PS} := \{\Phi_1, \ldots, \Phi_{19}\}$ and the characters in the basic set $\mathcal{BS}$ of ordinary characters given by the underlined characters in Table 1 turns out to be invertible over $\mathbb{Z}$. Hence $\mathcal{PS}$ is a basic set of projective characters.

3.3. **A collection of projective characters.** Let $\mathcal{P}$ be the set of projective characters obtained by inducing the projective indecomposable characters of the M_i, $i \in \{1, 3, 5, 6\}$, to G, by tensoring the 7-defect zero characters with the ordinary characters of $3.G$ and with the irreducible Brauer characters 45_1, 45_2, 406, 945_1, 945_2, 1 618, and by taking symmetric and skew squares of the defect zero characters.

The set $\mathcal{P}$ and the basic set $\mathcal{PS}$ of projective characters constructed in 3.2 are now used to find possible decompositions of the characters in $\mathcal{BS}$ into irreducible Brauer characters. This will successively lead to better basic sets of Brauer characters and finally to the set of irreducible Brauer characters. For a detailed discussion of the concepts and methods involved the reader is referred to [3, Chapter 3] or [11].

Let $\mathcal{BA} := \{\alpha_1, \ldots, \alpha_l\}$ denote the basis of the space of generalized Brauer characters dual to $\mathcal{PS}$, with respect to the pairing between the spaces of generalized Brauer characters and of generalized projective characters given by the usual scalar product. The α_i are called the Brauer atoms with respect to $\mathcal{PS}$, since every Brauer character ϕ can be written as $\phi = \sum_{i=1}^{l} n_i\alpha_i$, where $n_i \in \mathbb{Z}$, $n_i \geq 0$. Every irreducible constituent ϕ' of ϕ is of the form $\phi' = \sum_{i=1}^{l} n'_i\alpha_i$, where $n'_i \in \mathbb{Z}$, $0 \leq n'_i \leq n_i$. We may exclude ϕ' as a possible constituent by finding a projective character $\Phi \in \mathcal{P}$ having a negative scalar product with ϕ'. If we are able to exclude all possible candidate constituents this way, we will conclude that ϕ is irreducible.

Using the results of condensation or the analysis of submodule lattices we get further conditions on such possible constituents ϕ' of ϕ. We might know, for example, $\dim_F(V'e)$ for a module V' with Brauer character ϕ'. On the other hand, we know the 'condensed degrees' d_i of the Brauer atoms α_i, i. e. the scalar products of the restrictions of the α_i to K with the trivial character of K. We then necessarily have $\sum_{i=1}^{l} n_i' d_i = \dim_F(V'e)$.

3.4. **The permutation representation P of degree 122 760.** Using the MeatAxe and the methods described in [10], which are also implemented in [14], we compute the $\mathcal{K}$-submodule lattice of the condensed module Pe corresponding to P. It turns out that Pe is of F-dimension 1152 and has exactly 1512 submodules. Since 122 760 is not divisible by 7, the FG-module P has a uniquely determined direct summand isomorphic to the trivial module. Let P' denote the corresponding quotient module of P. Hence the $\mathcal{K}$-module $P'e$ is a quotient module of Pe and easily found using the MeatAxe.

$P'e$ turns out to have exactly 390 submodules, and its socle S is irreducible of dimension 90. By a Theorem of Zassenhaus, see e. g. [9, Theorem I.17.3], the FG-module P' has submodules whose Brauer characters are the ordinary constituents 10 944, 26 752 and 52 668 of the corresponding permutation character, see 2.3. These FG-modules condense to modules of dimensions 105, 256 and 488, respectively. It turns out that $P'e$ has unique $\mathcal{K}$-submodules of each of the dimensions 256 and 488, hence these are $eFGe$-submodules. Their intersection equals the socle S, hence S is an irreducible $eFGe$-submodule of $P'e$. We further observe that $P'e$ has exactly eight $\mathcal{K}$-submodules of dimension 105 all containing S as a maximal $\mathcal{K}$-submodule. By the Zassenhaus Theorem, at least one of them is an $eFGe$-submodule. It follows that the ordinary character 10 944 has modular constituents which condense to characters of degrees 90 and 15, respectively. Note that we do not know at this stage whether no simple B_1-module condenses to $\{0\}$. Hence we cannot immediately conclude that 10 944 has exactly two modular constituents.

With the method described in 3.3 we now look for possible constituents of 10 944 having condensed degree 15. It turns out that this search has a unique solution, the irreducible Brauer character 1 618 already known. Moreover, the irreducibility test also described in 3.3 shows that $9\,326 := 10\,944 - 1\,618$ is an irreducible Brauer character. By a further analysis of $P'e$, using similar techniques, we obtain the irreducible Brauer characters $7\,281_1$, $14\,169_1$ and 15 807. These are described by the following decompositions of three of the ordinary constituents of the permutation character of P: $10\,944 = 1\,618 + 9\,326$, $26\,752 = 1 + 1\,618 + 9\,326 + 15\,807$ and $32\,395_1 = 1 + 1\,618 + 9\,326 + 7\,281_1 + 14\,169_1$.

