LMS Short Course on Computational Group Theory

Lab session 8

Investigating Rubik's Cube using GAP

You can download a GAP-readable file with group generators for the Rubik's Cube group from

`http://tinyurl.com/MNGAPsess/rubik.g`

1. Let $g, h, k \in \mathrm{Sym}(12)$ be defined by

$$\begin{aligned} g &:= (1,2,3,4,5,6,7,8,9,10,11), \\ h &:= (3,7,11,8)(4,10,5,6), \\ k &:= (1,12)(2,11)(3,6)(4,8)(5,9)(7,10) \end{aligned}$$

   and let $U = \langle g, h, k \rangle \leq \mathrm{Sym}(12)$.

   a) What is $|U|$ and $[\mathrm{Sym}(12) : U]$?

   b) What are the orders of g, h and k and what other orders of elements occur in U?

   c) Are there orders of elements in $\mathrm{Sym}(12)$, which do not occur in U?

   d) What are the normal subgroups of U?

2. Use the group of the Rubik's Cube to answer the following questions:

   a) In how many different states can the Rubik's Cube be?

   b) Surely one cannot move a corner facet to an edge facet and vice versa. Is it possible to move any corner facet to any other corner facet, and any edge facet to any other edge facet?

   c) Is it possible to transform the Rubik's Cube, such that exactly one corner is turned and everything else remains unchanged?

   d) How could one solve the Rubik's Cube with the help of its group?

3. Now we want to investigate the structure of the Rubik's cube group R. These hints should help you with this.

   a) Which actions on the 8 corners does R allow?

   b) Let G be the subgroup of R that fixes all 8 corners. Which actions on the 3 facets of a corner does G allow?

   c) Let H be the subgroup of R that fixes all corner facets. This group only acts on the edges and the edge facets. Which actions does H allow on the 12 edges?

   d) Let L be the subgroup of H that fixes all 12 edges. This group only acts on the facets of the corners. Which actions does L allow here?