Furthermore, we obtain $7\,281_2 = 7\,281_1^x$, $14\,169_2 = 14\,169_1^x$, $14\,190_1 := 45_2^{[1,1,1]}$ and $14\,190_2 := 14\,190_1^*$. We now replace the basic set $\mathcal{BS}$ of Brauer characters by the following one, denoted by $\mathcal{BS}'$; Brauer characters already known to be irreducible are underlined.

$$\begin{array}{rrrrrrr} \underline{1}, & \underline{406}, & \underline{7\,281}_1, & \underline{9\,326}, & 13\,376_1, & \underline{14\,169}_1, & 14\,190_1, \\ & 25\,916_1, & 58\,311_1, & 58\,311_3, & \underline{1\,618}, & \underline{7\,281}_2, & \underline{15\,807}, \\ & 13\,376_2, & \underline{14\,169}_2, & 14\,190_2, & 37\,696\;, & 58\,311_2, & 175\,770. \end{array}$$

3.5. Decomposing the characters $342_2 \otimes 45_2$, $45_1^{[2,1]}$, $406^{[1,1]}$, $25\,916_2$ and $64\,790_1$ into $\mathcal{BS}'$ and applying the methods described in [3] to the resulting relations, we obtain the irreducible Brauer characters $1\,221_1$, $1\,221_2$, $12\,155_1$, $12\,155_2$ and 13 355, and Brauer characters $51\,029_1$ and $51\,029_2$ not yet known to be irreducible. For example, we have

$$342_2 \otimes 45_2 + \underline{1} + \underline{406} + 2 \cdot \underline{1\,618} + \underline{9\,236} = 14\,190_1 + \underline{14\,169}_1.$$

TABLE 7. Condensed dimensions for $\mathcal{BS}''$

$\mathcal{BS}''$	d	
$\underline{1}$	1	1
$\underline{406}$	5	5
$\underline{1\,221}_1$	10	10_1
$\underline{1\,221}_2$	10	10_2
$\underline{1\,618}$	15	15
$\underline{7\,281}_1$	67	67_1
$\underline{7\,281}_2$	67	67_2
$\underline{9\,326}$	90	90
$\underline{12\,155}_1$	106	106_1
$\underline{12\,155}_2$	106	106_2
$\underline{13\,355}$	121	121
$\underline{14\,169}_1$	129	129_1
$\underline{14\,169}_2$	129	129_2
$15\,807$	150	150
$37\,696$	337	$10_1 + 10_2 + 317$
$51\,029_1$	470	470_1
$51\,029_2$	470	470_2
$58\,311_1$	538	$2 \cdot 5 + 15 + 121 + 392$
$175\,770$	1607	$2 \cdot 15 + 150 + 392 + 1035$

Thus $1\,221_1 = 14\,190_1 - 1 - 406 - 2 \cdot 1\,618 - 9\,326$ is a Brauer character. Its irreducibility is proved with the criterion described in 3.3. We obtain the following new basic set $\mathcal{BS}''$ of Brauer characters.

$$\begin{array}{ccccccc} \underline{1}, & \underline{406}, & \underline{1\,221}_1, & \underline{7\,281}_1, & \underline{9\,326}, & \underline{12\,155}_1, & \underline{14\,169}_1, \\ & \underline{15\,807}, & 51\,029_1, & 58\,311_1, & \underline{1\,618}, & \underline{1\,221}_2, & \underline{7\,281}_2, \\ & \underline{13\,355}, & \underline{12\,155}_2, & \underline{14\,169}_2, & 37\,696, & 51\,029_2, & 175\,770. \end{array}$$

3.6. We have $342_1 \otimes 342_4 = 58\,311_1 + 58\,653_1$ as ordinary characters, where $58\,653_1$ is of defect zero and condenses to a character of degree 533. Furthermore, using the relation given by the decomposition of $406^{[2]}$ into $\mathcal{BS}''$ it can be shown that the irreducible Brauer character 13 355 is a constituent of $58\,311_1$. The Brauer characters $58\,311_1 - 13\,355$ and 13 355 condense to characters of degrees 417 and 121, respectively. It now follows from Table 5 that the Brauer character $58\,311_1$ has the $\mathcal{K}$-constituents $2 \cdot 5 + 15 + 121 + 392$.

Using similar arguments and Table 5 we can show that the FG-modules for the characters in $\mathcal{BS}''$ condense to $\mathcal{K}$-modules having constituents as given in the third column of Table 7. The second column of that table gives the corresponding condensed dimensions. The Proposition in 2.4 now implies that $S_k e \neq \{0\}$ for all simple B_1-modules S_k. This in turn proves that the Brauer characters $51\,029_1$ and $51\,029_2$ are irreducible.

3.7. Finally, the evaluation of $207\,360_1$, written in the basic set $\mathcal{BS}''$, gives the irreducible Brauer character 35 254. The relation given by $5\,598_1 \otimes 45_2$ and an analysis of the submodule lattice of the condensed module corresponding to $342_1 \otimes 342_4$ gives the irreducible Brauer character 42 526. And the relation given by $8\,865_2 \otimes 45_1$ yields the irreducible Brauer character 114 201. Note that $5\,598_1$ and $8\,865_2$ are irreducible Brauer characters belonging to block B_2; their construction only uses the Brauer character 406 in the principal block.

4. Remarks on the proof for $G.2$

4.1. Imitating the steps described in Sections 2.1, 2.2, we use the MeatAxe to construct irreducible matrix representations of degrees 406 and 1 618 for $G.2$, and

Table 8. A few scalar products

Φ	406_1	$1\,618_1$
$37\,696_1$	$-2/49$	$13/343$
$37\,696_2$	$19/49$	$62/343$
$169\,290_1$	$-18/49$	$-135/343$
$169\,290_2$	$-18/49$	$-184/343$
$207\,360_1$	$-33/49$	$29/343$
$207\,360_2$	$16/49$	$-216/343$
$207\,360_3$	$16/49$	$29/343$

Φ	406_1	$1\,618_1$
$207\,360_4$	$16/49$	$127/343$
$207\,360_5$	$16/49$	$372/343$
$207\,360_6$	$16/49$	$127/343$
$234\,080_1$	$-14/49$	$-7/343$
$234\,080_2$	0	$-105/343$
$234\,080_3$	$-14/49$	$-7/343$
$234\,080_4$	0	$-105/343$

compute the extensions $406_{1,2}$ and $1\,618_{1,2}$ of the Brauer characters 406 and 1 618 to $G.2$. Here the characters with index '1' are meant to be positive on class $2B$.

4.2. The irreducible Brauer characters of G which are not invariant under the action of the outer automorphism are $1\,221_1$, $7\,281_1$, $12\,155_1$, $14\,169_1$, $51\,029_1$ and their conjugates. They induce to irreducible Brauer characters of $G.2$. Then the same is true for the corresponding projective indecomposable characters. This gives us five of the 23 columns of the decomposition matrix. The remaining nine irreducible Brauer characters of the principal block of G are invariant, hence extendible to $G.2$. The corresponding projective indecomposable characters induce to the sum of two projective indecomposable characters.

To find these summands we check all possible splittings of such an induced projective character into a sum of two characters satisfying the following properties: The two summands vanish on 7-singular classes, they are obtained from each other by multiplying with the non-trivial extension of the trivial character of G and each summand has a non-negative integral scalar product with the Brauer characters $406_{1,2}$, $1\,618_{1,2}$, $406_1 \otimes 1\,618_2$ and $406_1^{[1,1]}$. For example, let Φ denote the projective character of $G.2$ induced from the projective indecomposable character corresponding to 35 254. Table 8 gives the scalar products of the ordinary constituents restricted to the 7-regular classes of Φ with the Brauer characters 406_1 and $1\,618_1$. It is easily deduced form these scalar products that Φ decomposes into the two characters given in columns 17 and 18 of Table 3.

This gives us the decompositions of seven of these induced characters into their projective indecomposable summands, namely those corresponding to the Brauer characters 1, 406, 9 326, 15 807, 35 254, 42 526 and 114 201, hence another 14 of the columns of the decomposition matrix.

4.3. To obtain the splitting of the induced projective character corresponding to 13 355, we use the projective indecomposable characters constructed so far and the tensor product $406_1 \otimes 58\,653_1$, which is a projective character since the $58\,653_1$ is a defect zero character. By far the hardest part finally is to obtain the splitting of the induced projective character corresponding to 1 618. This amounts to finding the multiplicities of $1\,618_1$ in the extensions of the invariant ordinary characters of G. To do this we have to use the MeatAxe again to analyze the tensor products

$406_1 \otimes 406_1$, again using condensation with respect to the subgroup K, and $1\,618_1 \otimes 1\,618_1$, this time using another condensation subgroup isomorphic to $L_2(11)$.

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(G. H., J. M.) Lehrstuhl D für Mathematik, RWTH Aachen, Templergraben 64, 52062 Aachen, Germany

(A. H.) Sommerville College, Oxford University, Oxford OX2 6HS, Great Britain

e-mail: hiss@math.rwth-aachen.de

e-mail: henke@maths.ox.ac.uk

e-mail: mueller@math.rwth-aachen.de